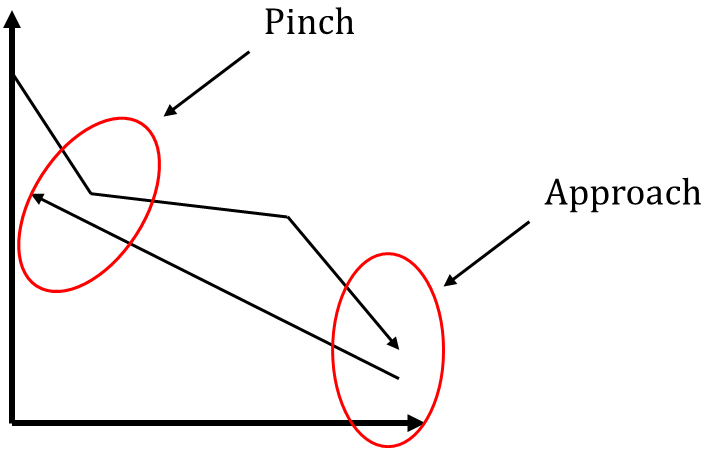
Thermodynamics 4
Tutorial 4 – Power Cycles 3 (Solutions)

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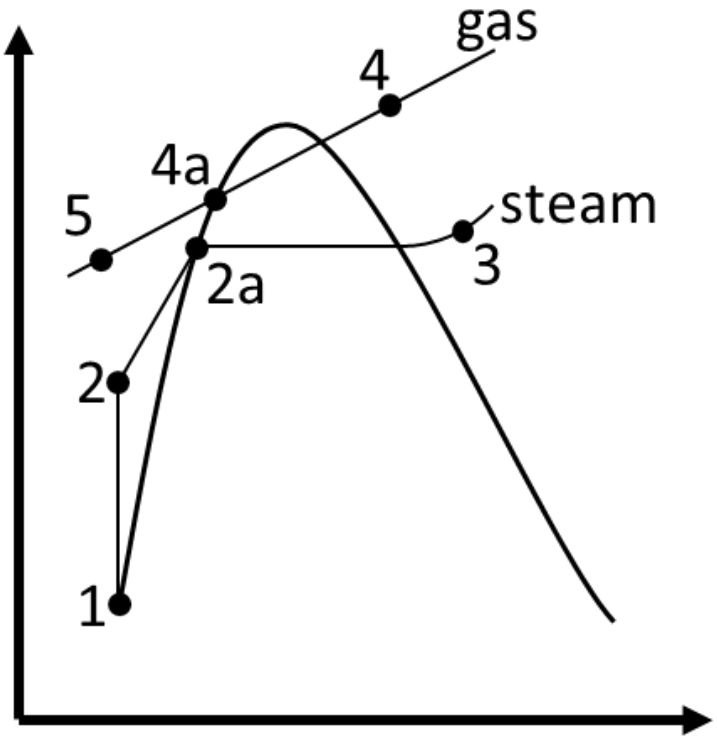
1) Sketch and label a pinch diagram for a closed feedwater heater. Label the pinch and approach. What factors influence these temperature differences?



The top line is the steam/water extracted from the turbine exit to condensate. The bottom line is the high-pressure water flowing towards the boiler.

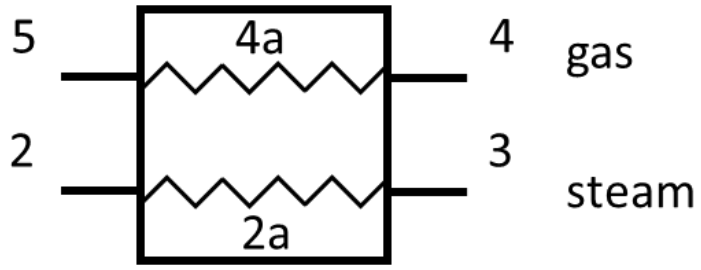
Heat transfer is driven by temperature difference, and in general, $\dot{Q} = cA\Delta T$ *where A is the surface area and* ΔT *is the difference in temperature between the fluids involved. To achieve the desired heat transfer rate, A and* ΔT *need to be balanced. Here, costs need to be considered because bigger A generally implies higher cost.*

2) A Rankine steam power plant is designed to operate with a high pressure of 3 MPa, a low pressure of 10 kPa, and a boiler exit temperature of 500°C. The high-temperature source for this cycle is a gas turbine that produces 175 kg/s of air at 600°C. The boiler operates as a counter-flowing heat exchanger where the temperature difference at the pinch point is 20°C (i.e., the smallest temperature difference between the gas and steam at any point in the heat exchanger is 20°C) and the gas can be assumed to be ideal with *Cp* = 1.15 kJ/kg·K.



Pinch point will be at 2a, $\Delta T_{pp} = 20°\text{C}$.

The diagram for the process is…



$$P_2 = 3\ MPa = P_3$$

$$P_1 = 10\ \text{kPa}$$

$$T_3 = 500°\text{C}$$

$$T_4 = 600°\text{C}$$

$$\dot{m}_g = 175\ \text{kg/s}$$

$$\dot{m}_{w,\max} = ?, \quad T_5 = ?$$

a) Show that subcooled water entering the boiler has a specific enthalpy of around 195 kJ/kg.

Find the enthalpies of steam/water from the tables…

$$h_1 = h_{sat@10kPa} = 191.8\ \text{kJ/kg}$$

$$h_2 = h_1 + w_p, \quad w_p = v_1 \times (P_2 - P_1) = 0.001010 \times (3000 - 10) = 3.02\ \text{kJ/kg}$$

$$h_2 = 191.8 + 3.02 = 194.9\ kJ/kg$$

Note that here, w_p could be neglected, but the following answers would be slightly different.

b) Find the temperatures at the pinch and, hence, the maximum water mass flow rate possible.

