Thermodynamics 4
Tutorial 3 – Power Cycles 2 (Solutions)

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1) Consider a 210-MW steam power plant that operates on a simple ideal Rankine cycle. Steam enters the turbine at 10 MPa and 500°C and is cooled in the condenser at a pressure of 10 kPa. Show the cycle on a T-s diagram with respect to saturation lines, and determine (a) the quality of the steam at the turbine exit, (b) the thermal efficiency of the cycle, and (c) the mass flow rate of the steam.

$$h_1 = h_{f\,@\,10\text{ kPa}} = 191.81 \text{ kJ/kg}$$

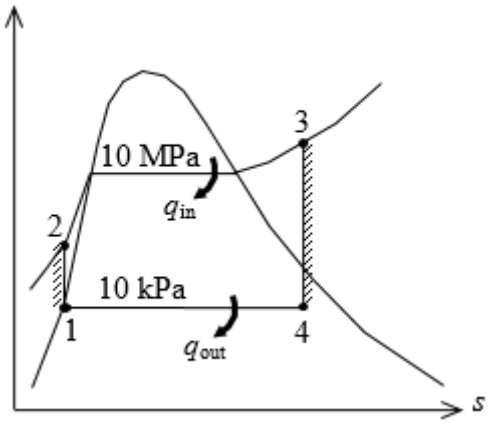
$$v_1 = v_{f\,@\,10\text{ kPa}} = 0.00101 \text{ m}^3/\text{kg}$$

$$w_{p,\text{in}} = v_1 \ (P_2 - P_1)$$

$$= \ 0.00101 \text{ m}^3/\text{kg} \ \ (10{,}000 - 10 \text{ kPa}) \left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right)$$

$$= 10.09 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{p,\text{in}} = 191.81 + 10.09 = 201.90 \text{ kJ/kg}$$



$$\left.\begin{array}{l} P_3 = 10 \text{ MPa} \\ T_3 = 500 \text{ °C} \end{array}\right\} \begin{array}{l} h_3 = 3375.1 \text{ kJ/kg} \\ s_3 = 6.5995 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\left.\begin{array}{l} P_4 = 10 \text{ kPa} \\ s_4 = s_3 \end{array}\right\} x_4 = \frac{s_4 - s_f}{s_{fg}} = \frac{6.5995 - 0.6492}{7.4996} = \mathbf{0.7934}$$

$$h_4 = h_f + x_4 h_{fg} = 191.81 + \ 0.7934 \ \ 2392.1 \ = 2089.7 \text{ kJ/kg}$$

$$q_{\text{in}} = h_3 - h_2 = 3375.1 - 201.90 = 3173.2 \text{ kJ/kg}$$

$$q_{\text{out}} = h_4 - h_1 = 2089.7 - 191.81 = 1897.9 \text{ kJ/kg}$$
$$w_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 3173.2 - 1897.9 = 1275.4 \text{ kJ/kg}$$

and

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{1275.4 \text{ kJ/kg}}{3173.2 \text{ kJ/kg}} = \mathbf{40.2\%}$$

$$\dot{m} = \frac{\dot{W}_{\text{net}}}{w_{\text{net}}} = \frac{210{,}000 \text{ kJ/s}}{1275.4 \text{ kJ/kg}} = \mathbf{164.7 \text{ kg / s}}$$

2) How do the following quantities change when a simple ideal Rankine cycle is modified with reheating? Assume the mass flow rate is maintained the same.

Pump work input: (a) increases, (b) decreases, (c) remains the same

Turbine work output: (a) increases, (b) decreases, (c) remains the same

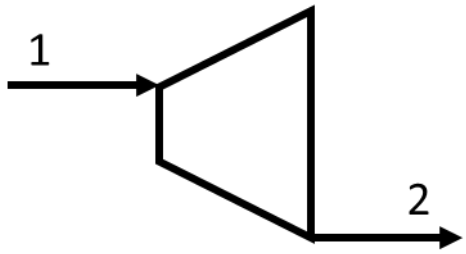
Heat supplied: (a) increases, (b) decreases, (c) remains the same

Heat rejected: (a) increases, (b) decreases, (c) remains the same

Moisture content at turbine exit: (a) increases, (b) decreases, (c) remains the same

When a simple ideal Rankine cycle is modified with reheating, the pump work remains the same, the moisture content decreases, and everything else increases.

