Thermodynamics 4
Tutorial 1 – Revision (Solutions)

Properties of Pure Substances

1) Properties of Pure Substances

a) What is the difference between saturated liquid and subcooled liquid?

Saturated liquid is at the saturation temperature and any heating of saturated liquid will cause some of the liquid to turn to vapour. When a subcooled liquid is heated, its temperature will increase and vapour will not be formed

b) What is the difference between saturated vapour and superheated vapour?

Saturated vapour exists at saturation temperature where it is about to condense. In a superheated state because T is greater than the saturation temperature for the pressure and the vapour is not about to condense

c) Why are temperature and pressure dependent properties in the saturated mixture region?

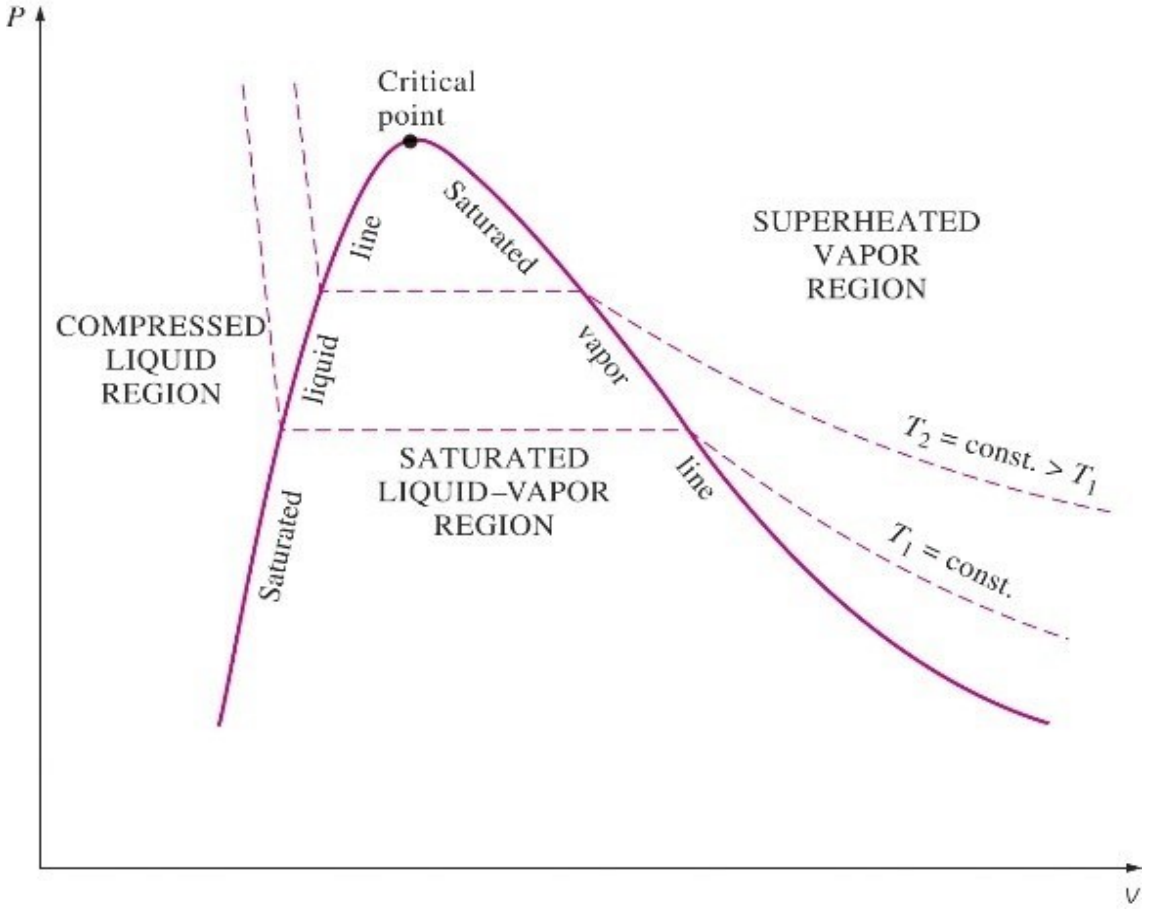
Temperature and pressure are dependent properties during a phase change process. This is because both temperature and pressure represent the energy of the system. If either pressure or temperature increases the energy stored in the system increases as well, which means that the latent heat needed to change the phase of matter from liquid to vapour will be smaller

d) Is it possible to have water vapour a 0°C?

*Yes, if pressure is lower than at the triple point (*0.61 kPa at 0.01°C*).*

e) Consider a piston cylinder device containing water at 1 MPa and 150°C (under these conditions, water is in the liquid phase). We decrease the pressure slowly (but steadily) and simultaneously keep the temperature constant at 150°C. Describe qualitatively what is going to happen and draw the corresponding P-v diagram for the process.

The process will be similar to the dotted line starting from the left hand side of the graph. Initially, decreasing pressure leads to an increase in volume, until saturated liquid conditions are reached. There is then a phase change (liquid to vapour) at constant pressure but with a further increase in volume. Once the phase change is complete, further pressure drop is possible in the superheated vapour region.



2) Ideal gas

a) Starting from the observations made by Boyle, Charles and Gay-Lussac (see Thermodynamics 2 notes) explain why, for an ideal gas:

$$Pv = RT$$

Where P is the pressure, v is the specific volume, T is the temperature and R is the gas constant (constant of proportionality).

In 1662, Boyle observed that the pressure of gases is inversely proportional to their volume. In 1802, Charles and Gay-Lussac determined that at low pressures, the volume of a gas is proportional to its temperature. This means that v is proportional to T/P or v=RT/P for an ideal gas. This can be rearranged to give the relationship Pv=RT.

b) Starting from the relationship in a), show that:

$$PV = mRT, \quad PV = NR_uT$$

By definition v=V/m, so Pv=RT -> PV/m=RT -> PV=mRT. Using the definition of molar mass, m=NM, so PV=mRT=NMRT.. Then, again by definition, R=R_u/M, so PV=NMRT=NM(R_u/M)T=NR_uT

c) A sealed, rigid container holds 2 kg of nitrogen at atmospheric pressure (1 atm) and 30°C. What is the volume of the container and how many moles of nitrogen are present? What is the molar mass of nitrogen? You can assume R_{N2} = 0.2968 kJ/kg·K and R_u = 8.314 kJ/kmol·K.

$m_1 = 2\ kg$, $P_1 = 1\ atm = 1.013\ bar = 1.013*10^5\ Pa$, $T_1 = 30°C = 303.15\ K$

$$PV = mRT \rightarrow V = \frac{mRT}{P} = \frac{2 * 296.8 * 303.15}{1.013 * 10^5} = 1.78\ m^3$$

From here there can be multiple approaches, but one option is:

$$PV = NR_uT \rightarrow N = \frac{PV}{R_uT} = \frac{1.013 * 10^5 * 1.78}{8314 * 303.15} = 0.072\ kmol$$

and then…

$$m = NM \rightarrow M = \frac{m}{N} = \frac{2\ kg}{0.072\ kmol} = 28\ kg/kmol$$

d) The container is heated so that the temperature of nitrogen increases by 100°C. What is the change in pressure in the container?