$$h_{2a} = h_{sat@3MPa} = 1008.4\ \text{kJ/kg}$$

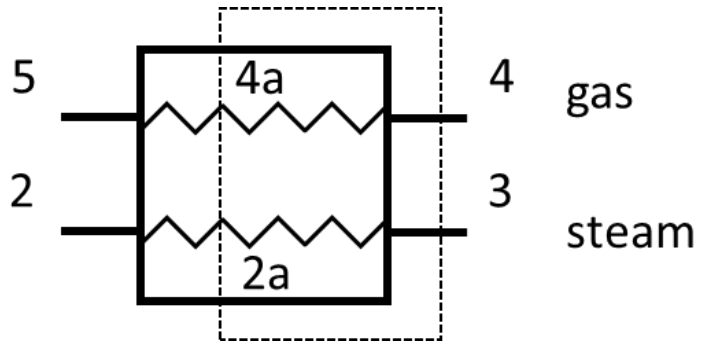
$$h_3 = h(3\ MPa, 500°C) = 3457\ \text{kJ/kg}$$

For the air side…

$$C_p = 1.15 \text{ kJ/kgK}$$

$$T_{4a} = T_{2a} + T_{pp} = T_{sat@3\,MPa} + 20°\text{C} = 253.9°\text{C}$$

So now consider a control volume such that…



$$\dot{m}_w(h_3 - h_{2a}) = \dot{m}_g(h_4 - h_{4a}) = \dot{m}_g C_p(T_4 - T_{4a})$$

$$\dot{m}_w = \frac{\dot{m}_q C_p(T_4 - T_{4a})}{h_3 - h_{2a}} = \frac{175 \times 1.15 \times (600 - 253.9)}{3457 - 1008.4} = 28.4 \text{ kJ/kg}$$

This problem can also be solved using air tables, but note that the answer may vary slightly (by about 2 *kg/s).*

c) Find the air exit temperature for this flow rate and confirm that this air exit temperature is thermodynamically possible.

Now consider the whole heat exchanger to find the exit gas temperature:

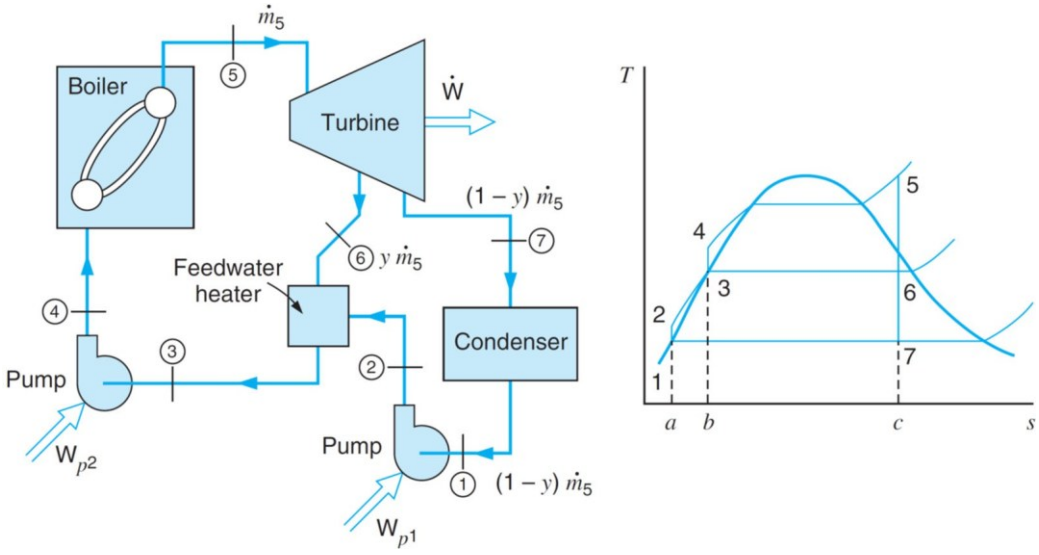
$$\dot{m}_w(h_3 - h_2) = \dot{m}_g(h_4 - h_5) = \dot{m}_g C_p(T_4 - T_5)$$

$$T_5 = T_4 - \frac{\dot{m}_w}{\dot{m}_g C_p}(h_3 - h_2) = 600 - \frac{28.4}{175 * 1.15} \times (3457 - 195) = 140°\text{C}$$

Here note that $T_5 > T_2$*, so this is thermodynamically OK.*

3) Consider an ideal steam regenerative cycle in which steam enters the turbine at 3.0 MPa, 400°C, and exhausts to the condenser at 10 kPa. Steam is extracted from the turbine at 0.8 MPa for an open FWH. The feedwater leaves the heater as a saturated liquid. The appropriate pumps are used for the water leaving the condenser and the FWH. Calculate the thermal efficiency of the cycle and the specific net work.

This is a standard Rankine cycle with an open FWH as shown in the figure below:



$P_1 = 10$ kPa $= 0.1$ bar;

$P_2 = P_3 = 0.8$ MPa $= 8$ bar;

$P_4 = P_5 = 3$ MPa $= 30$ bar; $T_5 = 400$°C

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(800 - 10) = 0.798 \text{ kJ/kg}$$

$$\Rightarrow\ h_2 = h_1 + w_{P1} = 191.81 + 0.798 = 192.61 \text{ kJ/kg}$$

C.V. FWH Call $\dot{m}_6 / \dot{m}_{tot} = y$ (the extraction fraction)

$$(1 - y)\, h_2 + y\, h_6 = 1\, h_3$$

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{721.1 - 192.61}{2891.6 - 192.61} = 0.1958$$

C.V Pump P2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001115(3000 - 800) = 2.45 \text{ kJ/kg}$$

$$h_4 = h_3 + w_{P2} = 721.1 + 2.45 = 723.55 \text{ kJ/kg}$$

CV Boiler: $q_H = h_5 - h_4 = 3230.82 - 723.55 = 2507.3$ kJ/kg

CV Turbine

Entropy Eq.: $s_7 = s_6 = s_5 = 6.9211$ kJ/kg K

P_6 , s_6 => $h_6 = 2891.6$ kJ/kg (superheated vapor)

$$s_7 = s_6 = s_5 = 6.9211 \quad \Rightarrow \quad x_7 = \frac{6.9211 - 0.6492}{7.501} = 0.83614$$

$$\Rightarrow\ h_7 = 191.81 + x_7\, 2392.82 = 2192.55 \text{ kJ/kg}$$

Turbine has full flow in HP section and fraction 1-y in LP section

$$\dot{W}_T / \dot{m}_5 = h_5 - h_6 + (1 - y)(h_6 - h_7)$$

$$w_T = 3230.82 - 2891.6 + (1 - 0.1988)(2891.6 - 2192.55) = 899.3 \text{ kJ/kg}$$