3) Steam enters a turbine at 8 bar and 400°C and exits at 0.05 bar. The isentropic efficiency of the turbine is 90%.



$P_1 = 8$ *bar,* $T_1 = 400°\text{C}$

$P_2 = 0.05$ *bar*

$\eta_{\text{turbine}} = 90\%$

Using the steam tables, find:

a) The enthalpy and entropy at the inlet of the turbine

From tables, $h_1 = 3267.6$ kJ/kg, $s_1 = 7.573$ kJ/kgK

b) The entropy of saturated liquid and saturated vapour at the outlet of the turbine

From tables at $P = 0.05$ *bar,* $s_f = 0.4763$ kJ/kgK, $s_g = 8.394$ kJ/kgK

c) Calculate the dryness fraction for a reversible expansion

$$x_{2s} = \frac{s_{2s} - s_f}{s_g - s_f}$$

where $s_{2s} = s_1$...

$$x_{2s} = \frac{s_1 - s_f}{s_g - s_f} = \frac{7.573 - 0.4763}{8.394 - 0.4763} = 0.896$$

d) The isentropic outlet enthalpy

$$x_{2s} = \frac{h_{2s} - h_f}{h_g - h_f}$$

$h_f = 137.77$ *kJ/kg,* $h_g = 2560.8$ *kJ/kg (Saturation properties at* 0.05 *bar)*

$$h_{2s} = h_f + x_{2s} \times \left(h_g - h_f\right) = 137.77 + 0.896 \times (2560.8 - 137.77) = 2309 \frac{\text{kJ}}{\text{kg}}$$

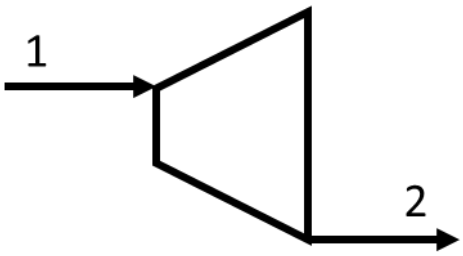
e) The actual turbine enthalpy and dryness fraction

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

$$h_2 = h_1 - \eta_t \times (h_1 - h_{2s}) = 3267.6 - 0.9 \times (3267.6 - 2309) = 2405 \frac{\text{kJ}}{\text{kg}}$$

$$x = \frac{h_2 - h_f}{h_g - h_f} = \frac{2405 - 137.77}{2560.8 - 137.77} = 0.936$$

4) Steam enters a turbine at 10 bar and 400°C and exits at 0.05 bar. The isentropic efficiency of the turbine is 90%.



$P_1 = 10\ bar,\ T_1 = 400°C$

$P_2 = 0.05\ bar$

$\eta_{turbine} = 90\%$

Find the isentropic enthalpy at the outlet of the turbine, the actual enthalpy at the outlet of the turbine, and the outlet dryness fraction. You might find it helpful to use a **Mollier diagram** (*h-s* diagram for steam) included with the steam tables on Learn.

The same methodology can be used here as in question one. Enthalpy and entropy at the inlet are found from the steam tables at $10\ bar, 400°C$ *as* $h_1 = 3264.4\ kJ/kg$, $s_1 = 7.467\ kJ/kg{\cdot}K$. *A Mollier diagram can also be used – sketch the process on the diagram to avoid some of the calculations. Sometimes, this can be quicker.*

Using the same calculations as for the question above, the intermediate results are as follows:

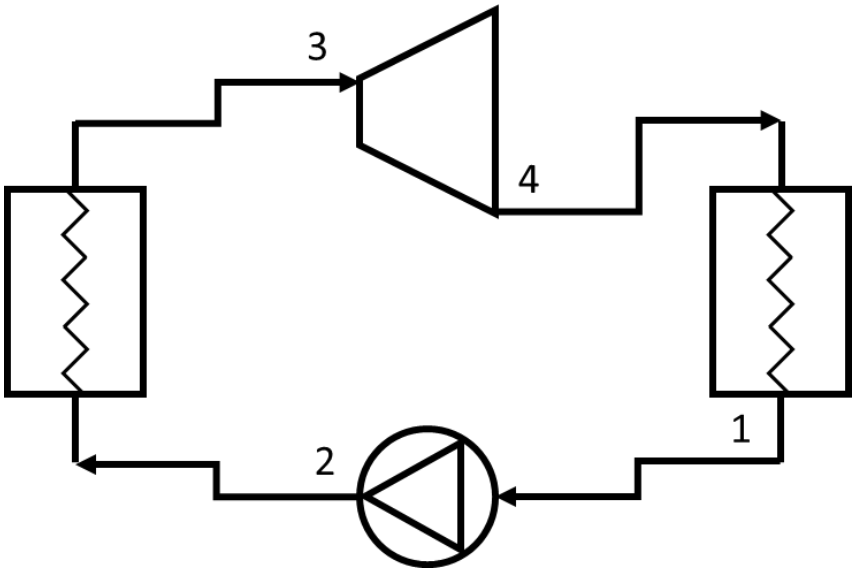
$x_{2s} = 0.883$

$h_{2s} = 2277\ \text{kJ/kg}$

$h_2 = 2376\ \text{kJ/kg}$

$x = 0.924$

5) Steam is generated at 100 bar and 600°C in a sub-critical boiler and is expanded in a turbine down to 0.1 bar.



$P_3 = 100$ bar, $T_3 = 600°C$

$P_4 = 0.1$ bar

a) For a turbine with an isentropic efficiency of 90% calculate the turbine work output.

$\eta_t = 90\%$

Need to find w_{34}, the work output of the turbine. From the steam tables at 100 *bar,* 600 *°C, the entropy and enthalpy are:*

$s_3 = 6.905$ kJ/kgK, $h_3 = 3625.8$ kJ/kg

Then we need to interpolate in steam tables for the value of enthalpy at 0.1 *bar and use knowledge of turbine efficiency to find h_4. From steam tables:*

$s_{4s} = s_3 = 6.905$ kJ/kgK

at 0.1 bar *... $s_f = 0.6942$* kJ/kgK, $s_g = 8.149$ kJ/kgK

$$x_{4s} = \frac{s_{4s} - s_f}{s_g - s_f} = \frac{6.905 - 0.6492}{8.149 - 0.6492} = 0.834$$

And also...

$$x_{4s} = \frac{h_{4s} - h_f}{h_g - h_f}$$

$$h_{4s} = h_f + x_{4s}(h_g - h_f) = 191.81 + 0.834 \times (2392.1) = 2187 \frac{\text{kJ}}{\text{kg}}$$

Now using the turbine efficiency expression…

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}}$$

$$h_4 = h_3 - \eta_t(h_3 - h_{4s}) = 3625.8 - 0.9 \times (3625.8 - 2187.1) = 2331 \text{ kJ/kg}$$

And finally calculating turbine work…

$$|w_{34}| = h_3 - h_4 = 3625.8 - 2331 = 1295 \text{ kJ/kg}$$

b) Calculate the efficiency of the corresponding Rankine cycle, assuming a pressure drop in the boiler of 20 bar and a pump efficiency of 80%.