We know that…

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \quad T_2 = 303.15 + 100 = 403.15\ K$$

Since the container is rigid, we also know that $V_1 = V_2$. It follows that…

$$P_2 = \frac{P_1T_2}{T_1} = \frac{1.013 * 10^5 * 403.15}{303.15} = 1.347 * 10^5\ Pa$$

$$\Delta P = P_2 - P_1 = (1.347 - 1.013) * 10^5 = 0.334\ bar$$

3) Explain the symbols and prove the relationship:

$$v = v_f + xv_{fg}$$

v – Average specific volume of a saturated mixture of liquid and vapour.

v_f – Specific volume of saturated liquid

v_g – Specific volume of saturated vapour

v_{fg} – Difference between the specific volume of saturated liquid and saturated vapour

x – The dryness fraction

See Thermodynamics 2 notes for the proof.

4) Determine the internal energy of refrigerant-134a at a temperature of 0°C and a quality of 60 %.

You can assume $u_{f@0°C} = 49.79\ kJ/kg$ and $u_{g@0°C} = 227.06\ kJ/kg$.

$$u = u_f + x(u_g - u_f) = 49.79 + 0.6 * (227.06 - 49.79) = 156.15\ kJ/kg$$

5) Find the internal energy of water at the given states for 7 MPa and plot the states on T-v, P-v and P-T diagrams:

a) P = 7 MPa, saturated vapour

Using Table 2, $u = u_{g@70bar} = 2580.9\ kJ/kg$

b) P = 7 MPa, saturated liquid

Using Table 2, $u = u_{f@70bar} = 1258.0\ kJ/kg$

c) P = 7 MPa, T = 600°C

Using Table 2, $T_{sat@70bar} = 285.8°C$*. Since* $600°C$ *is much above the saturation temperature at* 70 *bar, the state is superheated.*

Using Table 3, $u = 3261.0\ kJ/kg$

d) P = 7 MPa, T = 100°C

Using Table 1, $P_{sat@100°C} = 1.0142\ bar = 0.10142\ MPa$*. Since* $P > P_{sat}$*, the state is subcooled liquid. The approximate solution is* $u = u_{f@100°C} = 419.0\ kJ/kg$*.*

We can also use Table 3, $u = 417.01\ kJ/kg$*.*

e) P = 7 MPa, T = 455°C

Using Table 2, $T_{sat@70bar} = 285.8°C$*, since* $T > T_{sat}$*, the state is superheated. Now use linear interpolation of values for* 70 *bar from Table 3 to find u.*

T [°C]	*u [kJ/kg]*
440	2959.4
455	?
460	2998.1

Then using the above table to interpolate for the value of u at temperature 455 *°C.*

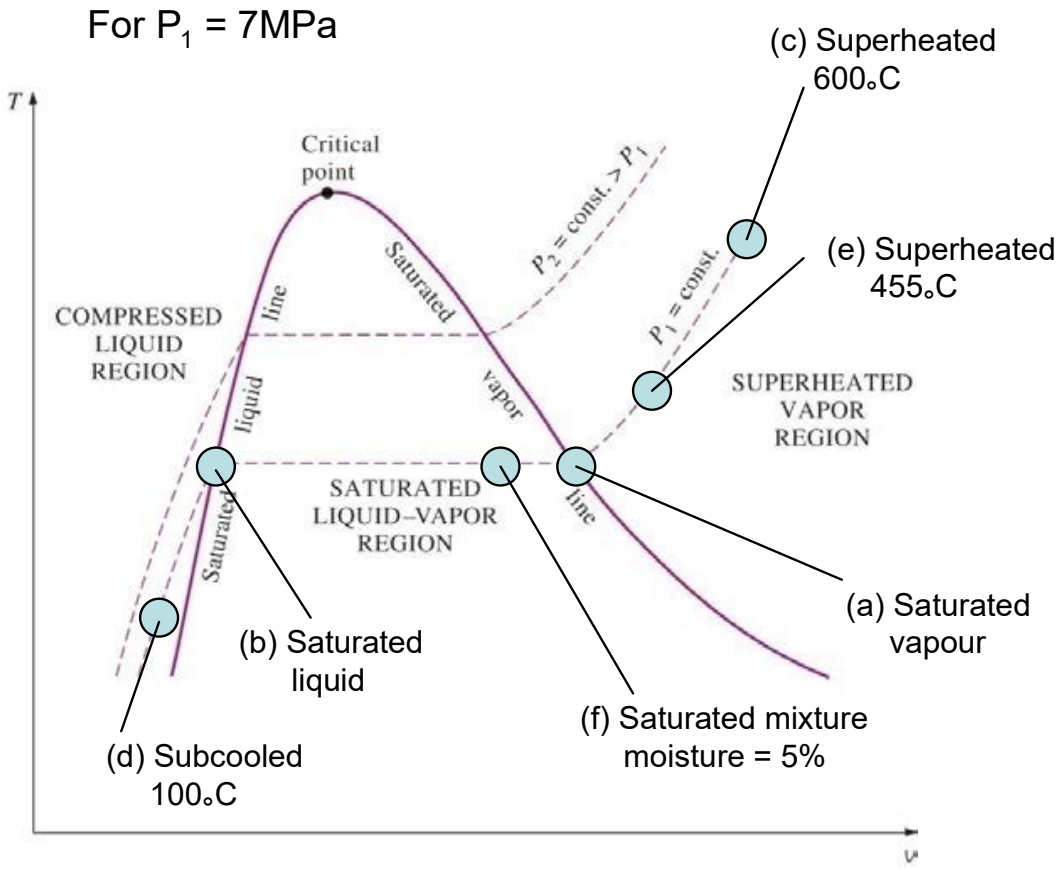
$$\frac{455 - 440}{460 - 440} = \frac{u - 2959.4}{2998.1 - 2959.4} \rightarrow u = 2988.4\ kJ/kg$$

f) P = 7 MPa, dryness fraction = 95 %

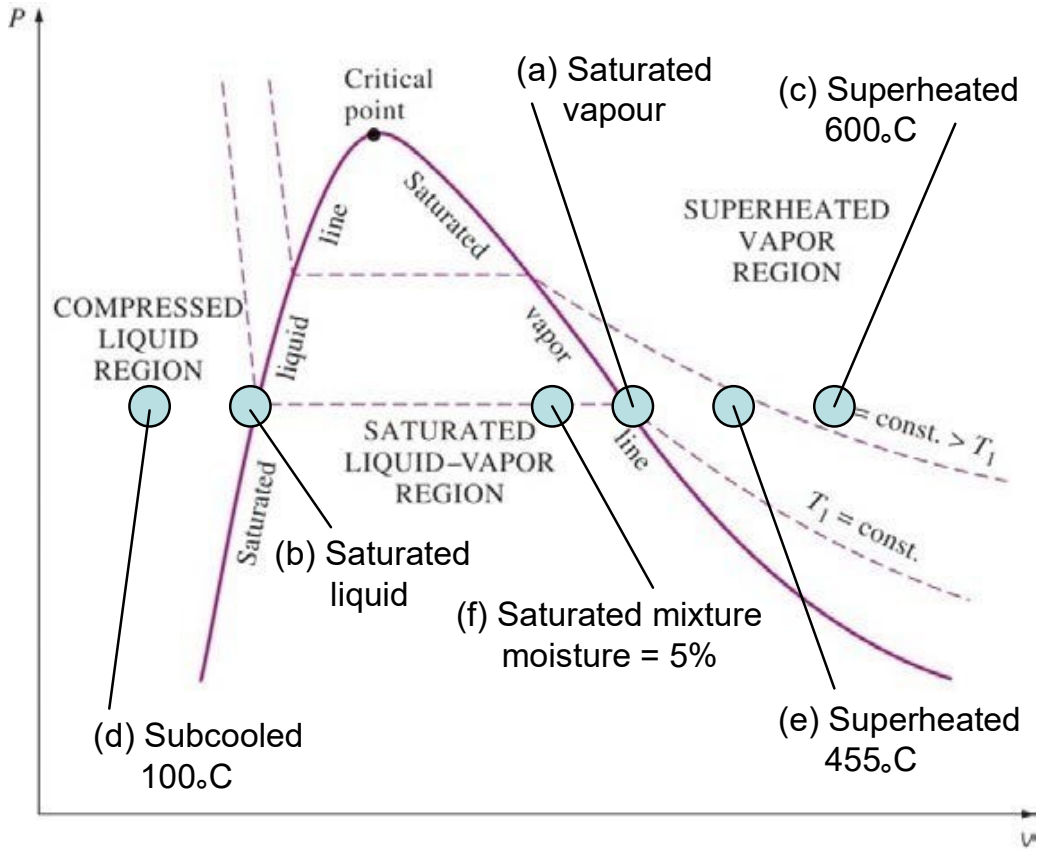
In the saturated region, $u = u_f + x(u_g - u_f)$. *And using Table 2,* $u_{f@70bar} = 1258.0\ kJ/kg$ and $u_{g@70bar} = 2580.9\ kJ/kg$, *so that…*

