Thermodynamics 4
Tutorial 5 – Ideal Gas Mixtures (Solutions)

Gas Mixtures

1) Amagat and Dalton Models

a) Describe the Amagat model for gas mixtures and define the volume fraction.

According to the Amagat model, each gas in the mixture is treated as being at the temperature T_{mix} and pressure P_{mix} of the mixture and occupying a volume V_i. The volume fraction is:

$$\sigma_i = \frac{V_i(T_{mix}, P_{mix})}{V_{mix}}$$

Where V_{mix} is the volume occupied by the mixture so that…

$$\sum V_i(T_{mix}, P_{mix}) = V_{mix}$$

b) Describe the Dalton model for gas mixtures and define the partial pressure.

According to the Dalton model, each gas in the mixture exists separately and independently at T_{mix} and occupies the entire volume V_{mix}. The partial pressure is then:

$$P_i = y_i P_{mix}$$

where y_i is the mole fraction and P_i is the partial pressure. Dalton's law is then:

$$\sum P_i(V_{mix}, T_{mix}) = P_{mix}$$

2) The Earth's Atmosphere

The Earth's atmosphere is 0.93% argon by volume. The molecular weight of Argon is 34.95 g/mol, and the apparent molecular weight of air is 28.97 g/mol. Find:

a) The fraction of the Earth's atmosphere that is argon, in terms of moles (mole fraction)

Argon is 0.93% *by volume, hence its volume fraction is* 0.0093.

We know that volume fraction = mole fraction…

Mole fraction of Argon $= 0.0093 = 9.3\times10^{-3}$

b) The fraction of the Earth's atmosphere that is argon, in terms of the mass (mass fraction)

Mass fraction is found as…

$$f_i = \frac{y_i M_i}{\sum y_j M_j} = \frac{0.0093 \times 34.95}{28.97} = 0.011$$

Note that the apparent molecular weight of the air already takes the Argon constituent into account.

c) The partial pressure of Argon in atmospheric air at 100k Pa

$$P_i = y_i P_{mix} = 0.0093 \times 100 = 0.93\ kPa$$

3) Mixing

Three pure gases (A, B, C) are initially kept in three separate compartments of a box. Each gas is initially at 300 K. There is 1 kg of gas A, 1.5 kg of gas B, and 2.5 kg of gas C. The heat capacities of the gases and their molecular weights are given in the table below. Find:

Note: Implicit assumption that all three gases are initially at the same pressure.

a) The entropy change upon mixing:

$$\Delta s_i = C_{p,i}\, ln\frac{T_2}{T_1} - R_i\, ln\frac{P_{i,2}}{P_{i,1}}$$

$$where\, P_{i,1} = y_{i,1}P_{m,1} \quad \text{and} \quad P_{i,2} = y_{i,2}P_{m,2}$$

We can then write,

$$\Delta S = -m_A R_A \ln(y_a) - m_B R_B \ln(y_B) - m_C R_C \ln(y_C)$$

$$R_A = \frac{R_{Universal}}{M_A} = \frac{8.314}{45} = 0.18475 \text{ kJ/kgK}$$

$$y_A = \frac{\frac{m_A}{M_A}}{\sum\frac{m_i}{M_i}} = \frac{\frac{1}{45}}{\frac{1}{45}+\frac{1.5}{52}+\frac{2.5}{61}} = 0.24142$$

$$R_B = \frac{R_{Universal}}{M_B} = \frac{8.314}{52} = 0.15988 \text{ kJ/kgK}$$

$$y_B = \frac{\frac{m_B}{M_B}}{\sum\frac{m_i}{M_i}} = \frac{\frac{1.5}{52}}{\frac{1}{45}+\frac{1.5}{52}+\frac{2.5}{61}} = 0.31337$$

$$R_C = \frac{R_{Universal}}{M_C} = \frac{8.314}{61} = 0.13629 \text{ kJ/kgK}$$

$$y_C = \frac{\frac{m_C}{M_C}}{\sum\frac{m_i}{M_i}} = \frac{\frac{2.5}{61}}{\frac{1}{45}+\frac{1.5}{52}+\frac{2.5}{61}} = 0.44522$$

Sanity check...

$$\sum(y_i) = 0.241 + 0.313 + 0.445 = 0.999 \cong 1$$

Substitute into the equation for ΔS *to get* $\Delta S = 0.817$ kJ/K

b) The apparent molar mass for the mixture

$$M_{mix} = \sum(y_i M_i) = 0.2414 \times 45 + 0.3134 \times 52 + 0.445 \times 61 = 54.3 \text{ g/mol}$$