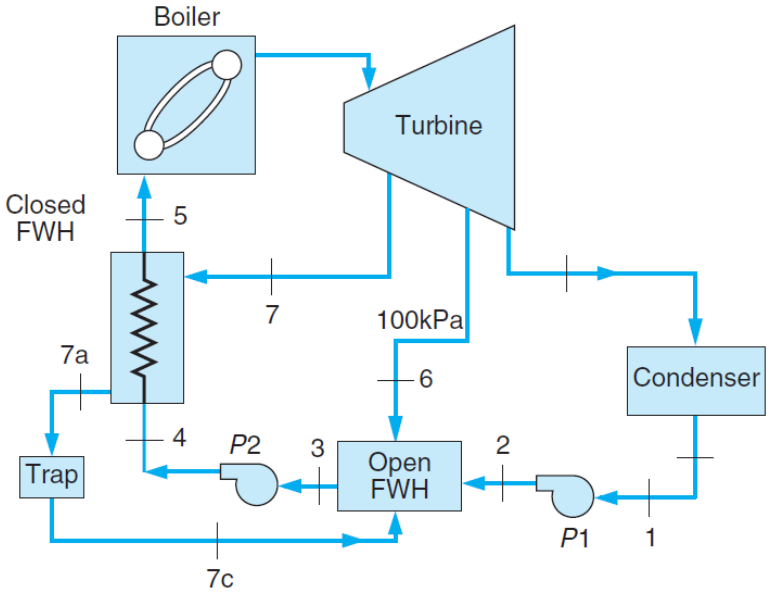
P2 has the full flow and P1 has the fraction 1-y of the flow

$$w_{net} = w_T - (1 - y)\, w_{P1} - w_{P2}$$

$$= 899.3 - (1 - 0.1988)0.798 - 2.45 = \mathbf{896.2\ kJ/kg}$$

$$\eta_{cycle} = w_{net} / q_H = 896.2 / 2507.3 = \mathbf{0.357}$$

4) Assume a variation of the cycle in Problem #3 with a closed FWH at 0.8 MPa and one open FWH at 100 kPa. A pump is used to bring the water leaving the condenser up to 100 kPa for an open FWH, and a second pump brings the feedwater up to 3.0 MPa, as shown in the figure. Condensate from the closed FWH is drained through a trap to the open FWH. Calculate the thermal efficiency of the cycle and the specific net work.



Analysis

This is a Rankine cycle with an open FWH and a closed FWH shown in the figure. Here, state 6 is out of the boiler, state 8 is extraction at 100 kPa, and state 9 is out of the turbine.

States

1: 10 kPa, x = 0: $h_1 = 191.81$ kJ/kg, $v_1 = 0.001010$ m^3/kg

3: 100 kPa, x = 0: $h_3 = 417.44$ kJ/kg, $v_3 = 0.001043$ m^3/kg

5: 3 MPa, $T = T_{7a} = 170.43^\circ$C: $h_5 \approx 721.1$ kJ/kg $(= h_{7a})$

6: 3 MPa, 400°C: $h_6 = 3230.82$ kJ/kg, $s_6 = 6.9211$ kJ/kg-K

7: 800 kPa, $s = s_6$: $h_7 = 2891.62$ kJ/kg (sup. vapor)

7a: 800 kPa, x = 0: $h_{7a} = h_f = 721.1$ kJ/kg

8: 100 kPa, $s = s_6$: $x_8 = (6.9211 - 1.3025)/6.0568 = 0.92765$

$h_8 = 417.44 + x_8\ 2258.02 = 2512.09$ kJ/kg

9: 10 kPa, $s = s_6$: $x_9 = (6.9211 - 0.6492)/7.501 = 0.83614$

$h_9 = 191.81 + x_9\ 2392.82 = 2192.55$ kJ/kg

C.V Pump P1

$$w_{P1} = h_2 - h_1 = v_1(P_2 - P_1) = 0.00101(100 - 10) = 0.0909 \text{ kJ/kg}$$
$$\Rightarrow\ h_2 = h_1 + w_{P1} = 191.81 + 0.0909 = 191.9 \text{ kJ/kg}$$

C.V Pump P2

$$w_{P2} = h_4 - h_3 = v_3(P_4 - P_3) = 0.001043(3000 - 100) = 3.025 \text{ kJ/kg}$$
$$h_4 = h_3 + w_{P2} = 417.44 + 3.025 = 420.465 \text{ kJ/kg}$$

Since the open FWH has a flow in from the trap we must do that first.

C.V. Closed FWH Call $\dot{m}_7 / \dot{m}_{tot} = y_1$ (the extraction fraction)

Energy Eq.: $0 = h_4 - h_5 + y_1(h_7 - h_{7a}) \Rightarrow y_1 = \dfrac{h_5 - h_4}{h_7 - h_{7a}} = \dfrac{721.1 - 420.465}{2891.62 - 721.1} = 0.1385$

CV Open FWH (3 flows in, 1 flow out)

Energy Eq.: $0 = y_2 h_8 + y_1 h_{7a} + (1 - y_1 - y_2) h_2 - h_3$

$$y_2 = \frac{h_3 - y_1 h_{7a} - (1 - y_1) h_2}{h_8 - h_2} = \frac{417.44 - y_1 \times 721.1 - (1 - y_1)\ 191.9}{2512.09 - 191.9} = 0.0656$$

CV Boiler: $q_H = h_6 - h_5 = 3230.82 - 721.1 = 2509.7 \text{ kJ/kg}$

CV Turbine $w_T = h_6 - y_1 h_7 - y_2 h_8 - (1 - y_1 - y_2) h_9$

$= 3230.82 - 0.1385 \times 2891.62 - 0.0656 \times 2512.09 - 0.7959 \times 2192.55$