For a Rankine cycle…

$$\eta_{Cycle} = \frac{w_{net}}{q_{in}} = \frac{|w_{34}| - |w_{12}|}{q_{23}}$$

We are assuming saturated liquid at the pump inlet…

$P_1 = P_4 = 0.1$ bar *(this is assuming there is no pressure drop in the condenser)*

$T_1 = T_{sat}$ at $P_1 = 45.81$°C

$h_1 = h_f$ at $P_1 = 191.81$ kJ/kg

$v_1 = v_f$ at $P_1 = 0.001010$ m^3/kg

… and $P_2 = P_3 + \Delta P_{Boiler} = 100 + 20 = 120$ bar

Then we can find the work of the pump…

$$w_{21} = \frac{v_1(P_2 - P_1)}{\eta_p} = \frac{0.001010 \times (120 - 0.1) \times 10^5}{0.8} = 15.1 \text{ kJ/kg}$$

$$h_2 = h_1 + w_{12} = 191.81 + 15.1 = 206.9 \text{ kJ/kg}$$

$$q_{23} = h_3 - h_2 = 3625.8 - 206.9 = 3419 \text{ kJ/kg}$$

$$\eta_{Cycle} = \frac{|w_{34}| - |w_{12}|}{q_{23}} = \frac{1295 - 15.1}{3419} = 0.374$$

c) Compare the result to the corresponding Carnot cycle efficiency.

To calculate the Carnot efficiency, we only need the temperatures T_c and T_h…

$$T_h = T_3 = 873 \text{ K}$$

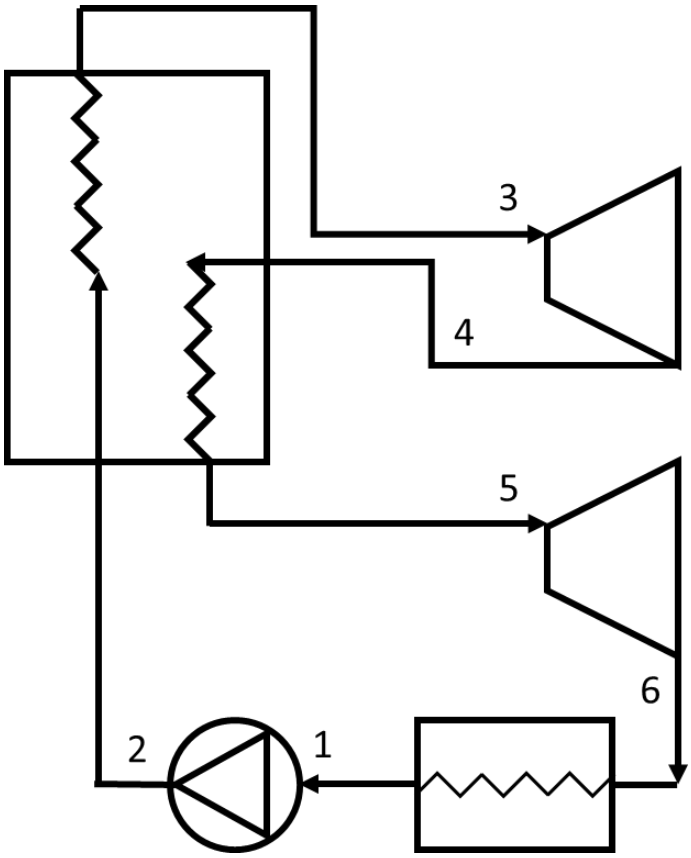
$$T_c = T_4 = 319 \text{ K}$$

$$\eta_{Carnot} = 1 - \frac{T_c}{T_h} = 1 - \frac{319}{873} = 0.635$$

Then to compare…

$$\frac{\eta_{Rankine}}{\eta_{Carnot}} = \frac{0.374}{0.635} = 0.59$$

6) Consider an ideal steam reheat cycle where steam enters the high-pressure turbine at 30 bar, 400°C, and then expands to 8 bar. It is then reheated to 400°C and expanded to 10 kPa in the low-pressure turbine.