c) The apparent specific heat at constant pressure of the mixture

Gas	**Molecular weight [g/mol]**	C_p **[kJ/kg·K]**
A	45	1.1
B	52	1.4
C	61	0.9

$$C_{P,mix} = \sum(f_i C_{P,i}), \quad f_i = \frac{m_i}{m_{total}}$$

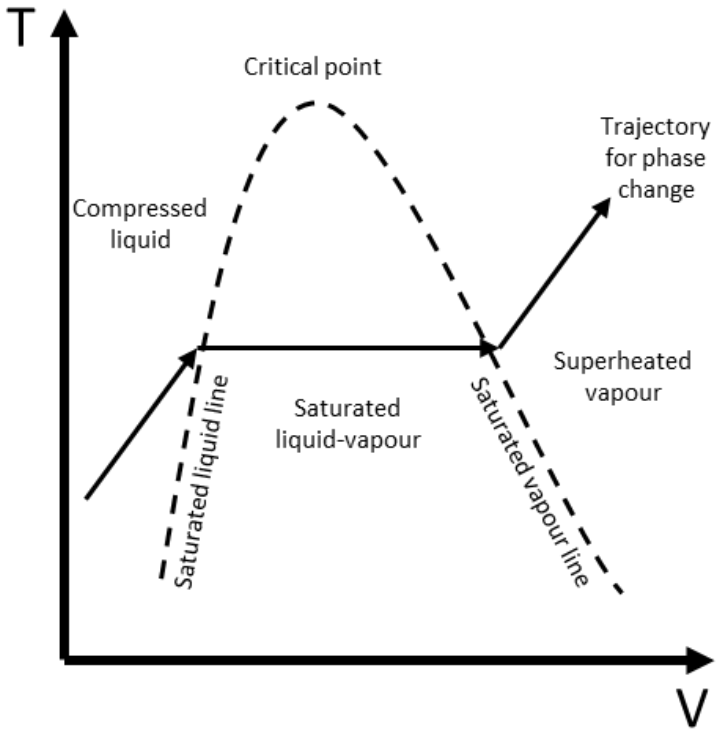
$$f_A = \frac{1}{1 + 1.5 + 2.5} = 0.2, \quad f_B = \frac{1.5}{5} = 0.3, \quad f_C = 0.5$$

$$C_{P,mix} = 0.2 \times 1.1 + 0.3 \times 1.4 + 0.5 \times 0.9 = 1.09 \text{ kJ/kgK}$$

Gas-Vapour Mixtures

4) Sketch a T-V graph (T on the y-axis, V on the x-axis) for liquid/vapour phases showing:

- the saturated liquid line
- the saturated vapour line
- the critical point
- the compressed liquid region
- the superheated vapour region
- the saturated liquid-vapour region
- a trajectory through the T-V space for a substance undergoing a phase transition from compressed liquid through the saturated liquid-vapour region into the superheated vapour region



5) Consider a mixture of air and water vapour at a temperature of 17°C. The relative humidity is 70% and the pressure of the mixture is 100 kPa. The molecular weight of water is 18.016 g/mol, and the apparent molecular weight of air is 28.97 g/mol.

What is the absolute humidity, and what is the dew point?

$$w = \frac{\frac{M_V}{M_{Air}}\phi P_g}{P_{mix} - \phi P_g} = \frac{0.622 P_v}{P - P_v}$$

$$\phi = 0.7$$

$$M_V = 18.016\ g/mol$$

$$M_{Air} = 28.91\ g/mol$$

$$P_{mix} = 100\ kPa$$

From the steam tables, the saturation pressure at 17℃ *can be interpolated from the values for* 15℃ *and* 20℃*…*

$$P_{sat@17℃} = 1.9586\ kPa$$

From that, we find absolute humidity by substituting…

$$w = \frac{\frac{18.016}{29.81} \times 0.7 \times 1.9586}{100 - 0.7 \times 1.9586} = 8.65 \times 10^{-3}$$

To find the dew point, we find the partial pressure of water vapour is:

$$0.7 \times 1.9586 \cong 1.37\ kPa$$

Cool the water while maintaining this pressure, and the water starts to condense at the saturation temperature. Interpolate between the saturation temperature at 1 *kPa and* 1.5 *kPa to find…*

$$T_{sat@1.37kPa} = 11.45℃$$

$$T_{dew} = 11.5℃$$

6) Consider an adiabatic saturator. The pressure of the mixture entering and leaving is 90 kPa. At the inlet, the temperature is 25°C, and at the exit, the temperature is 15°C. For dry air, the heat capacity at constant pressure is *Cp* = 1.004 kJ/kg·K.

If the mass flow rate for air is 1kg/s (dry air), find the mass flow rate of liquid water.

At the inlet…

$$T_1 = 25°\text{C}, \quad \dot{m}_{1a} = 1\ \text{kg/s}$$

At the outlet, there is saturated air…

$$T_2 = 15°\text{C}, \quad \phi_2 = 100\%$$

Conservation of mass dictates that…

$$\dot{m}_{1a} = \dot{m}_{2a}, \quad \dot{m}_{1v} + \dot{m}_l = \dot{m}_{2v}$$

Where $\dot{m}_{1v}$ is the inlet mass flowrate of vapour, $\dot{m}_{2v}$ is the outlet mass flowrate of vapour, $\dot{m}_l$ is the mass flow rate of liquid water and $\dot{m}_{1a}$ and $\dot{m}_{2a}$ are mass flowrates of air at inlet and outlet, respectively.