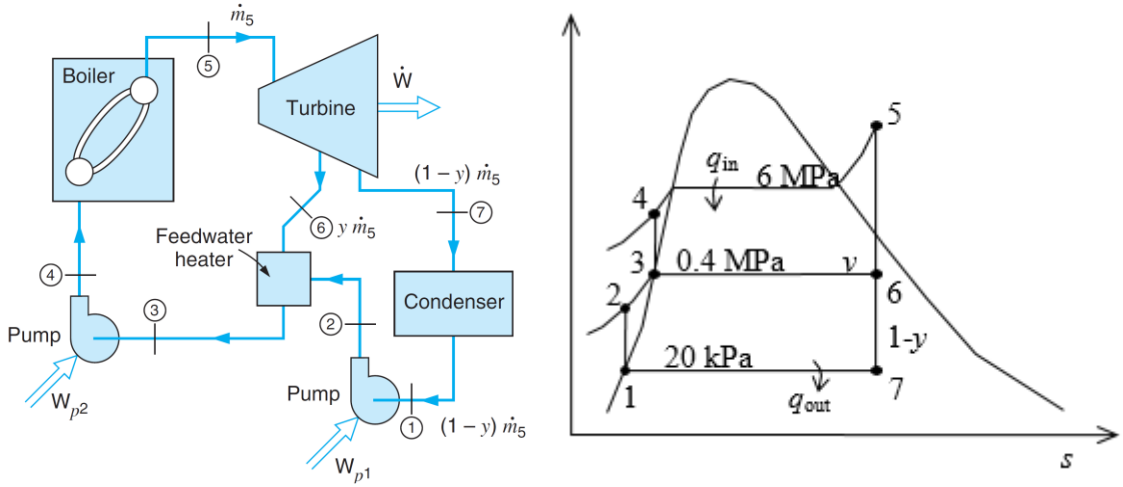
$= 920.487 \text{ kJ/kg}$

P2 has the full flow and P1 has the fraction $1 - y_1 - y_2$ of the flow

$$w_{net} = w_T - (1 - y_1 - y_2)\ w_{P1} - w_{P2}$$
$$= 920.487 - 0.7959 \times 0.0909 - 3.025 = \mathbf{917.39 \text{ kJ/kg}}$$
$$\eta_{cycle} = w_{net} / q_H = 917.39 / 2509.7 = \mathbf{0.366}$$

5) A steam power plant operates on an ideal regenerative Rankine cycle. Steam enters the turbine at 6 MPa and 450°C and is condensed in the condenser at 20 kPa. Steam is extracted from the turbine at 0.4 MPa to heat the feedwater in an open feedwater heater. Water leaves the feedwater heater as a saturated liquid. Show the cycle on a T-s diagram, and determine (a) the net work output per kilogram of steam flowing through the boiler and (b) the thermal efficiency of the cycle.

This is a standard Rankine cycle with an open FWH as shown in the figure below, along with the T-s diagram:



Assumptions

1 Steady operating conditions exist.

2 Kinetic and potential energy changes are negligible.

Analysis

(a) From the steam tables,

$$h_1 = h_{f\,@\,20\text{ kPa}} = 251.42\text{ kJ/kg}$$

$$v_1 = v_{f\,@\,20\text{ kPa}} = 0.001017\text{ m}^3/\text{kg}$$

$$w_{pI,\text{in}} = v_1\,(P_2 - P_1) = (0.001017\text{ m}^3/\text{kg})(400 - 20\text{ kPa})\left(\frac{1\text{ kJ}}{1\text{ kPa}\cdot m^3}\right) = 0.39\text{ kJ/kg}$$

$$h_2 = h_1 + w_{pI,\text{in}} = 251.42 + 0.39 = 251.81\text{ kJ/kg}$$

$$\left.\begin{array}{l} P_3 = 0.4\text{ MPa} \\ \text{sat.liquid} \end{array}\right\} \begin{array}{l} h_3 = h_{f\,@\,0.4\text{ MPa}} = 604.66\text{ kJ/kg} \\ v_3 = v_{f\,@\,0.4\text{ MPa}} = 0.001084\text{ m}^3/\text{kg} \end{array}$$

$$w_{pII,\,\text{in}} = v_3(P_4 - P_3) = (0.001084\ \text{m}^3/\text{kg})(6000 - 400\ \text{kPa})\left(\frac{1\ \text{kJ}}{1\ \text{kPa}\cdot m^3}\right)$$

$$= 6.07\ \text{kJ/kg}$$

$$h_4 = h_3 + w_{pII,\text{in}} = 604.66 + 6.07 = 610.73\ \text{kJ/kg}$$

$$\left.\begin{matrix} P_5 = 6\ \text{MPa} \\ T_5 = 450°C \end{matrix}\right\} \begin{matrix} h_5 = 3302.9\ \text{kJ/kg} \\ s_5 = 6.7219\ \text{kJ/kg}\cdot\text{K} \end{matrix}$$

$$\left.\begin{matrix} P_6 = 0.4\ \text{MPa} \\ s_6 = s_5 \end{matrix}\right\} \begin{matrix} x_6 = \dfrac{s_6 - s_f}{s_{fg}} = \dfrac{6.7219 - 1.7765}{5.1191} = 0.9661 \\ h_6 = h_f + x_6 h_{fg} = 604.66 + (0.9661)(2133.4) = 2665.7\ \text{kJ/kg} \end{matrix}$$

$$\left.\begin{matrix} P_7 = 20\ \text{kPa} \\ s_7 = s_5 \end{matrix}\right\} \begin{matrix} x_7 = \dfrac{s_7 - s_f}{s_{fg}} = \dfrac{6.7219 - 0.8320}{7.0752} = 0.8325 \\ h_7 = h_f + x_7 h_{fg} = 251.42 + (0.8325)(2357.5) = 2214.0\ \text{kJ/kg} \end{matrix}$$

The fraction of steam extracted is determined from the steady-flow energy balance equation applied to the feedwater heater. Noting that $\dot{Q} \cong \dot{W} \cong \Delta KE \cong \Delta PE \cong 0$,

$$\dot{E}_{in} - \dot{E}_{out} = \Delta \dot{E}_{system} \overset{\text{(steady)}}{\cancel{\;\;}\!\!\downarrow 0} = 0$$

$$\dot{E}_{in} = \dot{E}_{out}$$

$$\sum \dot{m}_i h_i = \sum \dot{m}_e h_e \longrightarrow \dot{m}_6 h_6 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \longrightarrow y h_6 + (1 - y) h_2 = 1(h_3)$$

where y is the fraction of steam extracted from the turbine ($= \dot{m}_6 / \dot{m}_3$). Solving for y,

$$y = \frac{h_3 - h_2}{h_6 - h_2} = \frac{604.66 - 251.81}{2665.7 - 251.81} = 0.1462$$