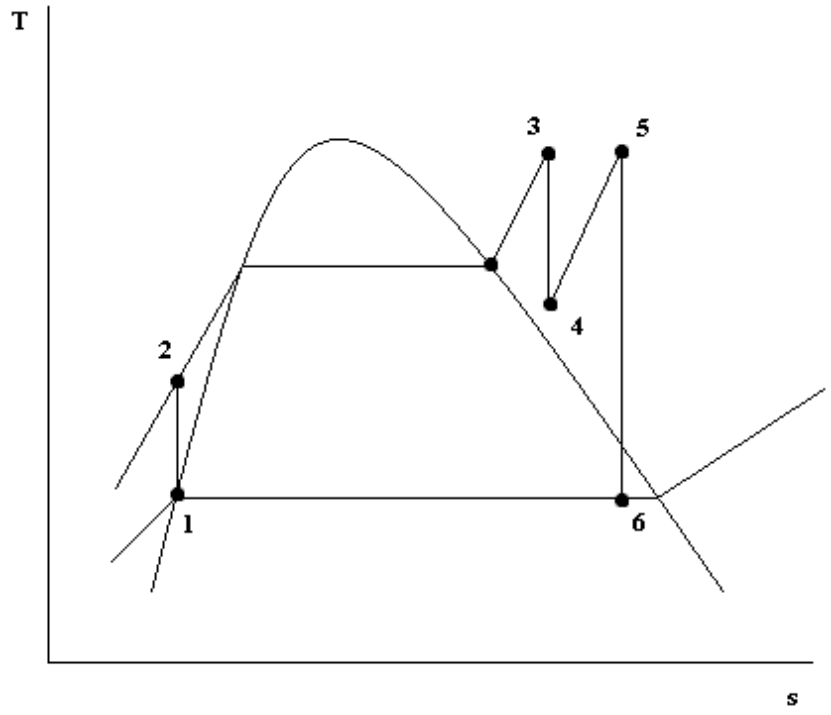
$P_3 = 30$ bar, $T_3 = 400°C$

$P_4 = 8$ bar

$T_5 = 400°C$

$P_6 = 10$ kPa

Calculate the cycle thermal efficiency and the moisture content (dryness fraction) of the steam leaving the low-pressure turbine.



To calculate the w and q values needed, we need to find the enthalpy at each state labelled in the cycle. Assuming there is no significant pressure drop and also ideal turbines and pump (efficiency = 100 *%)*

Starting with the pump…

$w_p = v_1(P_2 - P_1)$

$P_2 = P_3 = 30$ bar

$P_1 = P_6 = 10\ kPa = 0.1\ bar$

$v_1 = v_{sat}\ at\ P_1 = 0.001010$ m^3/kg

$w_p = 0.001010 \times (30 - 0.1) \times 10^5 = 3019.9 \frac{J}{kg} = 3.02$ kJ/kg

$w_p = h_2 - h_1$ *where* $h_1 = h_f\ at\ P_1 =$ **191.81** kJ/kg

Therefore… $h_2 = w_p + h_1 = 3.02 + 191.81 = 194.8$ kJ/kg

Now continuing with the boiler…

$h_3 = 3231.6$ *kJ/kg*

$q_{23} = h_3 - h_2 = 3231.6 - 194.8 = \mathbf{3036.8}$ kJ/kg

Now consider the turbines… starting with the high-pressure one

$|w_{hp}| = h_3 - h_4$ *where state* 3 *is* 30 bar, 400°C

$s_3 = s_4 = 6.923$ kJ/kgK, $h_3 = 3231.6$ kJ/kg *at state 3 and for an ideal turbine*

$P_4 = 8$ bar

To find h_4, *we need to interpolate from the steam tables…*

$\boldsymbol{T}$ [°C]	$\boldsymbol{h}$ [kJ/kg]	$\boldsymbol{s}$ [kJ/kgK]
220	2885.0	6.911
240	2928.9	6.999

$$h_4 = h_{220} + \frac{s_4 - s_{220}}{s_{240} - s_{220}} \times (h_{240} - h_{220})$$

$$h_4 = 2885.0 + \frac{6.923 - 6.911}{6.999 - 6.911} \times (2928.9 - 2885) = 2891 \text{ kJ/kg}$$

Then we can find the work done by the high-pressure turbine…

$|w_{hp}| = h_3 - h_4 = 3231.6 - 2891.0 = \mathbf{340.6}$ kJ/kg

Now the reheating…

$h_4 = 2891$ kJ/kg

At state 5… $T_5 = 400$°C, $P_5 = P_4 = 8$ bar

$h_5 = 3267.6$ kJ/kg

$s_5 = 7.573$ kJ/kgK *… these are needed for the low-pressure turbine calculations…*