$$w_2 = \frac{0.622\phi_2 P_g}{P - \phi_2 P_g}, \quad P_g = P_{sat@15°\text{C}} = 1.706\ \text{kPa}$$

$$w_2 = \frac{0.622 \times 1 \times 1.705}{90 - 1.705} = 0.01202$$

To find w_1 we need a different approach. For this case, we can write,

$$w_1\left(h_{v,1} - h_l\right) = C_P(T_2 - T_1) + w_2(h_{v,2} - h_l)$$

Where $h_{v,1}$ is the specific enthalpy of saturated vapour for water at 25°C *and h_l is the specific enthalpy of saturated liquid for water at* 15°C*.*

$$h_{v,1} = 2546.5\ \text{kJ/kg} \qquad (h_g\ at\ 25°\text{C})$$

$$h_l = 62.98\ \text{kJ/kg}$$

$$h_{v,2} - h_l = h_{fg,saturated\ water@15°\text{C}} = 2465.4\ \text{kJ/kg}$$

Now substitute to find w_1…

$$w_1 = \frac{1.004 \times (-10) + 0.01202 \times 2465.4}{2546.5 - 62.98} = 7.89 \times 10^{-3}$$

So we calculate…

$$\dot{m}_l = (0.01202 - 7.89 \times 10^{-3})\dot{m}_{1a} = 4.13 \times 10^{-3} \times 1 = 4.13 \times 10^{-3}\ \text{kg/s}$$

Gas-vapour Mixtures and Psychrometry

7) See Borgnakke & Sonntag, Example 11.3

8) Consider a room containing air at a pressure of 1 atmosphere, 80% relative humidity and 14°C. Using the psychrometric chart provided on the next page, find…

a) The specific humidity

Read from the right vertical axis, watch the units. $X = 8\ gm\ /\ kg\ of\ dry\ air$

b) The specific enthalpy (per mass of dry air)

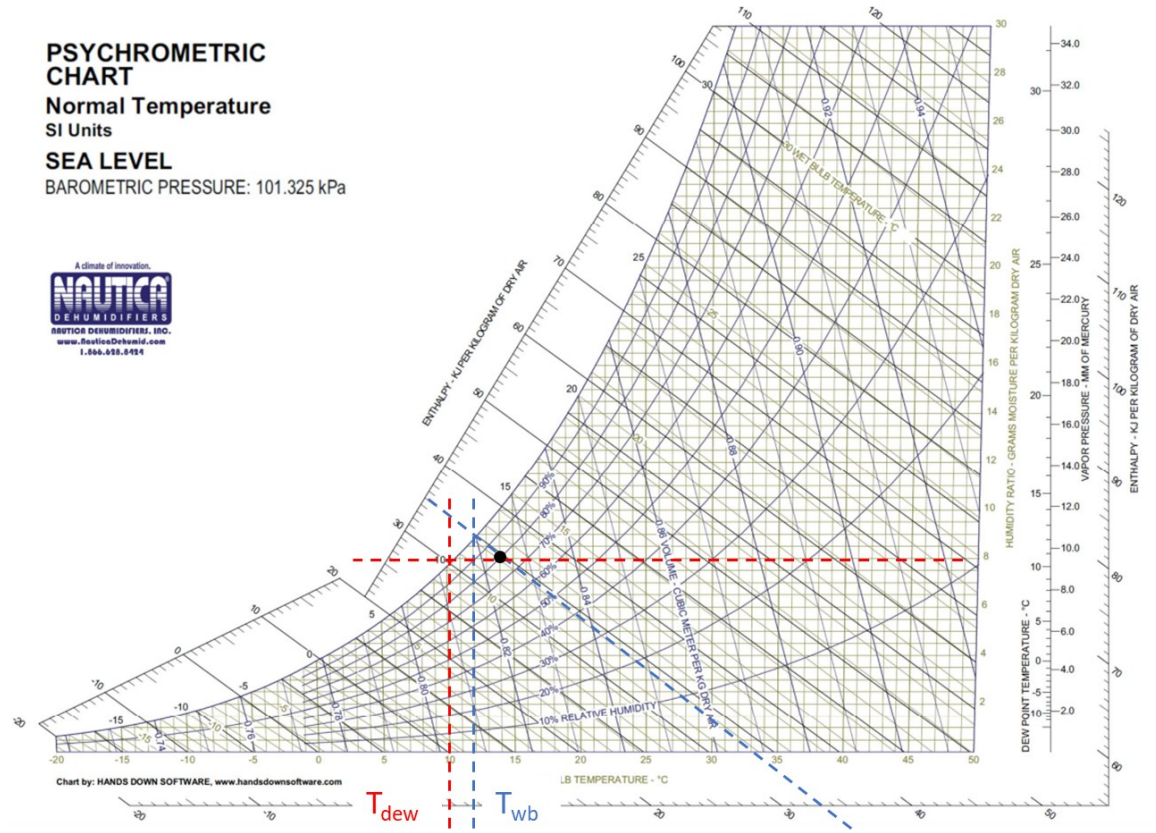
Read using isenthalpic lines going diagonally across the graph. $h = 34.2$ kJ/kg

c) The wet bulb temperature

Read by reading the temperature at which an isenthalpic line would reach $100\,\%$ *RH.* $T_{wb} = 12$°C

d) The dew point temperature

Read by extending a horizontal line from the point to the $100\,\%$ *RH line and read the temperature.* $T_{dew} = 10.6$°C



9) Air enters an air conditioning unit at 100 kPa, 40°C, and 5% relative humidity, at the steady rate of 1.0 m^3/s. Liquid water at 20°C is sprayed into the air in the AC unit at a rate of 20 kg/hr. The exit temperature is 10°C and the exit pressure is 100 kPa.