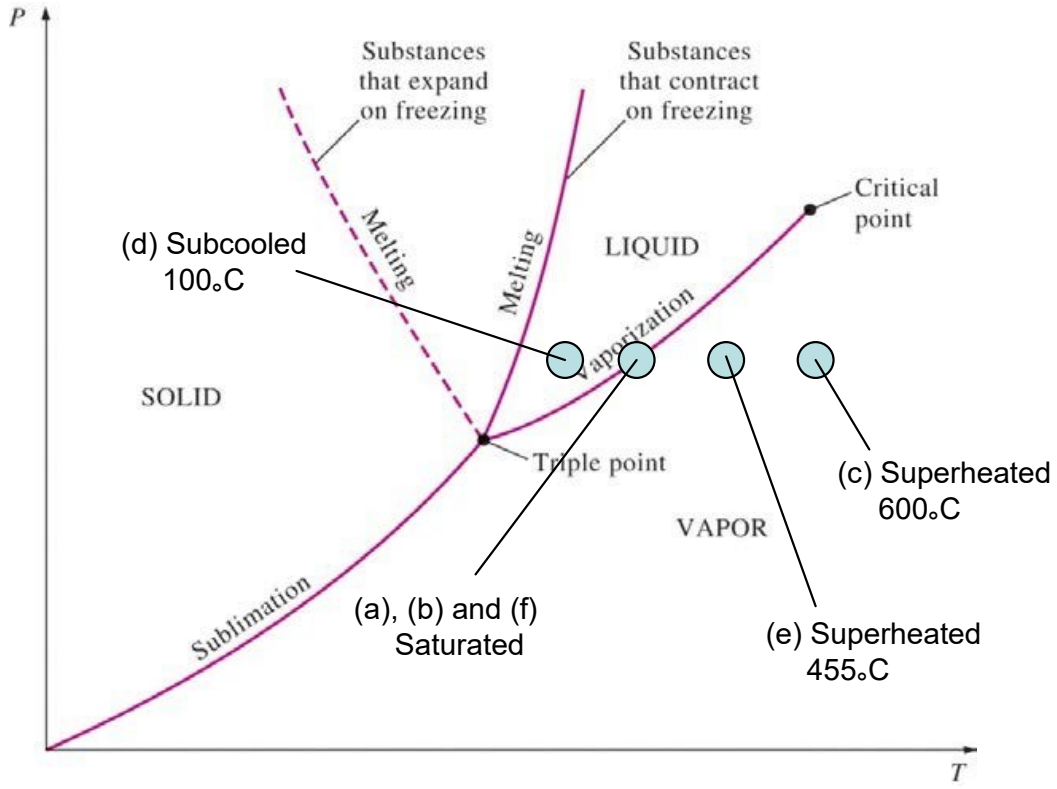
$$u = 1258 + 0.95 * (2580.9 - 1258) = 2541.8\ kJ/kg$$

Finally, the plots are as follows (not to scale):



For $T_1 = 285.8_\circ C$





6) Determine the enthalpy of 1.5 kg of water contained in a solid tank with volume 1.2 m^3 at 200 kPa.

Remember that we need two independent intensive properties to specify the state of a simple substance. Therefore, we calculate the specific volume…

$$v = \frac{V}{m} = \frac{1.2}{1.5} = 0.8\ m^3/kg$$

Now we need to determine the state of the water…

Using Table 2 for $200\ kPa$*…*

$$v_{f@2bar} = 0.001061\ m^3/kg$$

$$v_{g@2bar} = 0.8857\ m^3/kg$$

The specific volume is between the values of v_f *and* v_g *so the water is a saturated mixture of liquid and vapour and h will be given by…*

$$h = h_f + xh_{fg}$$

From Table 2 for $200\ kPa$*…*

$$h_{f@2bar} = 504.68\ kJ/kg$$

$$h_{fg@2bar} = 2201.6\ kJ/kg$$

And the quality x can be calculated using the Lever Rule:

$$x = \frac{v - v_f}{v_g - v_f} = \frac{0.8 - 0.001061}{0.8857 - 0.001061} = 0.903$$

Then…

$$h = h_f + xh_{fg} = 504.68 + 0.903 * 2201.6) = 2492.7\ kJ/kg$$

7) Complete the following table for properties of water. Describe the phase as compressed liquid, saturated mixture, or superheated vapour. If the state is a saturated mixture, give the quality. Sketch a T-v or P-v diagram for each state.

P [MPa]	v [m^3/kg]	T [°C]	Phase description and quality if applicable
0.62	0.42	300	*Superheated vapour*
0.85	0.2269	172.9	*Saturated vapour*
15.0	0.0157	400	*Superheated vapour*
1.00	0.001127	180	*Saturated liquid*
0.75	0.221	167.8	*Saturated mixture, $x = 0.864$*
0.01235	5.5	50	*Saturated mixture, $x = 0.457$*
0.20	0.095	120.2	*Saturated mixture, $x = 0.106$*
6.32	0.040	350	*Superheated vapour*
20.0	0.00136	300	*Subcooled liquid*
2.0	0.202	610	*Superheated vapour*
4.0	0.095	569	*Superheated vapour*
10.0	0.00118	220	*Subcooled liquid*
0.06017	0.580	86	*Saturated mixture, $x = 0.213$*
6.20	0.055	500	*Superheated vapour*
0.4	0.4624	143.6	*Saturated vapour*

The first law of thermodynamics

8) The first law of thermodynamics

a) What is the first law of thermodynamics?

The first law of thermodynamics is known as the conservation of energy principle. It states that energy can be neither created nor destroyed; it can only change forms.

b) What is the classical sign convention for heat and work?

According to the classical sign convention, heat transfer to a system and work done by a system are positive; heat transfer from a system and work done on a system are negative.

c) What is boundary work?

The system may do work on its surroundings because of moving boundaries due to expansion or compression processes that a fluid may experience in a piston-cylinder device. This work is boundary work. Different processes involving boundary work are outlined in Thermo2.

d) What is 'other' work? Give some practical examples.

Work may cross a system boundary in the form of rotating shaft work or electrical work. These work forms are called "other" work since they are work not associated with a moving boundary. Electrical work can include electrical heating. Common types of mechanical work are shaft work, spring work, work associated with stretching of a liquid film and work done to raise or accelerate a body.

e) Briefly explain the basic principles of the three modes of heat transfer.