Then,

$$q_{\text{in}} = h_5 - h_4 = 3302.9 - 610.73 = 2692.2\ \text{kJ/kg}$$

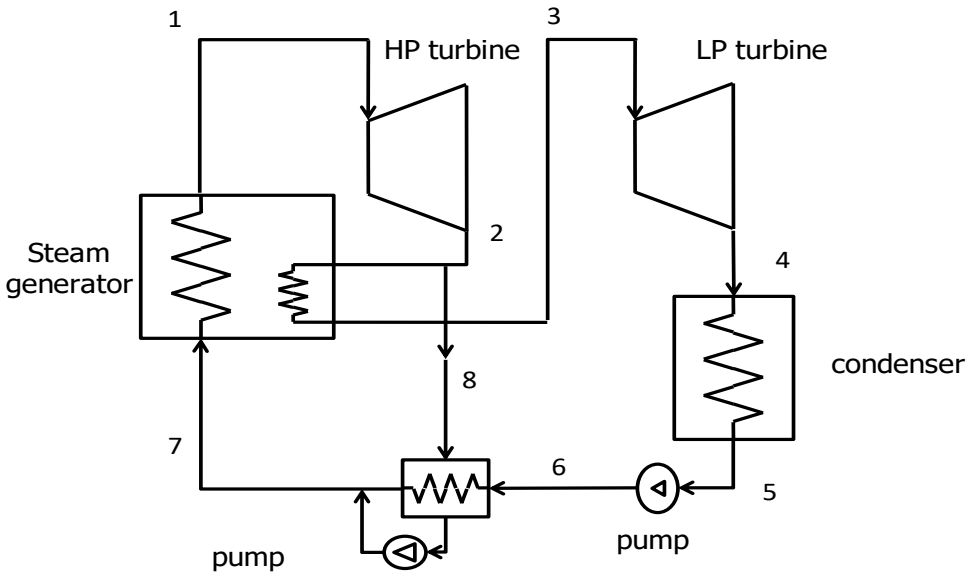
$$q_{\text{out}} = (1 - y)(h_7 - h_1) = (1 - 0.1462)(2214.0 - 251.42) = 1675.7\ \text{kJ/kg}$$

and $$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 2692.2 - 1675.7 = \mathbf{1016.5\ kJ/kg}$$

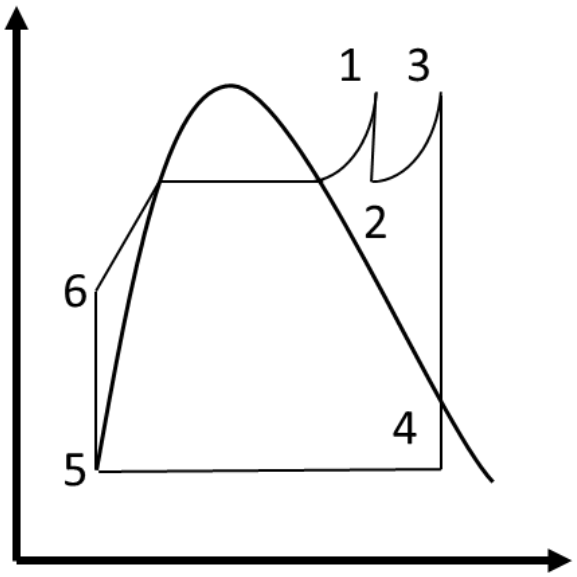
(b) The thermal efficiency is determined from

$$\eta_{\text{th}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{1675.7\ \text{kJ/kg}}{2692.2\ \text{kJ/kg}} = \mathbf{37.8\%}$$

6) The steam cycle (Example 2 of the lecture notes on Rankine cycles) is now upgraded with a closed feedwater heater taking steam from the exhaust of the high-pressure turbine at 70 bar, as shown in the diagram below. The feedwater heater has an approach temperature at the feedwater water inlet of 7.2K and a temperature pinch of 5.8K. Both pumps have an exit pressure of 320 bar, and it can be assumed that there are no significant pressure drops in the feedwater heater.



From Example 2 of the lecture note, we have:



T-s diagram of Example 2

$$P_1 = 300\ bar, \quad T_1 = 600°C\ (inlet\ of\ the\ high\ pressure\ turbine)$$

$$P_3 = 60 \text{ bar}, \quad T_3 = 600°\text{C} \ (inlet\ of\ the\ low\ pressure\ turbine)$$

$$\eta_t = 90\% \ (so\ the\ processes\ in\ 1-2\ and\ 3-4\ should\ not\ be\ vertical)$$

$$\eta_p = 80\%$$

$$\Delta P_{reheater} = 10 \text{ bar}, \quad \Delta P_{boiler} = 20 \text{ bar}$$