What is the relative humidity at the exit, and at what rate is heat rejected from the unit?

Conditions at the inlet…

$$\phi_1 = 5\%, \quad T_1 = 40°C, \quad P_{mix,1} = 100\ \text{kPa}, \quad \dot{V}_{air,1} = 1\ \text{m}^3/\text{s}$$

Conditions at the outlet…

$$\phi_2 = ?, \quad T_2 = 10°C, \quad P_{mix,2} = 100\ \text{kPa}$$

Liquid water…

$$T_{water} = 20°C, \quad \dot{m}_{water} = 20\ \text{kg/hr}$$

We need to find ϕ_2 and $\dot{Q}$ …

Conservation of mass:

$$\dot{m}_{air,1} = \dot{m}_{air,2} = \dot{m}_{air}$$

$$\dot{m}_{vapour,1} = \dot{m}_{vapour,2} - \dot{m}_{water}$$

Conservation of energy:

$$\dot{m}_{air,1}h_{air,1} + \dot{m}_{vapour,1}h_{vapour,1} + \dot{m}_{water}h_{water} = \dot{m}_{air,2}h_{air,2} + \dot{m}_{vapour,2}h_{vapour,2} - \dot{Q}$$

Find ϕ_2 first…

$$w_1\dot{m}_{air,1} + \dot{m}_{water} = w_2\dot{m}_{air,2}$$

$$w_2 = w_1 + \frac{\dot{m}_{water}}{\dot{m}_{air}}$$

$$w_1 = 0.622 \times \frac{P_{vapour,1}}{P_{air,1}} = 0.622 \times \frac{\phi_1 P_{g1}}{P_{air,1}} = 0.622 \times \frac{\phi_1 P_{g1}}{P - \phi_1 P_{g1}}$$

We can find P_{g1} from tables as the saturation pressure for water at 40°C*.*

$$P_{g1} = 7.3837 \text{ kPa}$$

Therefore, we can substitute and find w_1:

$$w_1 = 0.622 \times \frac{0.05 \times 7.3837}{100 - 0.05 \times 7.3837} = 2.304 \times 10^{-3}$$

$$\dot{m}_{water} = 5.55 \times 10^{-3}\ kg/s$$

$$\dot{m}_{air} = \frac{P_{air,1}\dot{V}_{air}}{RT_{air,1}} = \frac{(100 - 0.05 \times 7.3837) \times 1}{0.287 \times (40 + 273)} = 1.109 \text{ kg/s}$$

Therefore, we can find w_2:

$$w_2 = w_1 + \frac{\dot{m}_{water}}{\dot{m}_{air}} = 2.304 \times 10^{-3} + \frac{5.55 \times 10^{-3}}{1.109} = 7.31 \times 10^{-3}$$

$$w_2 = 0.622 \times \frac{\phi_2 P_{g2}}{P_{air,2}} = 0.622 \times \frac{\phi_2 P_{g2}}{P_{mix} - \phi_2 P_{g2}}$$

Rearrange to get:

$$\phi_2 = \frac{P_{mix} w_2}{0.622 P_{g2} + w_2 P_{g2}} = \frac{100 \times 7.31 \times 10^{-3}}{(0.622 + 7.31 \times 10^{-3}) \times 1.2276} = 94.5\%$$

Now to find $\dot{Q}$ ...

$$\dot{Q} = \dot{m}_{air}\left[w_1 h_{vapour,1} - w_2 h_{vapour,2} + h_{air,1} - h_{air,2}\right] + \dot{m}_{water} h_{water}$$

And also:

$$\Delta h_{air} = h_{air,1} - h_{air,2} = C_P \Delta T$$

$$\dot{Q} = (1.004 \times 30 + 0.0023 \times 2574.26 - 0.0073 \times 2519.74) \times 1.109 + \frac{20}{60^2} \times 83.94$$

$$\dot{Q} \cong 20 \text{ kW}$$

[note: h_g at 40°C = 2574.26 and h_g at 10°C =2519.74]

10) see Borgnakke & Sonntag, Example 11.6

11) see Borgnakke & Sonntag, Example 11.8

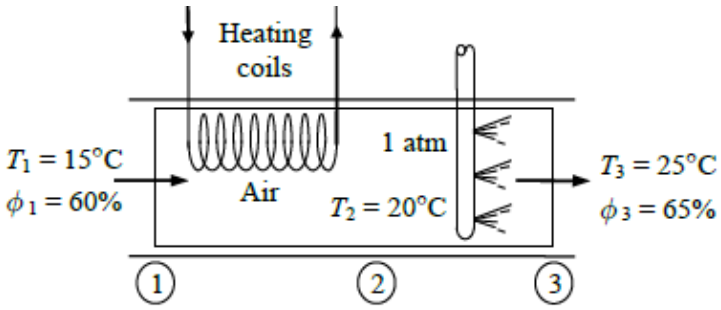
12) Air is first heated and then humidified by water vapour. The amount of steam added to the air and the amount of heat transferred to the air are to be determined.