Conduction – Similar to electricity conduction, heat can move through a material. An example would be heating a bar of metal on one end. The heat would then be conducted by the metal bar to the other parts of the bar so that it eventually has the same temperature everywhere.

Convection – This is heat transfer due to a moving medium that carries the heat. An example of this mode of heat transfer is when hot air is blown onto something to heat it up.

Radiation – Every body with a non-zero temperature radiates heat into its surroundings in the form of electromagnetic waves.

9) Three kilograms of nitrogen gas at 27°C, 0.15 MPa are compressed isothermally to 0.3 MPa in a piston-cylinder device.

Show that nitrogen can be expected to behave as an ideal gas during this process and determine the minimum work of compression, in kJ. For nitrogen: R = 0.2968 kJ/kg·K, T_{cr} = 126.2 K, P_{cr} = 3.39 MPa.

W_{net} = 184.5 kJ

Check the reduced temperature and pressure for nitrogen:

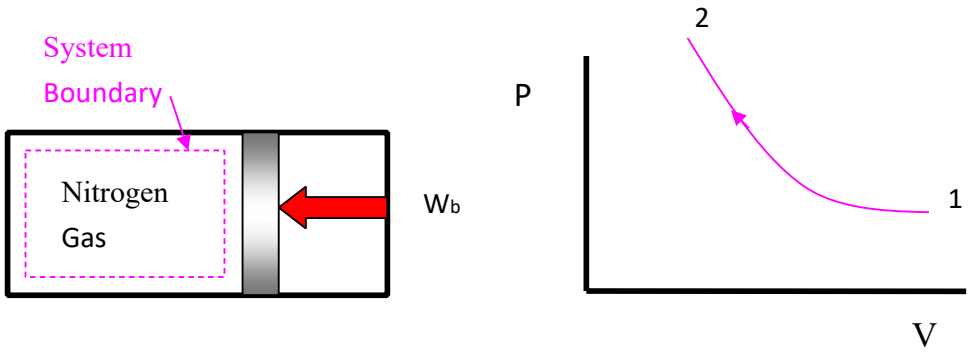
$$T_{R1} = \frac{T_1}{T_{cr}} = \frac{27 + 273}{126.2} = 2.38 = T_{R2}$$

$$P_{R1} = \frac{P_1}{P_{cr}} = \frac{0.15}{3.39} = 0.044$$

$$P_{R2} = 2P_{R1} = 0.088$$

Since $P_R << 1$ and $T_R > 2$ (or $T > 2\,T_{cr}$), nitrogen can be treated as an ideal gas and we use the ideal gas equation of state as the property relation.

$$PV = mRT$$



P-V diagram for T = constant

Then to determine the work of compression… The system is nitrogen contained in a piston-cylinder device and the process is an isothermal compression.

Work calculations:

$$W_{net,12} = (W_{net})_{other,12} + W_{b,12}$$

$$W_{b,12} = \int_1^2 PdV = \int_1^2 \frac{mRT}{V} dV = mRT \ln\left(\frac{V_2}{V_1}\right)$$

For an ideal gas in a closed system (constant mass), we have…

$$m_1 = m_2, \quad \frac{P_1V_1}{RT_1} = \frac{P_2V_2}{RT_2} \rightarrow \frac{V_2}{V_1} = \frac{P_1}{P_2}$$

Note that the temperatures cancel out because for an isothermal process $T_1 = T_2$.

$$W_{b,12} = mRT \ln\left(\frac{P_1}{P_2}\right) = 3 * 0.2968 * 300 * \ln\left(\frac{0.15}{0.30}\right) = -184.5\ kJ$$

Then the net work is:

$$W_{net,12} = 0 + W_{b,12} = -184.5\ kJ$$

To express this on a per unit mass basis…

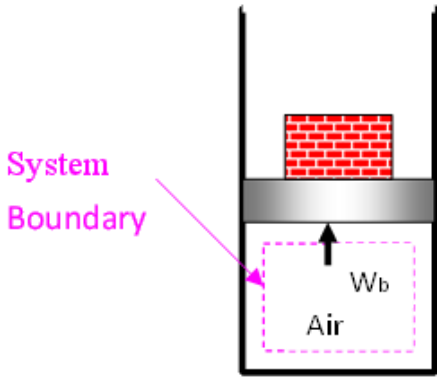
$$w_{net,12} = \frac{W_{net,12}}{m} = -61.5\ kJ/kg$$

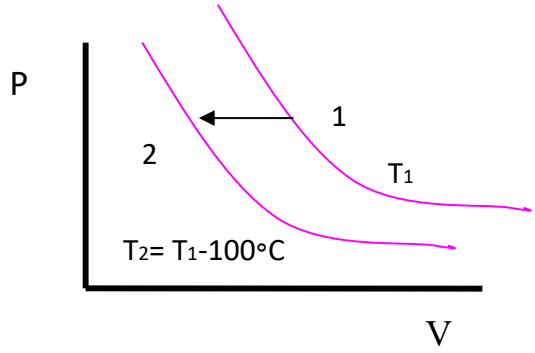
Note that the net work is negative because it is being done on the system during the compression process. The work is leaving the system.

10) Air undergoes a constant pressure cooling process in which the temperature decreases by 100°C.

What is the magnitude and direction of the work for this process? Assume that air behaves as an ideal gas and that R_{air} = 0.287 kJ/kg·K.

Property relation in this case is ideal gas so that $Pv = RT$. The process is an isobaric cooling.





P-V diagram for T = constant

The work calculations are:

$$W_{net,12} = 0 + W_{b,12} = \int_1^2 PdV = P(V_2 - V_1) = mR(T_2 - T_1)$$

And then the work per unit mass is:

$$w_{net,12} = \frac{W_{net,12}}{m} = R(T_2 - T_1) = 0.287(-100) = -28.7\ kJ/kg$$

11) Starting from the definition of enthalpy, show that $C_p = C_v + R$ for an ideal gas.

Starting with $h = u + Pv$, then differentiate…

$$dh = du + d(RT)$$

Using the definitions of C_p and C_v:

$$C_p dT = C_v dT + RdT$$

So:

$$C_p = C_v + R$$

a) How is the specific heat ratio k defined?

$$k = \frac{C_p}{C_v}$$

b) Find the relationships: (i) C_p in terms of k and R and (ii) C_v in terms of k and R.

The heat ratio is:

$$k = \frac{C_p}{C_v}$$

We have also shown that:

$$C_p - C_v = R$$

Dividing the second relationship by C_v gives:

$$\frac{C_p}{C_v} - 1 = \frac{R}{C_v}$$

Using the expression for the heat ratio:

$$k - 1 = \frac{R}{C_v}$$

Then, based on the heat ratio expression:

$$C_p = kC_v$$

Rearranging:

$$C_v = \frac{R}{k-1}$$

And substituting:

$$C_p = \frac{kR}{k-1}$$

c) Why does $C_p = C_v$ for solids and liquids?

Solids and liquids are treated as incompressible substances. That is, it is assumed that the density or specific volume of the substance is essentially constant during a process. Hence, the specific heats of incompressible substances are identical.

d) Complete the table below (for substances that can be assumed to be ideal gases, where these are properties at 300 K, $R_u = 8.314$ kJ/(kmol·K)).

Substance	Molar mass [kg/kmol]	R [kJ/kg·K]	C_p [kJ/kg·K]	C_v [kJ/kg·K]	k
Air	28.97	0.287	1.005	0.718	1.400
Carbon Dioxide	44.01	0.189	0.846	0.657	1.289
Hydrogen	2.016	4.124	14.31	10.18	1.405
Nitrogen	28.01	0.297	1.039	0.743	1.400
Oxygen	32.00	0.260	0.918	0.658	1.395

Note that there are various options for completing this table, this is just one example.