$q_{45} = h_5 - h_4 = 3267.6 - 2891 = \mathbf{376.6}$ kJ/kg

Now the low-pressure turbine…

$s_6 = s_5$, $P_6 = 0.1$ bar (10 kPa)

At 0.1 *bar…*

$s_f = 0.6492$ kJ/kgK, $s_g = 8.149$ kJ/kgK

$$x_6 = \frac{s_6 - s_f}{s_g - s_f} = \frac{7.573 - 0.6492}{8.149 - 0.6492} = 0.923$$

And also… $x_6 = \frac{h_6 - h_f}{h_g - h_f}$

$$h_6 = h_f + x_6(h_g - h_f) = 191.81 + 0.923 \times (2392.1) = 2400 \text{ kJ/kg}$$

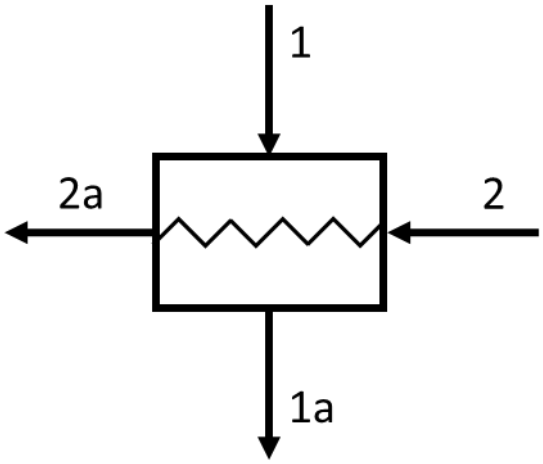
So finding low-pressure turbine work output…

$$|w_{lp}| = h_5 - h_6 = 3267.6 - 2400 = \mathbf{867.6} \text{ kJ/kg}$$

Finally, to get the efficiency of the cycle, the net work is divided by the total heat input into the system…

$$\eta_{cycle} = \frac{w_{net}}{q_{in}} = \frac{w_{hp} + w_{lp} - w_p}{q_{23} + q_{45}} = \frac{340.6 + 867.6 - 3.02}{3036.8 + 376.6} = 0.353$$

7) A closed feedwater heater (FWH) in a regenerative steam power cycle heats 20 kg/s of water from 100°C, 20 MPa to 200°C, 20 MPa. The extraction steam from the turbine enters the heater at 275°C, 4 MPa, and leaves at 4 MPa as a saturated liquid.



$\dot{m}_2 = \dot{m}_{2a} = 20$ kg/s

$T_1 = 275$°C, $P_1 = 4$ MPa $= 40$ bar

$T_{1a} = T_{sat}$ *at* P_1, $P_{1a} = 4$ MPa $= 40$ bar

$T_2 = 100$°C, $P_2 = 20$ MPa $= 200$ bar

$T_{2a} = 200$°C, $P_{2a} = 20$ MPa $= 200$ bar

What is the required mass flow rate of extraction steam?

Calculate the values of enthalpy for energy balance…

$h_2 = 434.1$ kJ/kg

$h_{2a} = 860.4$ kJ/kg

$h_{1a} = 1087.4$ kJ/kg *… all of these values are from the steam tables*

The value for h_1 *needs to be interpolated from the tables…*

T[°C]	h [kJ/kg]
260	2837.2
275	h_1
280	2902.9

$$h_1 = 2837.2 + \frac{275 - 260}{280 - 260} \times (2902.9 - 2837.2) = 2886.5 \text{ kJ/kg}$$

Then…

$$\dot{m}_1(h_1 - h_{1a}) = \dot{m}_2(h_{2a} - h_2)$$

$$\dot{m}_1 = \frac{\dot{m}_2(h_{2a} - h_2)}{h_1 - h_{1a}} = \frac{20 \times (860.4 - 434.1)}{2886.5 - 1087.4} = 4.74 \text{ kg/s}$$