Assumptions

1 This is a steady-flow process, and thus the mass flow rate of dry air remains constant during the entire process. ($\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$)

2 Dry air and water vapour are ideal gases.

3 The kinetic and potential energy changes are negligible.



Properties: The inlet and the exit states of the air are completely specified, and the total pressure is 1 atm. The properties of the air at various states are determined from the psychrometric chart to be

$$h_1 = 31.1 \text{ kJ/kg dry air}$$

$$\omega_1 = 0.0064 \text{ kg } H_2O\text{/kg dry air } (= \omega_2)$$

$$h_2 = 36.2 \text{ kJ/kg dry air}$$

$$h_3 = 58.1 \text{ kJ/kg dry air}$$

$$\omega_3 = 0.0129 \text{ kg } H_2O\text{/kg dry air}$$

Analysis

(a) The amount of moisture in the air remains constant; it flows through the heating section ($\omega_1 = \omega_2$), but increases in the humidifying section ($\omega_3 > \omega_2$). The amount of steam added to the air in the heating section is

$$\Delta\omega = \omega_3 - \omega_2 = 0.0129 - 0.0064 = \mathbf{0.0065 \text{ kg } H_2O \text{ / kg dry air}}$$

(b) The heat transfer to the air in the heating section per unit mass of air is

$$q_{in} = h_2 - h_1 = 36.2 - 31.1 = \mathbf{5.1 \text{ kJ / kg dry air}}$$

13) (i) To achieve a higher level of comfort. Very dry air can cause dry skin, respiratory difficulties, and increased static electricity. (ii) To drop its relative humidity to more desirable levels.

14) Humid air at 101.3 kPa, 39°C dry bulb, and 50 percent relative humidity is cooled at constant pressure to a temperature 10°C below its dew-point temperature. (a) Sketch the system diagram and the psychrometric diagram for the process. (b) If it has been determined that the rate of heat transfer from the atmospheric air is 1340 kW, what is the inlet volume flow rate of atmospheric air, in m^3/s, for this process?

Assumptions:

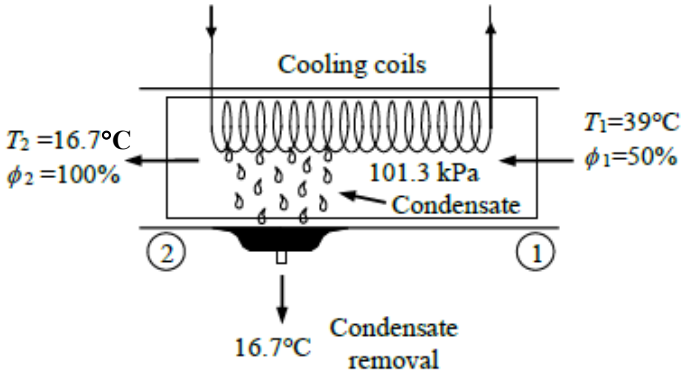
1. This is a steady-flow process, and thus the mass flow rate of dry air remains constant during the entire process ($\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$).

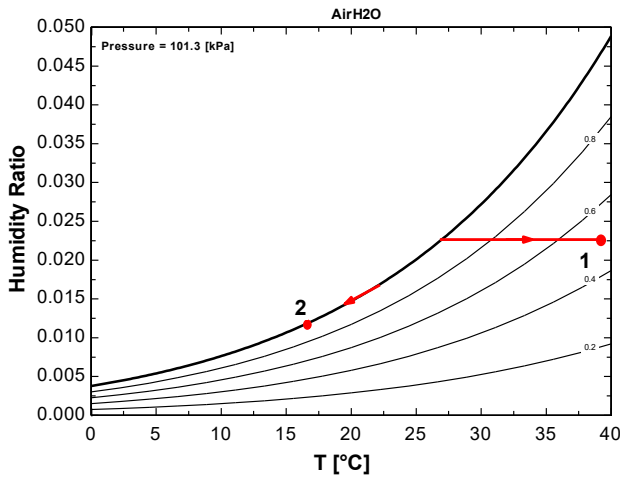
2. Dry air and water vapour are ideal gases.

3. The kinetic and potential energy changes are negligible.

Properties:

(a) The schematic of the cooling and dehumidification process and the process on the psychrometric chart are given below.





The inlet and the exit states of the air are completely specified, and the total pressure is 101.3 kPa (1 atm). The properties of the air at the inlet and exit states are determined from the psychrometric chart to be

$$T_{\text{dp}} = 26.7°\text{C}$$

$$h_1 = 96.5 \text{ kJ/kg dry air}$$

$$\omega_1 = 0.0222 \text{ kg H}_2\text{O/kg dry air}$$

$$v_1 = 0.916 \text{ m}^3\text{/kg dry air}$$

and

$$T_2 = T_{\text{dp}} - 10 = 26.7 - 10 = 16.7°\text{C}$$

$$\phi_2 = 1.0$$

$$h_2 = 46.9 \text{ kJ/kg dry air}$$

$$\omega_2 = 0.0119 \text{ kg H}_2\text{O/kg dry air}$$

Also,

$$h_w \cong h_{f@16.7°\text{C}} = 70.10 \text{ kJ/kg}$$

Analysis:

Applying the water mass balance and energy balance equations to the combined cooling and dehumidification section,

Water Mass Balance:

$$\sum \dot{m}_{w,i} = \sum \dot{m}_{w,e} \longrightarrow \dot{m}_{a1}\omega_1 = \dot{m}_{a2}\omega_2 + \dot{m}_w$$

$$\dot{m}_w = \dot{m}_a(\omega_1 - \omega_2)$$

Energy Balance:

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\sum \dot{m}_i h_i = \dot{Q}_{out} + \sum \dot{m}_e h_e$$

$$\dot{Q}_{\text{out}} = \dot{m}_{a1}h_1 - (\dot{m}_{a2}h_2 + \dot{m}_w h_w)$$

$$= \dot{m}_a(h_1 - h_2) - \dot{m}_w h_w$$

$$= \dot{m}_a(h_1 - h_2) - \dot{m}_a(\omega_1 - \omega_2)h_w$$

Solving for the mass flow rate of dry air,

$$\dot{m}_a = \frac{\dot{Q}_{\text{out}}}{(h_1 - h_2) - (\omega_1 - \omega_2)h_w} = \frac{1340 \text{ kW}}{(96.5 - 46.9) \text{ kJ/kg} - (0.0222 - 0.0119)(70.1 \text{ kJ/kg})} = 27.4 \text{ kg/s}$$

The volume flow rate is then determined from

$$\dot{V}_1 = \dot{m}_a v_1 = (27.4 \text{ kg/s})(0.916 \text{ m}^3/\text{kg}) = \mathbf{25.1} \text{ m}^3/\text{s}$$

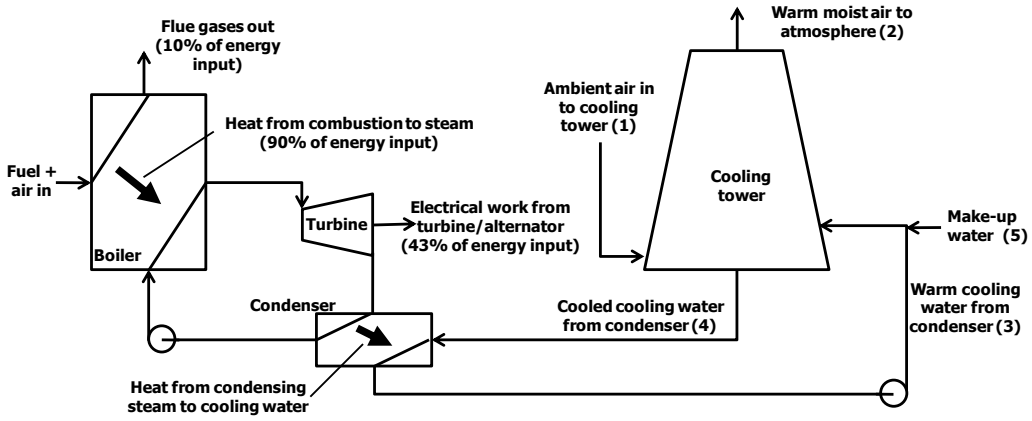
15) Wet cooling tower

How much water does an 800 MW supercritical power plant with wet cooling evaporate per second at an ambient air dry bulb temperature of 20ºC and an RH of 60%?

As indicated in the figure below, assume the plant efficiency is 43% (LHV basis) and the boiler efficiency 90% and that all heat losses take place in the main condenser. Water circulating through the condenser is fed to the cooling tower at 32ºC and leaves the tower at 17ºC and the air/vapour mix leaves *saturated* at 30ºC. Make-up water at 20ºC is added to the incoming water to the tower to maintain the required water flow circulated through the condenser. Water liquid and vapour enthalpies can all be assumed to be equal to saturated values.

Use the following data:

- Molar mass of water: 18.01 g/mol
- Molar mass of dry air: 28.96 g/mol
- *Cp* of dry air: 1.01 kJ/kg/K



a) What flow rate of cooling water is required?

Heat input into the boiler…

$$\dot{Q}_B = \frac{800}{43\%} = 1860.5 \text{ MW}$$

Heat input into the steam cycle…

$$\dot{Q}_{steam} = \dot{Q}_B * 90\% = 1674.45 \text{ MW}$$

1st Law of thermodynamics, energy balance on the steam cycle gives the condenser heat output as:

$$\dot{Q}_{condenser} = \dot{Q}_{steam} - 800 \text{ MW} = 874.45 \text{ MW}$$

The temperatures and enthalpies across the water loop are:

$$T_3 = 32℃, \quad h_{f3} = 134.1 \text{ kJ/kg}$$

$$T_4 = 17℃, \quad h_{f4} = 71.4 \text{ kJ/kg}$$

$$T_5 = 20℃, \quad h_{f5} = 83.9 \text{ kJ/kg}$$

Conservation of mass in the water condenser:

$$\dot{m}_{f3} = \dot{m}_{f4}$$

Mass flow of water passing through the condenser:

$$\dot{m}_{f3} = \frac{\dot{Q}_{condenser}}{h_{f3} - h_{f4}} = \frac{874.45}{134.1 - 71.4} = 13936.1 \text{ kg/s}$$

b) Per kg of dry air flowing through the cooling tower:

i) How much moisture enters with the ambient air?