For air…

$$R = \frac{R_u}{M} \rightarrow M = \frac{R_u}{R} = \frac{8.314}{0.287} = 28.97\ kg/kmol$$

$$C_v = \frac{R}{k-1} = \frac{0.287}{1.4-1} = 0.718\ kJ/kg{\cdot}K$$

$$C_p = C_v + R = 0.718 + 0.287 = 1.005\ kJ/kg{\cdot}K$$

For CO_2…

$$k = \frac{C_p}{C_v} = \frac{0.846}{0.657} = 1.288$$

$$C_p = C_v + R \rightarrow R = C_p - C_v = 0.846 - 0.657 = 0.189\ kJ/kg{\cdot}K$$

$$R = \frac{R_u}{M} \rightarrow M = \frac{R_u}{R} = \frac{8.314}{0.189} = 43.99\ kg/kmol$$

For H_2...

$$R = \frac{R_u}{M} = \frac{8.314}{2.016} = 4.124\ kJ/kg{\cdot}K$$

$$C_p = C_v + R = 10.18 + 4.124 = 14.30\ kJ/kg{\cdot}K$$

$$k = \frac{C_p}{C_v} = \frac{14.30}{10.18} = 1.405$$

For N_2...

$$R = \frac{R_u}{M} = \frac{8.314}{28.01} = 0.297\ kJ/kg{\cdot}K$$

$$C_v = C_p - R = 1.039 - 0.297 = 0.742\ kJ/kg{\cdot}K$$

$$k = \frac{C_p}{C_v} = \frac{1.039}{0.742} = 1.400$$

For O_2...

$$k = \frac{C_p}{C_v} \rightarrow C - v = \frac{C_p}{k} = \frac{0.918}{1.395} = 0.658\ kJ/kg{\cdot}K$$

$$R = C_p - C_v = 0.918 - 0.658 = 0.260\ kJ/kg{\cdot}K$$

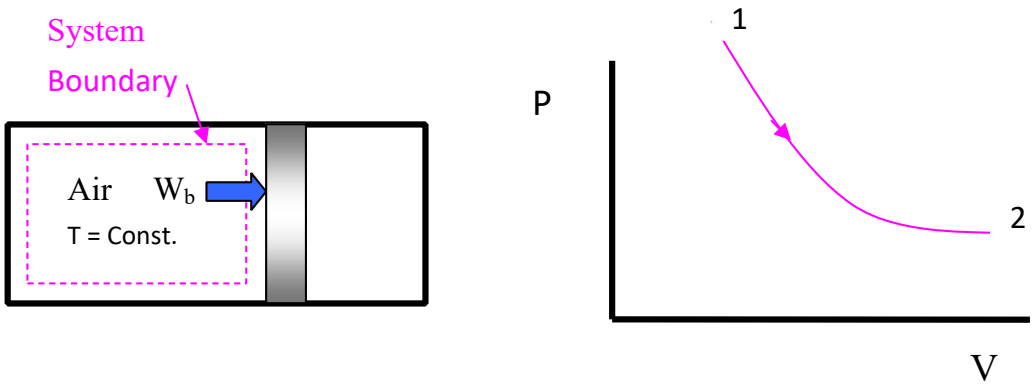
$$M = \frac{R_u}{R} = \frac{8.314}{0.260} = 31.98\ kg/kmol$$

12) Air is expanded isothermally at 100°C from 0.4 MPa to 0.1 MPa.

Find the ratio of the final to initial volume, the heat transfer, and the work. Assume air behaves as an ideal gas and use the ideal gas property relations with constant specific heats and R = 0.287 kJ/(kg·K).

The system is air contained in a piston-cylinder device (closed system). The process is isothermal. Assume air is an ideal gas and use the ideal gas property relations with constant specific heats.

$$PV = mRT, \quad \Delta u = C_v \Delta T = C_v(T_2 - T_1)$$



P-V diagram for T = constant

Conservation of energy equations:

$$E_{in} - E_{out} = \Delta E$$

$$Q_{net} - W_{net} = \Delta U$$

The system mass is constant but is not given and cannot be calculated, therefore calculate work and heat transfer per unit mass.

$$W_{net,12} = (W_{net})_{other,12} + W_{b,12}$$

$$W_{b,12} = \int_1^2 PdV = \int_1^2 \frac{mRT}{V} dV = mRT \ln\left(\frac{V_2}{V_1}\right)$$

For an ideal gas system with constant mass, we have:

$$m_1 = m_2, \quad \frac{P_1 V_1}{RT_1} = \frac{P_2 V_2}{RT_2}$$

For an isothermal process, $T_1 = T_2$ and the R cancels out.

$$\frac{V_2}{V_1} = \frac{P_1}{P_2} = \frac{0.4}{0.1} = 4$$

Then the expression for work per unit mass becomes:

$$w_{b,12} = 0.287 * (100 + 273) \ln(4) = 148.3\ kJ/kg$$

Then the expression for the net work per unit mass is:

$$w_{net,12} = 0 + w_{b,12} = 148.4\ kJ/kg$$

Now we continue with the conservation of energy to find the heat transfer. Remember here that $T_2 = T_1 =$ constant. Therefore:

$$\Delta U_{12} = m\Delta u_{12} = mC_v(T_2 - T_1) = 0$$

So the heat transfer per unit mass is:

$$q_{net} = \frac{Q_{net}}{m}$$

$$q_{net} - w_{net} = \Delta u = 0$$

$$q_{net} = w_{net} = 148.4\ kJ/kg$$

During an isothermal expansion, the heat transfer to the air is equal to the work done.

13) What makes a device steady-state and steady-flow?

See Thermodynamics 2 notes. Many energy conversion devices operate steadily over long periods of time. A process during which a fluid flows through a control volume steadily is called a steady-flow process. Under these conditions, the mass content of the control volume is constant with time:

$$\frac{dm_{cv}}{dt} = \Delta\dot{m}_{cv} = 0$$

Under steady-state, steady-flow conditions the mass and energy contents of a control volume remain constant, and the fluid properties at an inlet or exit also remain constant when the process operates under steady-state conditions.

a) State the conservation of mass for a general control volume and show how this is applied to a general steady-state, steady-flow device.

See Thermodynamics 2 notes.

b) What is flow work and how does it affect the energy of a flowing fluid?

See Thermodynamics 2 notes.

c) State the conservation of energy for a general control volume and show how it is applied to a steady-state, steady-flow device with one inlet and one exit.

See Thermodynamics 2 notes.

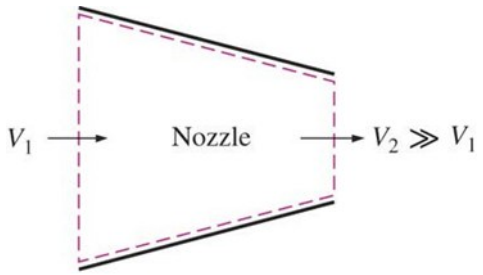
d) Why are kinetic and potential energy often neglected in calculations related to steady-state, steady-flow devices?

See Thermodynamics 2 notes.

14) Steam at 0.4 MPa, 300°C enters an adiabatic nozzle with a low velocity and leaves at 0.2 MPa with a quality of 90 %.

Find the exit velocity in m/s.

