Thermodynamics 4
Tutorial 2 – Power Cycles 1 (Solutions)

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1) Back work ratio is the ratio of the compressor (or pump) work input to the turbine work output. It is usually between 0.40 and 0.6 for gas turbine engines.

2) In gas turbine engines, a gas is compressed, and thus the compression work requirements are very large since the steady-flow work is proportional to the specific volume.

3) As a result of turbine and compressor inefficiencies, (a) the back work ratio increases, and (b) the thermal efficiency decreases.

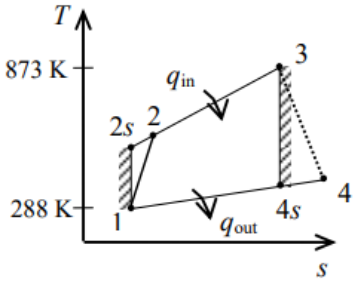
4) **Consider a simple Brayton cycle using air as the working fluid; it has a pressure ratio of 12; has a maximum cycle temperature of 600°C; and operates the compressor inlet at 100 kPa and 15°C. Which will have the greatest impact on the back work ratio: a compressor isentropic efficiency of 80 percent or a turbine isentropic efficiency of 80 percent? Use constant specific heats at room temperature.**

Assumptions

1. Steady operating conditions exist.
2. The air-standard assumptions are applicable.
3. Kinetic and potential energy changes are negligible.
4. Air is an ideal gas with constant specific heats.

Properties The properties of air at room temperature are $c_p = 1.005\ \text{kJ/kg} \cdot K$ and $k = 1.4$.

Analysis For the compression process,



$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (288\text{ K})(12)^{0.4/1.4} = 585.8\text{ K}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)} \quad \longrightarrow \quad T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_C}$$

$$= 288 + \frac{585.8 - 288}{0.80}$$

$$= 660.2\text{ K}$$

For the expansion process,

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = (873\text{ K})\left(\frac{1}{12}\right)^{0.4/1.4} = 429.2\text{ K}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{c_p(T_3 - T_4)}{c_p(T_3 - T_{4s})} \quad \longrightarrow \quad T_4 = T_3 - \eta_T(T_3 - T_{4s})$$

$$= 873 - (0.80)(873 - 429.2)$$

$$= 518.0\text{ K}$$

The isentropic and actual work of the compressor and turbine are

$$W_{\text{Comp},s} = c_p(T_{2s} - T_1) = (1.005\text{ kJ/kg}\cdot K)(585.8 - 288)K = 299.3\text{ kJ/kg}$$

$$W_{\text{Comp}} = c_p(T_2 - T_1) = (1.005\text{ kJ/kg}\cdot K)(660.2 - 288)K = 374.1\text{ kJ/kg}$$

$$W_{\text{Turb},s} = c_p(T_3 - T_{4s}) = (1.005\text{ kJ/kg}\cdot K)(873 - 429.2)K = 446.0\text{ kJ/kg}$$

$$W_{\text{Turb}} = c_p(T_3 - T_4) = (1.005\text{ kJ/kg}\cdot K)(873 - 518.0)K = 356.8\text{ kJ/kg}$$

The back work ratio for an 80% efficient compressor and an isentropic turbine case is

$$r_{\text{bw}} = \frac{W_{\text{Comp}}}{W_{\text{Turb},s}} = \frac{374.1\text{ kJ/kg}}{446.0\text{ kJ/kg}} = 0.8387$$

The back work ratio for the 80% efficient turbine and isentropic compressor case is

$$r_{\text{bw}} = \frac{W_{\text{Comp},s}}{W_{\text{Turb}}} = \frac{299.3\text{ kJ/kg}}{356.8\text{ kJ/kg}} = 0.8387$$

The two results are identical.