To calculate per unit mass dry air…

$$\dot{m}_a = 1 \text{ kg/s}$$

$$P_{total} = 1.01325 \text{ bar}$$

$$T_1 = 20℃, \quad \phi_1 = 60\%$$

$$P_{g1} = 2339.2\ Pa$$

$$P_{v1} = \phi_1 P_{g1} = 2339.2 \times 0.6 = 1403.5 \text{ Pa}$$

$$M_v = 18.01\ g/mol$$

$$M_a = 28.96\ g/mol$$

$$w_1 = \frac{m_{v1}}{m_a} = \frac{0.622 P_{v1}}{P - P_{v1}}$$

$$\dot{m}_{v1} = \frac{\dot{m}_a \times P_{v1} \times M_v}{(P_{total} - P_{v1}) \times M_a} = 8.735 \text{ g/kgs}$$

ii) How much moisture leaves at the top of the tower?

$$T_2 = 30℃, \quad \phi_2 = 100\%$$

$$P_{g2} = 4246.7\ Pa, \quad P_{v2} = \phi_2 P_{g2} = 4246.7 \text{ Pa}$$

$$\dot{m}_{v2} = \frac{\dot{m}_a \times P_{v2} \times M_v}{(P_{total} - P_{v2}) \times M_a} = 27.205 \text{ g/s}$$

iii) What flow of make-up water is required?

The water loss per kg of dry air is equal to the make-up water flow.

$$\dot{m}_{f5} = \dot{m}_{v2} - \dot{m}_{v1} = 18.469 \text{ g/s}$$

iv) Write an energy balance for the tower that includes only the mass flow rates of cooling water and dry air, and hence calculate the mass flow rate of dry air.

Energy balance for the cooling tower (assume there is no fan work):

$$\dot{m}_{f3}h_{f3} + \dot{m}_{air}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{f5}h_{f5} = \dot{m}_{air}h_{a2} + \dot{m}_{v2}h_{v2} + \dot{m}_{f4}h_{f4}$$

$$T_1 = 20°\text{C}, \quad h_{g1} = 2537.5 \text{ kJ/kg}$$

$$T_2 = 30°\text{C}, \quad h_{g2} = 2556.6 \text{ kJ/kg}$$

$$C_{Pa} = 1.01 \text{ kJ/kg·K}$$

$$\Longrightarrow \dot{m}_{f3}h_{f3} + \dot{m}_{air}h_{a1} + \dot{m}_{air}w_1h_{v1} + \dot{m}_{air}\frac{\dot{m}_{f5}}{\dot{m}_{air}}h_{f5} = \dot{m}_{air}h_{a2} + \dot{m}_{air}w_2h_{v2} + \dot{m}_{f4}h_{f4}$$

$$\Longrightarrow \dot{m}_{f3}h_{f3} - \dot{m}_{f4}h_{f4} = \dot{m}_{air}h_{a2} + \dot{m}_{air}w_2h_{v2} - \dot{m}_{air}h_{a1} - \dot{m}_{air}w_1h_{v1} - \dot{m}_{air}\frac{\dot{m}_{f5}}{\dot{m}_{air}}h_{f5}$$

$$\Longrightarrow \dot{m}_{f3}(h_{f3} - h_{f4}) = \dot{m}_{air}h_{a2} - \dot{m}_{air}h_{a1} + \dot{m}_{air}w_2h_{v2} - \dot{m}_{air}w_1h_{v1} - \dot{m}_{air}\frac{\dot{m}_{f5}}{\dot{m}_{air}}h_{f5}$$

$$\Longrightarrow \dot{m}_{f3}(h_{f3} - h_{f4}) = \dot{m}_{air}\left(h_{a2} - h_{a1} + w_2h_{v2} - w_1h_{v1} - \frac{\dot{m}_{f5}}{\dot{m}_{air}}h_{f5}\right)$$

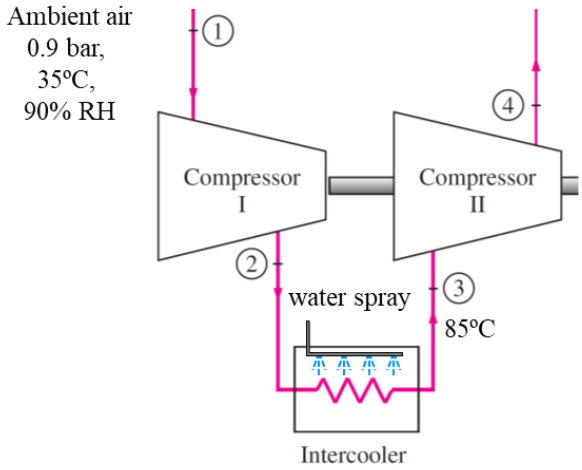
$$\therefore \dot{m}_{air} = \frac{\dot{m}_{f3}(h_{f3} - h_{f4})}{C_{Pa}(T_2 - T_1) + \frac{\dot{m}_{v2}}{\dot{m}_a}h_{g2} - \frac{\dot{m}_{v1}}{\dot{m}_a}h_{g1} - \frac{\dot{m}_{f5}}{\dot{m}_a}h_{f5}} = 15640.2 \text{ kg/s}$$

c) Use the mass flow rate of dry air to calculate the make-up water flow.

$$\frac{\dot{m}_{f5}}{\dot{m}_a}\dot{m}_{air} = 288.9 \text{ kg/s}$$

16) Spray intercooling

A spray intercooler follows the first stage of a gas turbine compressor with an isentropic efficiency of 85%. The exit pressure from the stage is 4 bar, and the pressure drop in the intercooler can be neglected.