The control volume is the nozzle. Use property relations from steam tables. For the process, assume a steady state, steady flow process.



Use the conservation principles for mass and energy. According to the sketched control volume, the mass crosses the control surface, but no work or heat transfer crosses the control surface. Hence, neglecting potential energies, we have:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \rightarrow \dot{m}_1 = \dot{m}_2 = \dot{m}$$

$$\dot{E}_{in} = \dot{E}_{out} \rightarrow \dot{m}\left(h_1 + \frac{\vec{V}_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{\vec{V}_2^2}{2}\right)$$

Neglecting the inlet kinetic energy, the exit velocity is

$$\vec{V}_2 = \sqrt{2(h_1 - h_2)}$$

Now we need to use the steam tables to find the enthalpies.

For state 1, $T_1 = 300°C$ *and* $P_1 = 0.4\ MPa = 4\ bar$*. From table 3,* $h_1 = 3067.1\ kJ/kg$*.*

For state 2, $P_2 = 0.2\ MPa = 2\ bar, x_2 = 0.9$*. From table 2,* $h_f = 504.68\ kJ/kg$ *and* $h_{fg} = 2201.6\ kJ/kg$*. Therefore:*

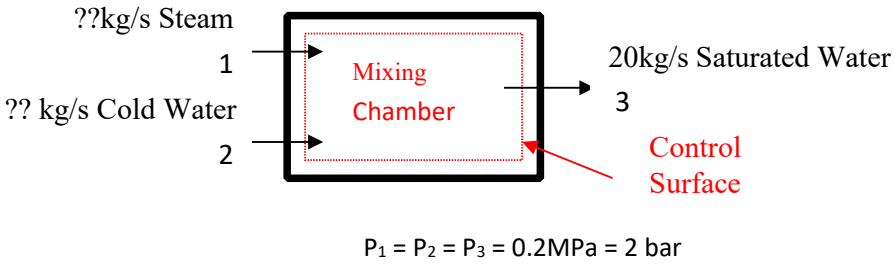
$$h_2 = h_f + xh_{fg} = 504.68 + 0.9 * 2201.6 = 2486.1\ kJ/kg$$

Now we just substitute the values into the expression above…

$$\vec{V}_2 = \sqrt{2(h_1 - h_2)} = 2\sqrt{2 * (3067.1 - 2486.1)} * \frac{1000\frac{m^2}{s^2}}{\frac{kJ}{kg}} = 1078\ m/s$$

15) Steam at 0.2 MPa, 300°C enters a mixing chamber and is mixed with cold water at 20°C, 0.2 MPa to produce 20 kg/s of saturated liquid water at 0.2 MPa.

What are the required steam and cold water flow rates?



Here the control volume is the mixing chamber, the properties need to be found from the steam tables. For the process, assume a steady state, steady flow, adiabatic mixing, and no work.

Apply conservation principles for mass and energy:

$$\sum \dot{m}_{in} = \sum \dot{m}_{out} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3 \rightarrow \dot{m}_2 = \dot{m}_3 - \dot{m}_1$$

$$\dot{Q}_{in} + \sum \left[\dot{m}_e \left(h_e + \frac{\vec{V}_e^2}{2} + gz_e\right)\right] = \dot{W}_{net} + \sum \left[\dot{m}_i \left(h_i + \frac{\vec{V}_i^2}{2} + gz_i\right)\right]$$

where the sum with the e indices is the sum of all exits and the sum with i indices is the sum of all inlets.

According to the sketched control volume, mass crosses the control surface. Neglecting kinetic and potential energies and noting that the process is adiabatic with no work, we have for two inlets and one exit the following expression:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \rightarrow \dot{m}_1 h_1 + (\dot{m}_3 - \dot{m}_2) h_2 = \dot{m}_3 h_3$$

$$\dot{m}_1 (h_1 - h_2) = \dot{m}_3 (h_3 - h_2)$$

$$\dot{m}_1 = \frac{\dot{m}_3 (h_3 - h_2)}{h_1 - h_2}$$

Now we use the steam tables to find the enthalpies:

At state 1, $T_1 = 300°C, P_1 = 2\ bar$. *From table 3,* $h_1 = 3072.1\ kJ/kg$.

At state 2, $T_2 = 20°C, P_2 = 2\ bar$. *Subcooled liquid approximation:* $h_2 = h_{f@20°C} = 83.92\ kJ/kg$ *or from table 3* $h_2 = 84.11\ kJ/kg$.

At state 3, P_3 *is* 2 *bar and it is a saturated liquid. Therefore* $h_3 = h_{f@2bar} = 504.68\ kJ/kg$.

Now substitute into the equation above to get mass flowrate $\dot{m}_1$.

$$\dot{m}_1 = \frac{\dot{m}_3 (h_3 - h_2)}{h_1 - h_2} = 20 * \frac{504.68 - 84.11}{3072.1 - 84.11} = 2.82\ kg/s$$

$$\dot{m}_2 = \dot{m}_3 - \dot{m}_1 = 17.18\ kg/s$$

The second law of thermodynamics

16) The only significant energy input into a power cycle is 150 MW of heat energy. The cycle produces 50 MW of net work.

Determine the cycle thermal efficiency and the heat rejected by the cycle to the surroundings.

$$\eta_{th} = \frac{W_{net,out}}{Q_H} = \frac{50}{150} = 33.3\ \%$$

$$W_{net,out} = Q_H - Q_L \rightarrow Q_L = Q_H - W_{net,out} = 150 - 50 = 100\ MW$$

17) A Carnot heat engine receives 500 kJ of heat per cycle from a high-temperature heat reservoir at 652°C and rejects heat to a low-temperature heat reservoir at 30°C.

Determine the thermal efficiency of this Carnot engine and the amount of heat rejected to the low-temperature heat reservoir.

In this case, the Carnot thermal efficiency can be expressed like so:

$$\eta_{th,rev} = 1 - \frac{T_L}{T_H} = 1 - \frac{30 + 273}{652 + 273} = 67.2\ \%$$

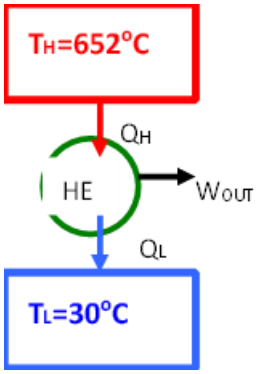
To evaluate the heat lost…

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} = \frac{30 + 273}{652 + 273} = 0.328$$

$$Q_L = Q_H * 0.328 = 500 * 0.328 = 164\ kJ$$

18) An inventor claims to have developed a refrigerator with a COP of 13.5 that maintains the refrigerated space at 2°C while operating in a room where the temperature is 25°C.

Evaluate the maximum possible (Carnot) COP of a refrigerator operating in these conditions. Is the inventor's claim true?



$$COP_R = \frac{Q_L}{Q_H - Q_L} = \frac{T_L}{T_H - T_L} = \frac{2 + 273}{25 - 2} = 11.96$$

The claim is false, since no refrigerator can have a lower COP than that of a reversed Carnot device.

19) 3150 kJ of heat is transferred into a cycle at 440°C. 1294.46 kJ of heat is rejected from the same cycle at 20°C. Heat transfer occurs at a constant temperature.