$$P_4 = P_5 = 0.05\ bar$$

Low pressure turbine…

$$h_3 = 3659 \text{ kJ/kg}$$

$$h_4 = 2333 \text{ kJ/kg}$$

$$w_{34} = h_3 - h_4 = 1326 \text{ kJ/kg}$$

High pressure turbine…

$$h_1 = 3447 \text{ kJ/kg}$$

$$h_2 = 3064 \text{ kJ/kg}$$

$$w_{12} = h_1 - h_2 = 383 \text{ kJ/kg}$$

Boiler… $P_6 = 320 \text{ bar}, P_1 = 300 \text{ bar}$

$$h_6 = 178 \text{ kJ/kg}$$

$$q_{61} = h_1 - h_6 = 3269 \text{ kJ/kg}$$

Re-heater…

$$q_{32} = h_3 - h_2 = 595 \text{ kJ/kg}$$

Pump…

$$w_{56} = h_6 - h_5 = 40 \text{ kJ/kg}$$

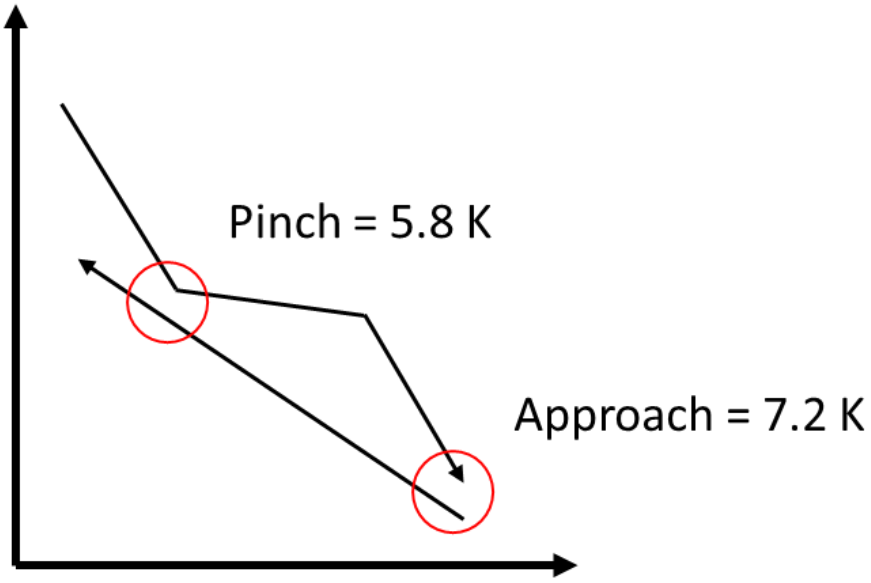
Then…

$$w_{total} = w_{12} + w_{34} - w_{56} = 1669 \text{ kJ/kg}$$

$$q_{total} = q_{23} + q_{61} = 3864 \text{ kJ/kg}$$

$$\eta_{Rankine} = \frac{w_{total}}{q_{total}} = 43\%$$

a) Draw the pinch diagram for the feedwater heater.



The bottom line is the high-pressure water flowing towards the boiler, and the top line is the steam/condensate flowing from the steam cycle.

b) Using the steam tables, find the specific enthalpy of both streams at the approach and the pinch.

First consider the approach… The boiler inlet $P_7 = 320$ bar. *Assuming there is no significant pressure drop in the feedwater heater, this means* $P_6 = 320$ bar. *Then at state* 6, *assume saturated liquid (this is after the condensate outlet) or (if taking into account pump work) subcooled liquid.*

$h_6 = 178$ kJ/kg *as previously, with* $T_6 = 32.88°\text{C} = 306.03$ K *as a reasonable approximation, so treated as if there was no pump as saturated liquid at* $P_5 = 0.05$ bar *to find* T_6 *. Could also interpolate in tables for subcooled temperature.*

Now consider there is some pre-heating in the feedwater heater before the flow enters the boiler.

The steam exiting the feedwater heater is at a higher temperature than water at the approach – therefore heat is being transferred from the steam to the water exiting the condenser.

$$T_{8a} = 306.03 + 7.2 = 313.2\ \text{K} = 40°\text{C}$$

Then find h_{8a} *from the tables at* $P_2 = 70$ bar *and* 40°C*…*

$$h_{8a} = 173.7\ \text{kJ/kg}$$

Although we are more interested in overall inlet/outlet temperatures, we know more about the system at the pinch point. Therefore, it is useful to include that in our thinking. At the pinch point, the steam/condensate extracted from the steam cycle is saturated vapour at 70 *bar.*

$T_{g,pinch} = 285.8°\text{C}\ (= T_{sat}\ at\ 70\ bar)$, $\quad h_{g,pinch} = 2772.6\ kJ/kg$ (from Table 2)

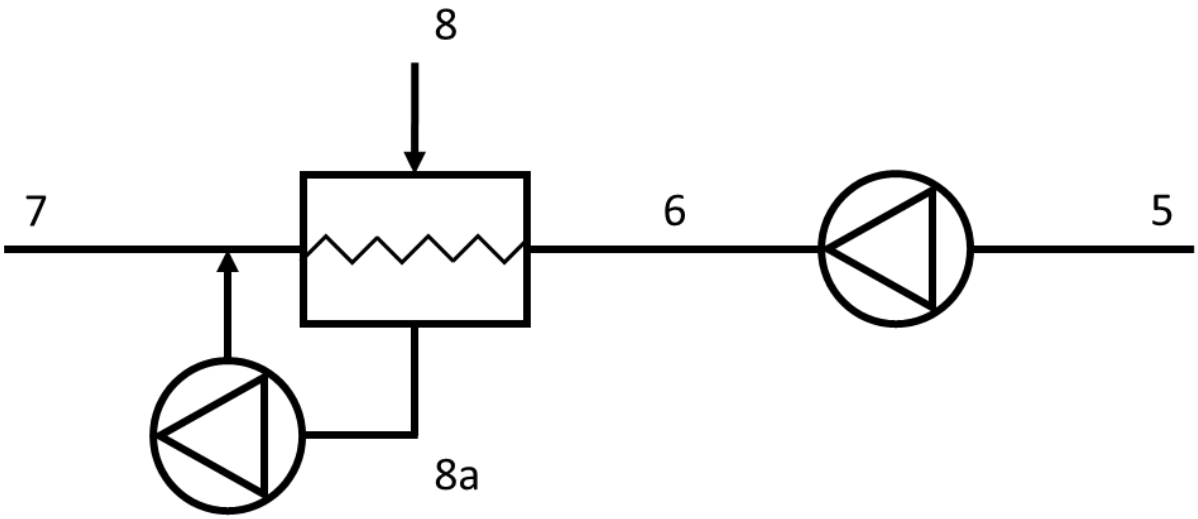
The water is cooled so:

$T_{w,pinch} = 285.8 - 5.8 = 280°\text{C}$, $\quad P_{w,pinch} = 320$ bar, $\quad \rightarrow h_{w,pinch} = 1229.5$ kJ/kg

The enthalpy of the water at the pinch point is found in the corresponding steam tables.

c) Calculate the mass fraction relative to the boiler mass flow of steam entering the feedwater heater.