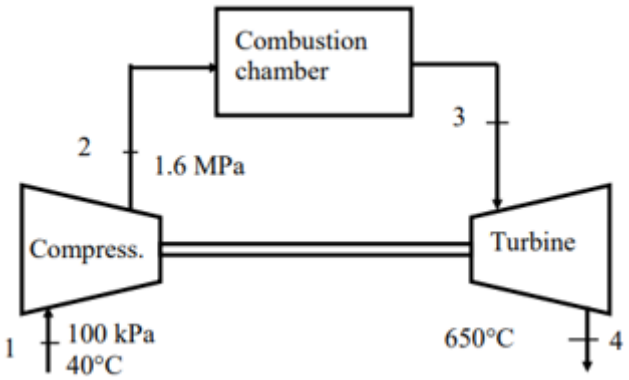
5) A gas-turbine power plant operates on the simple Brayton cycle between the pressure limits of 100 and 1600 kPa. The working fluid is air, which enters the compressor at 40°C at a rate of 850 m3/min and leaves the turbine at 650°C. Assuming a compressor isentropic efficiency of 85 percent and a turbine isentropic efficiency of 88 percent, determine (a) the net power output, (b) the back work ratio, and (c) the thermal efficiency. Use constant specific heats with c_v = 0.821 kJ/kg·K, c_p = 1.108 kJ/kg·K, and k = 1.35.

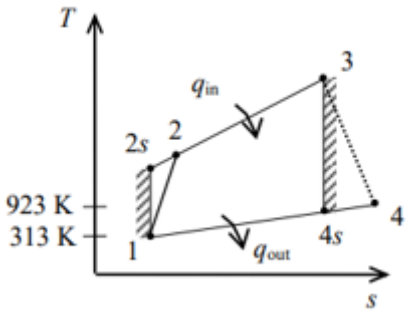
A gas-turbine plant operates on the simple Brayton cycle. The net power output, the back work ratio, and the thermal efficiency are to be determined.

Assumptions

1. The air-standard assumptions are applicable.
2. Kinetic and potential energy changes are negligible.
3. Air is an ideal gas with variable specific heats.

Properties The gas constant of air is $R = 0.287\ \text{kJ/kg} \cdot K$ (Table A-1). Other properties of air are given as $c_p = 1.108\ \text{kJ/kg}.K$, $c_v = 0.821\ \text{kJ/kg}.K$, and $k = 1.35$.





Analysis

(a) For the compression process,

$$r_p = \frac{P_2}{P_1} = \frac{1600\ \text{kPa}}{100\ \text{kPa}} = 16$$

$$T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (313\ \text{K})(16)^{0.35/1.35} = 642.3\ \text{K}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1}$$

$$= \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)} \quad \longrightarrow \quad T_2 = T_1 + \frac{T_{2s} - T_1}{\eta_C} = 313 + \frac{642.3 - 313}{0.85} = 700.4\ \text{K}$$

For the expansion process,

$$T_{4s} = T_3\left(\frac{P_4}{P_3}\right)^{(k-1)/k} = T_3\left(\frac{1}{16}\right)^{0.35/1.35} = 0.4873T_3$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{c_p(T_3 - T_4)}{c_p(T_3 - T_{4s})} \quad \longrightarrow \quad T_4 = T_3 - \eta_T(T_3 - T_{4s})$$

$$923 = T_3 - (0.88)(T_3 - 0.4873T_3)$$

$$T_3 = 1682\ \text{K}$$

The mass flow rate is determined from

$$\dot{m} = \frac{P_1\dot{V}_1}{RT_1} = \frac{(100\ \text{kPa})(850/60\ \text{m}^3/s)}{(0.287\ \text{kPa}\cdot m^3/\text{kg}\cdot K)(40 + 273\ \text{K})} = 15.77\ \text{kg/s}$$

The net power output is

$$\dot{W}_{\text{C,in}} = \dot{m}c_p(T_2 - T_1) = (15.77\ \text{kg/s})(1.108\ \text{kJ/kg}\cdot K)(700.4 - 313)K$$

$$= 6769\ \text{kW}$$

$$\dot{W}_{\text{T,out}} = \dot{m}c_p(T_3 - T_4) = (15.77\ \text{kg/s})(1.108\ \text{kJ/kg}\cdot K)(1682 - 923)K$$

$$= 13{,}257\ \text{kW}$$

$$\dot{W}_{\text{net}} = \dot{W}_{\text{T,out}} - \dot{W}_{\text{C,in}} = 13{,}257 - 6769 = 6488\ kW$$