Is the Clausius inequality satisfied and is the cycle reversible or irreversible? Calculate the net work and cycle efficiency for this cycle.

$$\oint \frac{\delta Q_{net}}{T} \leq 0$$

$$\int \left(\frac{\delta Q_{net}}{T}\right)_{in} + \int \left(\frac{\delta Q_{net}}{T}\right)_{out} \leq 0$$

$$\frac{Q_{in}}{T_{in}} + \frac{-Q_{out}}{T_{out}} \leq 0$$

$$\frac{3150}{440 + 273} + \frac{-1294.46}{20 + 273} \leq 0$$

$$4.418 - 4.418 = 0$$

So the inequality is satisfied in this case. Now calculate the work and efficiency of the cycle:

$$W_{net} = Q_{in} - Q_{out} = 3150 - 1294.46 = 1855.4\ kJ$$

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{1855.54}{3150} = 58.9\ \%$$

20) Complete the table using steam tables.

Pressure	T [°C]	State	Specific entropy s [kJ/kg·K]
5 MPa	120	*Subcooled*	1.523
1 MPa	50	*Subcooled*	0.701
1.8 MPa	100	*Subcooled*	1.306
40 kPa	75.86	*Wet*, x=0.9	7.005
40 kPa	75.86	*Wet*, x=0.926	7.1794

Line 1: Look up values in Table 3

Line 2: Interpolation in Table 3, halfway values between 40°C *and* 60°C*.*

$$\frac{0.572 + 0.8307}{2} = 0.701\ kJ/kgK$$

Line 3: Interpolation in Table 3

Line 4: Use information from Table 2

$$s_f = 1.0259\ kJ/kgK$$

$$s_g = 7.669\ kJ/kgK$$

$$s_{0.9} = s_f + 0.9(s_g - s_f) = 7.005\ kJ/kgK$$

Line 5: Use information from table 2

$$s_f = 1.0259\ kJ/kgK$$

$$s_g = 7.669\ kJ/kgK$$

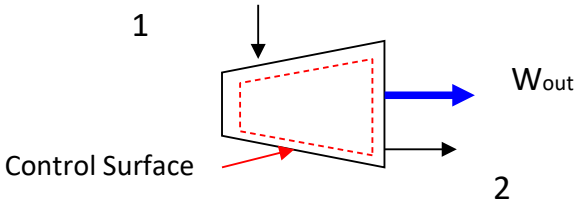
$$s = 7.1794\ kJ/K$$

$$x = \frac{s - s_f}{s_g - s_f} = 0.926$$

21) Steam at 1 MPa, 600°C expands in a turbine to 0.01 MPa.

If the process is isentropic, find the final temperature, final specific enthalpy of the steam and the turbine's specific work. You will need to use steam tables to solve this problem.

The control volume here is formed by the turbine. Use properties found from steam tables. The process is isentropic, steady state, steady flow. There is only one entrance and one exit. Neglect kinetic energy and potential energy.



Apply the conservation of mass principles…

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_3$$

Apply the first law of conservation of energy. Assume that the process is isentropic due to being adiabatic and reversible and also that changes in potential and kinetic energy are negligible. In this case, the steady flow equation is:

$$\dot{Q}_{net} - \dot{W}_{net} = \dot{m}(h_2 - h_1)$$

$$\dot{W}_{net} = \dot{m}(h_1 - h_2)$$

$$w_{net} = \frac{\dot{W}_{net}}{\dot{m}_{net}} = h_1 - h_2$$

Now we use the steam tables to find the temperature and enthalpy and hence the specific work of the turbine. Using Table 3:

$$h_1 = 3698.6\ kJ/kg$$

$$s_1 = 8.031\ kJ/kgK$$

This is an isentropic process, so $s_2 = s_1 = 8.031\ kJ/kg$. *Now at* $P_2 = 0.01\ MPa = 0.1\ bar$. *Use Table 2…*

$$s_f = 0.6492\ kJ/kgK$$

$$s_g = 8.149\ kJ/kgK$$

This tells us that the steam is wet at the turbine exit, since $s_f < s < s_g$. $T_2 = 45.8°C$ *since steam is saturated (as seen in Table 2).*

$$x_2 = \frac{s - s_f}{s_g - s_f} = \frac{8.301 - 0.6492}{8.149 - 0.6492} = 0.984$$

$$h_2 = h_f + xh_g = 191.8 + 0.984 * 2392.1 = 2546\ kJ/kg$$

$$w_{out} = h_1 - h_2 = 3699 - 2546 = 1153\ kJ/kg$$

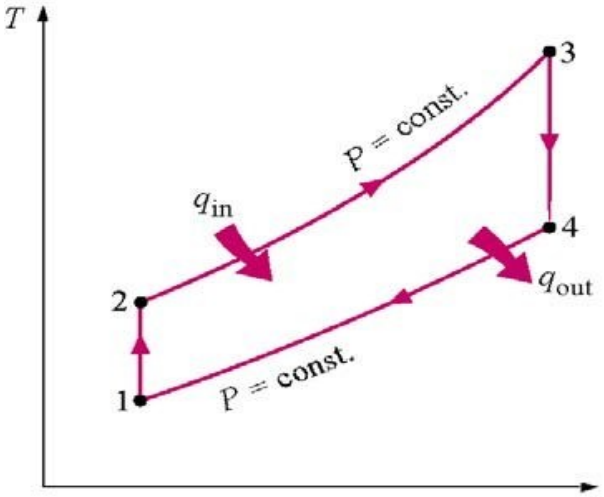
Analysis of thermodynamic cycles

For all of the questions, air-standard assumptions are used.

For the ideal Brayton cycle, assume cold-standard air: C_v = 0.718 kJ/kg·K, C_p = 1.005 kJ/kg·K, R = 0.287 kJ/kg·K, and k = 1.4.

22) An ideal closed Brayton cycle has a pressure ratio of 8.56. Air enters the compressor at 300 K and has a temperature of 1100 K at the turbine inlet.

a) Draw the T-s diagram of the Brayton cycle.



(a) T-s diagram

b) Determine:

$$r_p = 8.56, \quad T_1 = 300\ K, \quad T_3 = 1100\ K$$

i) The gas temperature at the exits of the compressor and the turbine

$$r_p = \frac{P_2}{P_1} = \frac{P_3}{P_4}$$

Both the compressor and the turbine are isentropic, so can apply these expressions:

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}, \quad T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}}$$

$$T_2 = T_1 r_p^{\frac{k-1}{k}} = 554\ K, \quad T_4 = T_3 \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}} = 596\ K$$

ii) Work input to the compressor and work output of the turbine

$$w_c = C_p(T_2 - T_1) = 255\ kJ/kg$$

$$w_t = C_p(T_4 - T_3)$$

$$w_t = -507\ kJ/kg$$

iii) The back work ratio

The expression for the back work ratio is

$$bwr = -\frac{w_c}{w_t} = 0.504$$

iv) The net heat input and net heat output of the heat exchangers

$$q_{in} = C_p(T_3 - T_2) = 549\ kJ/kg$$

$$q_{out} = C_p(T_4 - T_1) = 297\ kJ/kg$$

v) The thermal efficiency of the power cycle.

$$\eta = 1 - \frac{q_{out}}{q_{in}} = 0.459$$

Or can also use...

$$w_{net} = w_c + w_t = -252\ kJ/kg$$

$$\eta = -\frac{w_{net}}{q_{in}} = 0.459$$

23) An ideal closed Brayton cycle has a pressure ratio of 6 and maximum temperature of 1200 K. If the heat input (process 2-3) is 700 kJ/kg.