Define y as the mass fraction used for the hot side of the feedwater heater and total flow as the flow entering the boiler…



At the outlet, 7 *…*

$$\dot{m}_7 = 1$$

At points 8 *and* 8a*…*

$$\dot{m}_8 = \dot{m}_{8a} = y$$

At point 6*…*

$$\dot{m}_6 = 1 - y$$

Consider the enthalpy balance between the pinch and approach…

$$\dot{m}_{steam,pinch} = y, \quad h_{steam,pinch} = 2772.6 \text{ kJ/kg}$$

$$\dot{m}_{8a} = y, \quad h_{8a} = 173.7 \text{ kJ/kg}$$

$$\dot{m}_6 = 1 - y, \quad h_6 = 178 \text{ kJ/kg}$$

$$\dot{m}_{water,pinch} = 1 - y, \quad h_{water,pinch} = 1229.5 \text{ kJ/kg}$$

$$\dot{m}_{steam}\left(h_{steam,pinch} - h_{8a}\right) = \dot{m}_{water}(h_{water,pinch} - h_6)$$

Here, just substitute known values…

$$y(2772.6 - 173.7) = (1 - y)(1229.5 - 178)$$

$$2598.9y = 1051.5(1 - y)$$

$$y = 0.29$$

d) Calculate the new cycle efficiency and compare it to the efficiency of the cycle without feedwater heating (i.e. Example of the lecture notes on Rankine cycles).

We know a lot of the quantities from the previous parts of this question…

$$h_8 = h_2 = 3064 \text{ kJ/kg}$$

We also know…

$$\dot{m}_6(h_{6a} - h_6) = \dot{m}_8(h_{8a} - h_8) \rightarrow h_{6a} = h_6 + \frac{\dot{m}_8}{\dot{m}_6}(h_8 - h_{8a})$$

$$h_{6a} = 178 + \frac{0.29}{0.71}(3064 - 173.7) = 1359 \text{ kJ/kg}$$

$$P_6 = 320 \text{ bar}$$

From the tables, this means that T_{6a} is somewhere between 300℃ *and* 320℃*.*

	300 bar	**400 bar**
300℃	1328.7	1325.4
320℃	1433.5	1426.0

Now consider the changes and impact on the cycle efficiency. First, there is a change in the enthalpy of steam entering the boiler…

$$\dot{m}_6 h_6 + \dot{m}_8 h_8 = \dot{m}_7 h_7$$

$$(1 - y)h_6 + yh_8 = h_7$$

$$h_7 = (1 - 0.29)178 + 0.29 * 3064 = 1015 \text{ kJ/kg}$$

And the boiler exit $h_1 = 3446.9\ kJ/kg$ as previously. For reheating, the mass flow rate is reduced because only steam entering the low-pressure turbine is reheated (not bled off for feedwater heating). h_2 and h_3 are unchanged.

$$q = (h_1 - h_7) + (1 - y)(h_3 - h_2)$$

$$q = (3447 - 1015) + (1 - 0.29)(3659 - 3064) = 2854.5 \text{ kJ/kg}$$

Similarly, the high-pressure turbine work is unchanged, but the low-pressure turbine work is reduced due to the reduced mass flow…

$$w_{LP} = (1 - y)(h_3 - h_4) = (1 - 0.29)(3659 - 2333) = 941.5 \text{ kJ/kg}$$

Also, the work of the pump after the condenser is reduced due to the reduced mass flow…

$$w_{Pump} = (1 - y)(h_6 - h_5) = (1 - 0.29)(40) = 28.4 \text{ kJ/kg}$$

(Note: You could add here the work of the pump for the condensed steam coming out of the FWH; however, it is reasonable to neglect it due to the tiny fraction of work it would use.)

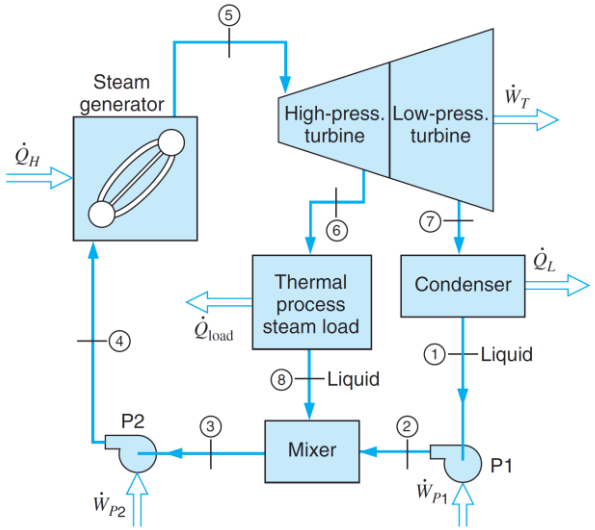
$$w_{total} = 941.5 + 383 - 28.4 = 1296.1 \text{ kJ/kg}$$

Then we calculate the efficiency…

$$\eta_{Rankine} = \frac{w_{total}}{q_{total}} = \frac{1296.1}{2854.5} = 45.4\%$$

Note that in this calculation, the pump work of the second pump is neglected. Over the cycle with reheat, but no feedwater heating, the net power produced is reduced, but the efficiency is increased.

7) **Combined Heat and Power:** A cogenerating steam power plant, as shown below, operates with a boiler output of 25 kg/s steam at 7 MPa, 500°C. The condenser operates at 7.5 kPa, and the process heat is extracted as 5 kg/s from the turbine at 500 kPa, state 6 and after use is returned as saturated liquid at 100 kPa, state 8. Assume all components are ideal and find the temperature after pump 1, the total turbine output, and the total process heat transfer.



Pump 1:

Inlet state is saturated liquid: $h_1 = 168.79$ kJ/kg, $v_1 = 0.001008$ m^3/kg

$$w_{P1} = \int v\, dP = v_1 (P_2 - P_1) = 0.001008 \text{ m}^3/\text{kg} (100 - 7.5) \text{ kPa} = 0.093 \text{ kJ/kg}$$

$$w_{P1} = h_2 - h_1 \Rightarrow h_2 = h_1 + w_{P1} = 168.88 \text{ kJ/kg}, \quad T_2 = \mathbf{40.3°C}$$

Turbine:

$h_5 = 3410.3$ kJ/kg, $s_5 = 6.7974$ kJ/kg K

$P_6, s_6 = s_5 \Rightarrow x_6 = 0.9952, \quad h_6 = 2738.6$ kJ/kg

$P_7, s_7 = s_5 \Rightarrow x_7 = 0.8106, \quad h_7 = 2119.0$ kJ/kg

$$\dot{W}_T = \dot{m}_5 (h_5 - h_6) + 0.80\dot{m}_5 (h_6 - h_7) = 25 (3410.3 - 2738.6) + 20 (2738.6 - 2119) = 16\,792.5 + 12\,392 = \mathbf{29.185 \text{ MW}}$$

$$\dot{Q}_{proc} = \dot{m}_6 (h_6 - h_8) = 5(2738.6 - 417.46) = \mathbf{11.606 \text{ MW}}$$

8) **Combined Gas–Steam Power Cycle:** Consider the combined gas–steam power cycle shown below. The topping cycle is a gas-turbine cycle that has a pressure ratio of 8. Air enters the compressor at 300 K and the turbine at 1300 K. The isentropic efficiency of the compressor is 80 percent, and that of the gas turbine is 85 percent. The bottoming cycle is a simple ideal Rankine cycle operating between the pressure limits of 7 MPa and 5 kPa. Steam is heated in a heat exchanger by the exhaust gases to a temperature of 500°C. The exhaust gases leave the heat exchanger at 450 K. **Determine (a) the ratio of the mass flow rates of the steam and the combustion gases and (b) the thermal efficiency of the combined cycle.** Assume that the gas-turbine cycle and the steam cycle are already analysed, and the following results are already obtained.

Gas cycle:

$h_{4'} = 880.36$ kJ/kg ($T_{4'} = 853$ K)

$q_{in} = 790.58$ kJ/kg $w_{net} = 210.41$ kJ/kg $\eta_{th} = 26.6\%$

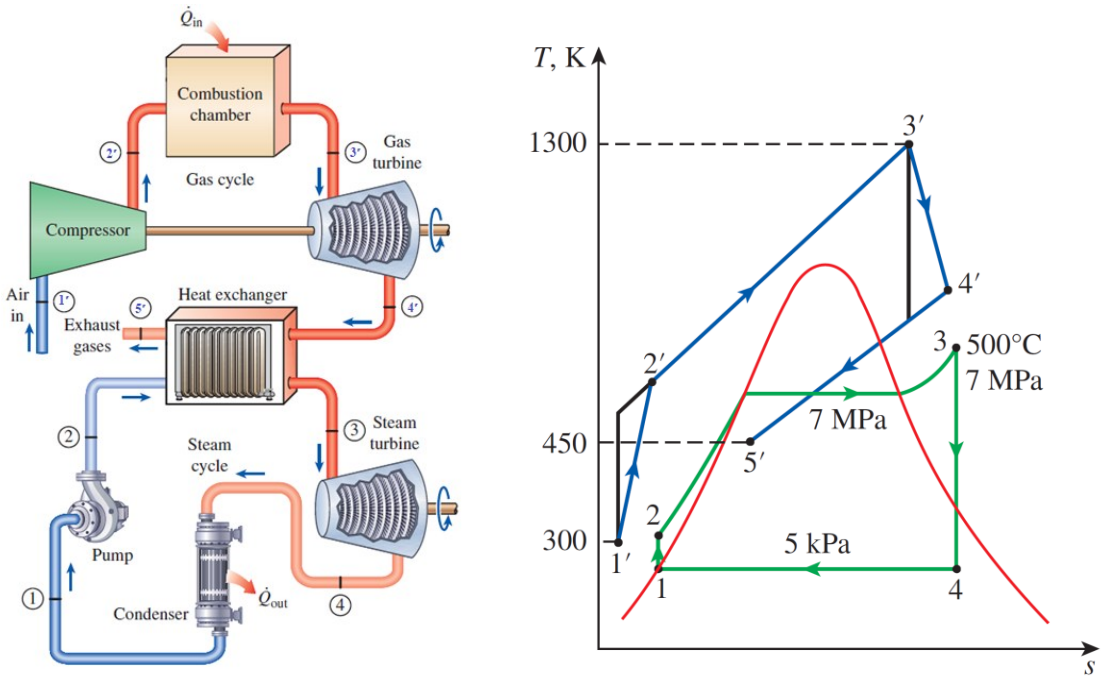
$h_{5'} = h$ @ 450 K = 451.80 kJ/kg

Steam cycle:

$h_2 = 144.78$ kJ/kg ($T_2 = 33°C$)

$h_3 = 34\,11.4$ kJ/kg ($T_3 = 500°C$)

$w_{net} = 1331.4$ kJ/kg $\eta_{th} = 40.8\%$



(a) The ratio of mass flow rates is determined from an energy balance on the heat exchanger:

$$\dot{E}_{in} = \dot{E}_{out}$$
$$\dot{m}_g h_{5'} + \dot{m}_s h_3 = \dot{m}_g h_{4'} + \dot{m}_s h_2$$
$$\dot{m}_s(h_3 - h_2) = \dot{m}_g(h_{4'} - h_{5'})$$
$$\dot{m}_s(3411.4 - 144.78) = \dot{m}_g(880.36 - 451.80)$$

Thus, $\frac{\dot{m}_s}{\dot{m}_g} = y = 0.131$

That is, 1 kg of exhaust gases can heat only 0.131 kg of steam from 33 to 500°C as they are cooled from 853 to 450 K. Then the total net work output per kilogram of combustion gases becomes

$$\begin{aligned} w_{net} &= w_{net,gas} + y w_{net,steam} \\ &= (210.41 \text{ kJ/kg gas}) + (0.131 \text{ kg steam/kg gas})(1331.4 \text{ kJ/kg steam}) \\ &= 384.8 \text{ kJ/kg gas} \end{aligned}$$

Therefore, for each kg of combustion gases produced, the combined plant will deliver 384.8 kJ of work. The net power output of the plant is determined by multiplying this value by the mass flow rate of the working fluid in the gas-turbine cycle.

(b) The thermal efficiency of the combined cycle is determined from

$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{384.8 \text{ kJ/kg gas}}{790.6 \text{ kJ/kg gas}} = 0.487 \text{ or } \mathbf{48.7\%}$$

Note: *This combined cycle converts to useful work 48.7 percent of the energy supplied to the gas in the combustion chamber. This value is considerably higher than the thermal efficiency of the gas-turbine cycle (26.6 percent) or the steam-turbine cycle (40.8 percent) operating alone.*