(b) The back work ratio is

$$r_{\text{bw}} = \frac{\dot{W}_{\text{C,in}}}{\dot{W}_{\text{T,out}}} = \frac{6769\ \text{kW}}{13{,}257\ \text{kW}} = 0.511$$

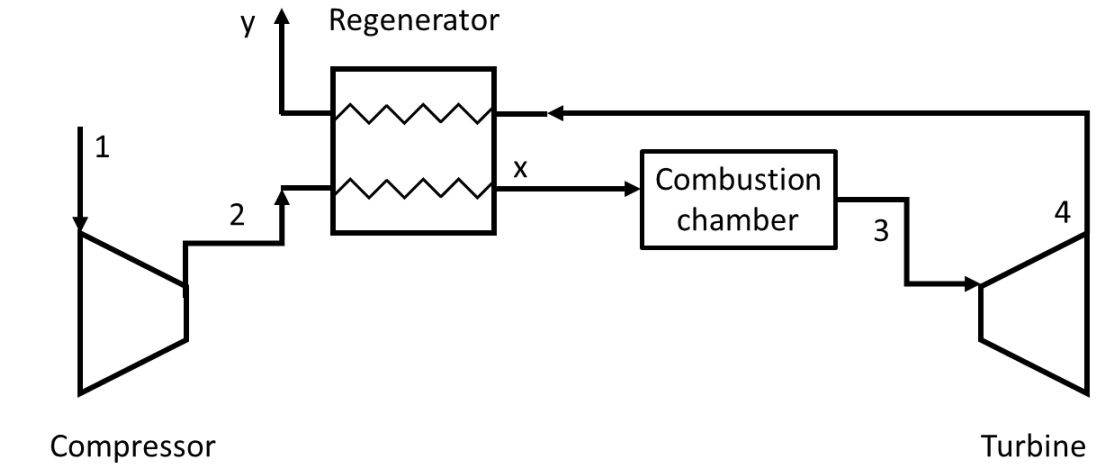
(c) The rate of heat input and the thermal efficiency are

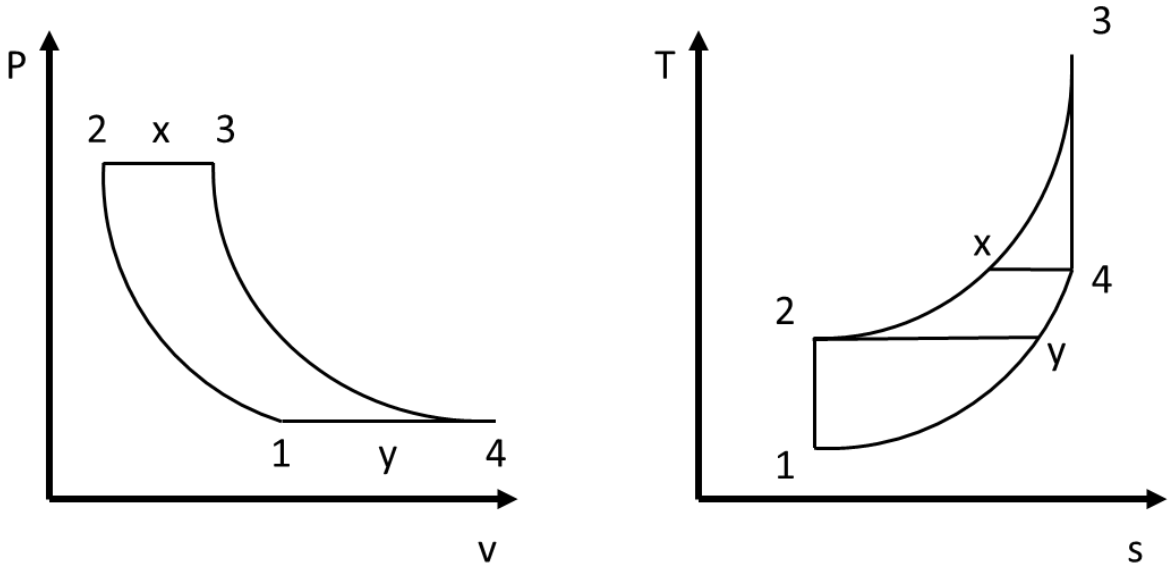
$$\dot{Q}_{\text{in}} = \dot{m}c_p(T_3 - T_2) = (15.77\ \text{kg/s})(0.821\ \text{kJ/kg} \cdot K)(1682 - 700.4)K$$
$$= 17{,}147\ \text{kW}$$