$q_{in} = 700\ kJ/kg$, $T_3 = 1200\ K$, $r_p = 6$

Determine:

a) The compressor exit temperature

$$q_{in} = C_p(T_3 - T_2) \rightarrow T_2 = T_3 - \frac{q_{in}}{C_p} = 503\ K$$

b) The compressor inlet temperature

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \rightarrow T_1 = \frac{T_2}{\left(r_p\right)^{\frac{k-1}{k}}} = 302\ K$$

c) The turbine exit temperature

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} \rightarrow T_4 = T_3\left(\frac{1}{r_p}\right)^{\frac{k-1}{k}} = 719\ K$$

d) The back work ratio

$$w_c = C_p(T_2 - T_1) = 203\ kJ/kg$$
$$w_t = C_p(T_4 - T_3) = -483\ kJ/kg$$
$$bwr = -\frac{w_c}{w_t} = 0.42$$

e) The thermal efficiency of the power cycle

$$q_{in} = C_p(T_3 - T_2) = 700\ kJ/kg$$
$$q_{out} = C_p(T_4 - T_1) = 420\ kJ/kg$$
$$w_{net} = w_c + w_t = -280\ kJ/kg$$
$$\eta = 1 - \frac{q_{out}}{q_{in}} = -\frac{w_{net}}{q_{in}} = 0.401$$

24) What is the Carnot vapour cycle and why is it not used in reality?

See Thermo2 lecture notes.

a) What is the simple ideal Rankine cycle and how does it address the practical difficulties associated with implementing the Carnot vapour cycle?

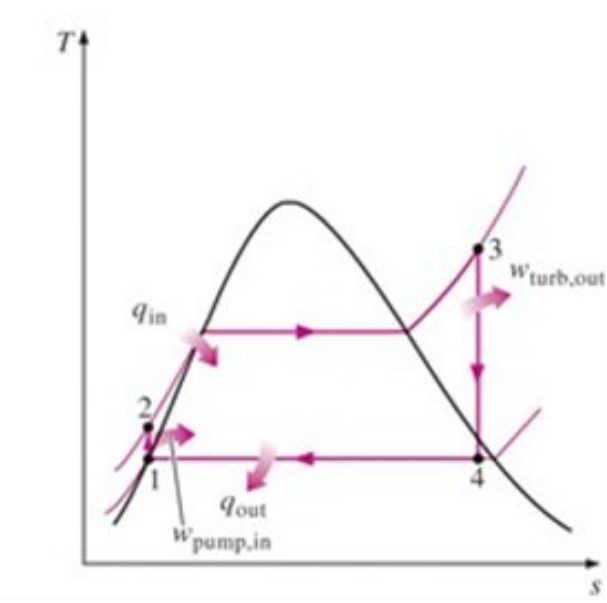
See Thermo2 lecture notes.

b) What approaches can be used to improve the efficiency of the simple ideal Rankine cycle?

See Thermo2 lecture notes.

25) Consider a steam power plant operating on the simple ideal Rankine cycle. Steam enters the turbine at 3 MPa and 350°C and is condensed in the condenser at a pressure of 75 kPa.

Determine the thermal efficiency of this cycle using steam tables to find water/steam properties.



From the question we know that $P_2 = P_3 = 30\ bar, P_1 = P_4 = 0.75\ bar$.

To determine thermal efficiency, we will need to calculate the enthalpy of steam at different points in the cycle to calculate work/heat transfers. To determine enthalpy, we will also need to consider the state of the steam at different points in the cycle.

The pump work…

$$v_1 = v_f = 0.001037\ m^3/kg$$

$$h_1 = h_f = 384.365\ kJ/kg$$

$$h_2 = h_1 + w_{pump}$$

$$w_{pump} = v_1(p_2 - p_1) = 3.03\ kJ/kg$$

$$h_2 = 387.399\ kJ/kg$$

Heat input into the boiler…

$$h_3 = h_{reg2}(P_3, T_3) = 3.116 * 10^3\ kJ/kg$$

$$q_{in} = h_3 - h_2 = 2.728 * 10^3\ kJ/kg$$

$$s_3 = s_{reg2}(P_3, T_3) = 6.744\ kJ/kgK$$

Work output of the turbine…

$$s_4 = s_3, \quad P_4 = 0.75\ bar$$

$$s_f = 1.2130\ kJ/kgK$$

$$s_g = 7.456\ kJ/kgK$$

$$x_4 = \frac{s_4 - s_f}{s_g - s_f} == 0.886$$

$$h_{f4} = 384.365\ kJ/kg$$

$$h_{g4} = 2.662 * 10^3 kJ/kg$$

$$h_4 = h_{f4} + x_4\left(h_{g4} - h_{f4}\right) = 2.403 * 10^3\ kJ/kg$$

$$w_{turb} = h_3 - h_4 = 712.998\ kJ/kg$$

Net work…

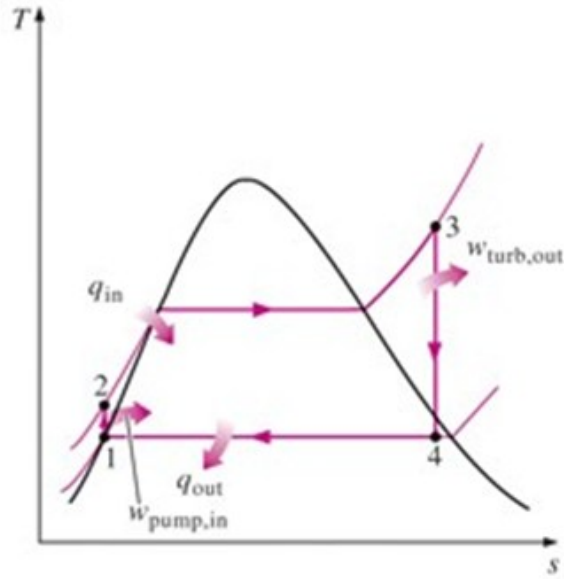
$$w_{net} = w_{turb} - w_{comp} = 709.965\ kJ/kg$$

Efficiency…

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 0.26 = 1 - \frac{q_{out}}{q_{in}}, \quad q_{out} = h_4 - h_1 = 2.018 * 10^3\ kJ/kg$$

26) Wet steam in an ideal Rankine cycle exits the turbine at 30 kPa with a dryness fraction of 90%. The pump work is 5 kJ/kg and 3000 kJ/kg of heat are transferred into the cycle in the boiler.

Calculate the net work done by the cycle and the cycle net thermal efficiency using steam tables to find water/steam properties.



From the question, we know… $P_4 = 30\ kPa, x_4 = 90\ \%,\ w_{pump} = 5\ kJ/kg$ *and* $q_{boiler} = 3000\ kJ/kg$*. As in the previous question, analyse each process in the steam cycle, in turn, to determine the system properties at different points in the cycle and hence the net work and the cycle efficiency.*

Pump work…

$$P_1 = P_4 = 30\ kPa = 0.3\ bar$$

$$h_1 = h_{f1} = 289.229\ kJ/kg$$

$$h_2 = w_{pump} + h_1 = 294.229\ kJ/kg$$

Heat input into boiler…

$$q_{boiler} = h_3 - h_2 \rightarrow h_3 = q_{boiler} + h_2 = 3.294 * 10^3\ kJ/kg$$

Work done by the turbine…

$$h_4 = h_f + x_4 h_{fg} = h_f + x_4\left(h_{g4} - h_{f4}\right) = 2.391 * 10^3\ kJ/kg$$

$$w_{turbine} = h_3 - h_4 = 903.21\ kJ/kg$$

Net work…

$$w_{net} = w_{turbine} - w_{pump} = 898.21\ kJ/kg$$

Thermal efficiency…

$$\eta_{th} = \frac{w_{net}}{q_{boiler}} = 0.299$$