Data is as follows:

Ambient pressure: 0.9 bar, Ambient temperature: 35°C, RH (or ϕ) = 90%

Cooling water temperature: 20°C

In the compressor, air and water vapour can be modelled as ideal gases with constant specific heats, with the following properties used to calculate average mixture properties:

Cp_a = 1.05 kJ/kg·K, R_a = 0.287 kJ/kg·K, M_a = 28.96 g/mol,

Cp_v = 1.95 kJ/kg·K, R_v = 0.461 kJ/kg·K, M_v = 18.01 g/mol

In the spray cooler, more accurate results can be obtained by modelling the air as an ideal gas, but the water/vapour as having enthalpies equal to the saturation values.

a) Calculate the compressor work for the air/vapour mixture for a dry air flow of 100 kg/s

$$\eta_c = 85\%, \quad P_1 = 0.9 \text{ bar}, \quad T_1 = 35°\text{C}, \quad \dot{m}_a = 100 \text{ kg/s}$$

$$P_{g1} = 5629.6 \text{ Pa}, \quad \phi_1 = 90\%, \quad P_{v1} = \phi_1 P_{g1} = 5065.8 \text{ Pa}$$

$$\dot{m}_{v1} = \frac{\dot{m}_a \times P_{v1} \times M_v}{(P_{total} - P_{v1}) \times M_a} == 3.709 \text{ kg/s}$$

$$\dot{m}_g = \dot{m}_a + \dot{m}_{v1} = 103.709 \text{ kg/s}$$

$$C_{P_g} = \frac{\dot{m}_a C_{P_a} + \dot{m}_{v1} C_{P_{v1}}}{\dot{m}_g} = 1.082 \text{ kJ/kg·K}$$

$$R_g = \frac{\dot{m}_a R_a + \dot{m}_{v1} R_v}{\dot{m}_g} = 0.293 \text{ kJ/kg·K}$$

$$k_g = \frac{C_{P_g}}{C_{P_g} - R_g} = 1.372$$

$$P_2 = 4 \text{ bar}, \quad P_1 = 0.9 \text{ bar}$$

$$T_{2s} = \left(\frac{P_2}{P_1}\right)^{\frac{k_g - 1}{k_g}} \times T_1, \quad T_1 = 308.15 \text{ K}, \quad \rightarrow \quad T_{2s} = 461.63 \text{ K}$$

$$T_2 = \frac{T_{2s} - T_1}{\eta_c} + T_1, \quad T_2 = 488.71 \text{ K} = 215.6℃$$

$$W_{12} = \dot{m}_g C_{P_g} (T_2 - T_1) = 20.3 \text{ MW}$$

b) Calculate the water spray flow required to achieve a cooled gas temperature of 85ºC

$$T_3 = 85℃$$

Saturation enthalpy of water vapour at T_2 and T_3 using interpolation of steam tables.

$$T_2 = 215.6℃, \quad h_{g2} = 2799.6 \text{ kJ/kg}$$

$$T_3 = 85℃, \quad h_{g3} = 2651.3 \text{ kJ/kg}$$

The saturation enthalpy of water at 20℃:

$$h_w = 83.9 \text{ kJ/kg} \qquad (h_f \text{ at } 20℃)$$

Using the steady flow energy equation around the control volume consisting of the spray intercooler:

$$\dot{m}_2 h_2 + \dot{m}_w h_w = \dot{m}_3 h_3$$

$$\Longrightarrow \dot{m}_{a2} h_{a2} + \dot{m}_{v2} h_{v2} + \dot{m}_w h_w = \dot{m}_{a3} h_{a3} + \dot{m}_{v3} h_{v3}$$

$$\Longrightarrow \dot{m}_{a2} h_{a2} + \dot{m}_{v2} h_{v2} + \dot{m}_w h_w = \dot{m}_{a3} h_{a3} + (\dot{m}_{v2} + \dot{m}_w) h_{v3}$$

$$\therefore \dot{m}_w = \frac{\dot{m}_a C_{P_a}(T_2 - T_3) + \dot{m}_{v1}\left(h_{g2} - h_{g3}\right)}{h_{g3} - h_w} = 5.554 \text{ kg/s}$$

c) Verify that this should give no droplets of water to damage the next compressor stages.

Compare the partial pressure of water in the gas mixture with the saturation pressure.

Mass flow at spray outlet:

$$\dot{m}_{v3} = \dot{m}_{v1} + \dot{m}_w$$

Mole fraction of water in the mixture:

$$y_w = \frac{\frac{m_{v3}}{M_v}}{\frac{m_{v3}}{M_v} + \frac{m_a}{M_a}} = 13\%$$

No pressure drop in the intercooler:

$$P_3 = P_2$$

Partial pressure of water in the gas mixture:

$$P_{v3} = y_w P_3 = 0.519 \text{ bar}$$

Saturation pressure of water vapour at T_3:

$$P_{g3} = 0.579 \text{ bar}$$

Mole fraction of water at saturation:

$$\frac{P_{g3}}{P_3} = 14.5\%$$

Mole fraction of water:

$$\frac{P_{v3}}{P_3} = 13\%$$

Relative humidity:

$$RH = \frac{P_{v3}}{P_{g3}} = 89.6\%$$