$$\eta_{th} = \frac{\dot{W}_{\text{net}}}{\dot{Q}_{\text{in}}} = \frac{6488\ \text{kW}}{17{,}47\ \text{kW}} = 0.378 = 37.8\%$$

6) A Brayton cycle with an ideal regenerator has inlet conditions of 290 K and 90 kPa. The highest pressure and temperature are 1170 kPa and 1800 K, respectively.

Using the cold-air value for heat capacity C_p = 1.004 kJ/kg·K, find the specific heat transfer and the cycle efficiency.





$P_1 = P_4 = 90\ kPa,\ T_1 = 290\ K$

$P_2 = P_3 = 1170\ kPa,\ T_3 = 1800\ K$

First: Want to find the work done in the compressor stage, assuming adiabatic compression…

Assuming $k = 1.4$*…*

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290 \times \left(\frac{1170}{90}\right)^{\frac{(1.4-1)}{1.4}} = 603.5\ \text{K}$$

$$w_c = h_2 - h_1 = C_p\Delta T = 1.004 \times (603.5 - 290) = 314.74\ \text{kJ/kg}$$

Now we want to find the heat added in combustion…

$$q_h = h_3 - h_x = C_p(T_3 - T_x)$$

From the diagrams, we can tell that $T_x = T_4$*. Assume the expansion in the turbine is adiabatic and reversible. Then:*

$$T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}} = 1800 \times \left(\frac{90}{1170}\right)^{\frac{(1.4-1)}{1.4}} = 864.97\ \text{K}$$

Hence…

$$q_h = h_3 - h_x = C_p(T_3 - T_x) = 1.004 \times (1800 - 864.97) = 938.8\ \text{kJ/kg}$$

Where q_h *is the specific heat transfer requested.*

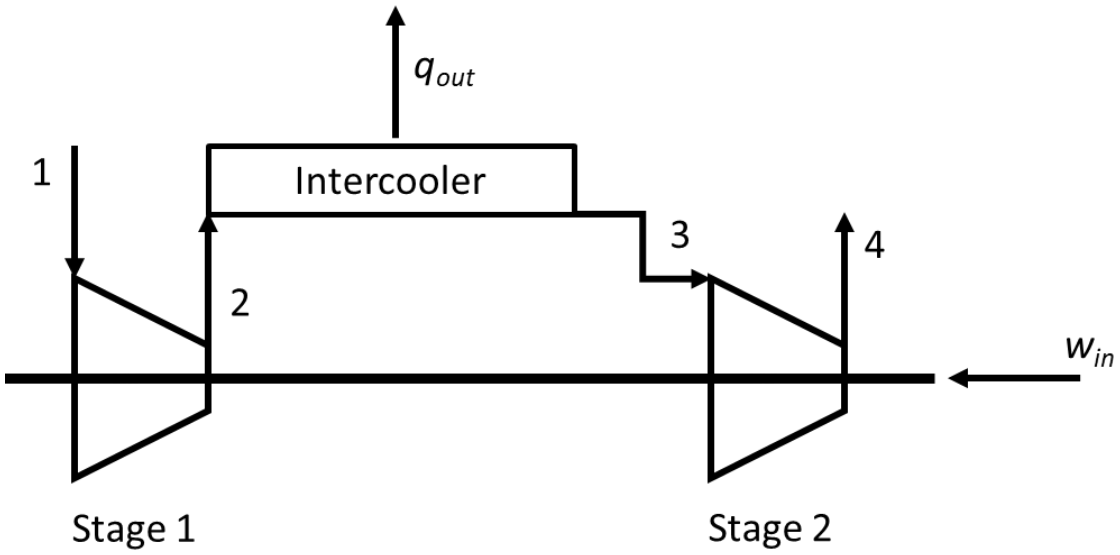
In this case, turbine work $w_t = q_h$*.*

Net work = turbine work – compressor work

Efficiency = Net work/heat transfer into the system

$$\eta = \frac{938.8 - 314.8}{938.8} = 0.666$$

7) A two-stage air compressor has an intercooler between the two stages as shown below. The inlet state is 100 kPa, 290 K, and the final exit pressure is 1.6 MPa. Assume that the constant-pressure intercooler cools the air to the inlet temperature so that $T_1 = T_3$. It can be shown that the optimal pressure P_2 for minimum total compressor work is given by $P_2 = (P_1P_4)^{1/2}$ (you may assume this without the need for proof).



Find the specific compressor work for both stages and the intercooler heat transfer that would give optimal P_2.

$P_1 = 100$ kPa, $T_1 = 290$ K

$P_4 = 1.6$ MPa,

From what is given… $P_2 = (P_1P_4)^{\frac{1}{2}} = (100 \times 1600)^{\frac{1}{2}} = 400$ kPa *for optimum performance.*

Finding the work done by the first compressor stage…

$$w_{c1} = h_2 - h_1 = C_p(T_2 - T_1)$$

We know everything aside from T_2. For optimum performance, the compression must be reversible and adiabatic, we can use…

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290 \times \left(\frac{400}{100}\right)^{\frac{1.4-1}{1.4}} = 430.9 \text{ K}$$

Then…

$$w_{c1} = C_p(T_2 - T_1) = 1.004 \times (430.9 - 290) = 141.5 \text{ kJ/kg}$$

Now need to find the heat transfer for the intercooler. Assume that the intercooler is isobaric so that $P_2 = P_3$. We also know that $T_3 = T_1$.

$$q_{cool} = -(h_3 - h_2) = -C_p(T_3 - T_2) = C_p(T_2 - T_1) = w_{c1} = 141.5 \text{ kJ/kg}$$

Now to find work done by the second compressor stage…

Similarly, we need to find T_4 as the stage 2 outlet using the reversible, adiabatic expression…

$$T_4 = T_3 \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}}$$

But here $P_3 = P_2$ and $T_3 = T_1$, which results in $T_4 = T_2$ and $T_3 = T_1$.

Which means that $w_{c1} = q_{cool} = w_{c2} =$ 141.5 kJ/kg.

8) A two-stage compressor in a gas turbine brings atmospheric air at 100 kPa, 17°C to 500 kPa, then cools it in an intercooler to 27°C at constant pressure. The second stage brings the air to 2500 kPa. Assume both stages are adiabatic and reversible.

Find the combined specific work of the compressor stages. Compare that to the specific work for the case of no intercooler (only one compression from 100 kPa to 2500 kPa)

$P_1 = 100\ kPa,\ T_1 = 290\ K$

$P_2 = 500$ kPa

$P_3 = P_2,\ T_3 = 300\ K$

$P_4 = 2500$ kPa

Conservation of energy for a control volume around the first compressor stage…

$$w_{c1} = h_2 - h_1 = C_p(T_2 - T_1)$$

Where, for an adiabatic, reversible process…

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = 290 \times \left(\frac{500}{100}\right)^{\frac{0.4}{1.4}} = 459.3\ \text{K}$$

Therefore…

$$w_{c1} = 1.004 \times (459.3 - 290) = 169.97\ \text{kJ/kg}$$

For the second compressor stage, the process is the same just with different numbers…

$$w_{c2} = C_p(T_4 - T_3), \quad T_4 = T_3\left(\frac{P_4}{P_3}\right)^{\frac{(k-1)}{k}} = 300 \times \left(\frac{2500}{500}\right)^{\frac{2}{7}} = 175.8\ kJ/kg$$

Hence the total work $w_c = w_{c1} + w_{c2} = 345.8$ kJ/kg.

Consider the case with only one compressor stage…

$$T_5 = T_1\left(\frac{P_5}{P_1}\right)^{\frac{k-1}{k}} = 290 \times \left(\frac{2500}{100}\right)^{\frac{2}{7}} = 727.5\ \text{K}$$

And therefore, the work of a single stage would be…

$$w_{1stage} = C_p(T_5 - T_1) = 1.004 \times (727.5 - 290) = 439\ \text{kJ/kg}$$

So we can see that using an intercooler and two stages requires less specific work compared to using a single-stage compressor without an intercooler.

9) Show that *dh* = *vdP* for an adiabatic, reversible process.

See lecture notes…

10) Recalling that $C_p - C_v = R$ and that $k = C_p/C_v$, show that $(kR)/(k\text{-}1) = C_p$, where C_p is the specific heat capacity at constant pressure, C_v is the specific heat capacity at constant volume, k is the specific heat ratio, and R is the universal gas constant.

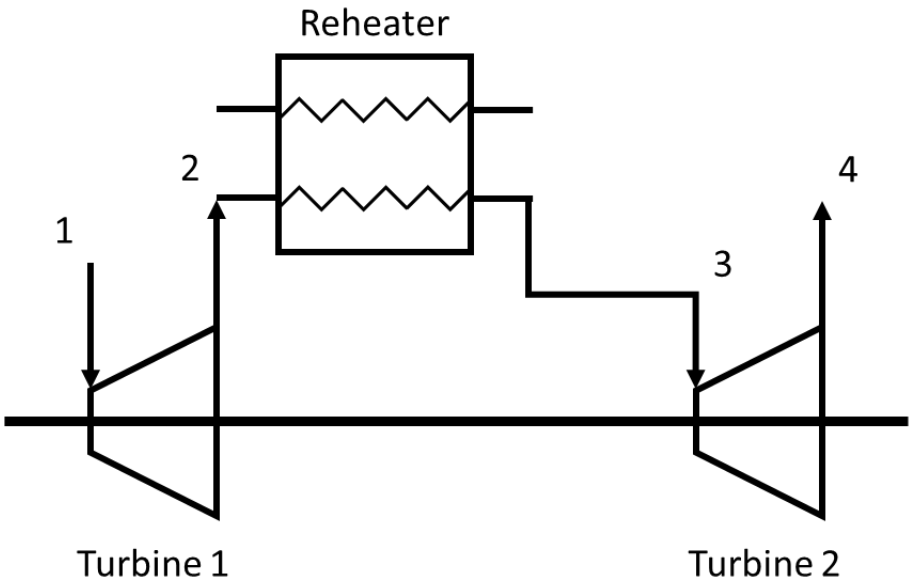
$$C_p - C_v = R, \quad \frac{C_p}{C_v} = k$$

Rearrange to get…

$$C_p - \frac{C_p}{k} = R$$

$$C_p = \frac{R}{1 - \frac{1}{k}} = \frac{kR}{k-1}$$

11) Using the development for an intercooled compressor in the lecture slides, demonstrate that for two consecutive turbines with reheating, the overall turbine work is maximised when the pressure ratio across the turbines is identical, assuming reheating to the high-pressure turbine inlet temperature in an isobaric steady flow process.



Starting with the work output of the high-pressure turbine…

$$w_{t1} = \dot{m}C_p(T_1 - T_2)$$

For a reversible, adiabatic expansion in the turbine…

$$w_{t1} = \dot{m}C_pT_1\left(1 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}\right)$$

Similarly, for the low-pressure turbine…

$$w_{t2} = \dot{m}C_p(T_3 - T_4) = \dot{m}C_pT_3\left(1 - \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}}\right)$$

In reheating we assume that $T_1 = T_3$ and $P_3 = P_2$…

Then the total work of the two turbines is:

$$w_{total} = \dot{m}C_pT_1\left(2 - \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} - \left(\frac{P_4}{P_2}\right)^{\frac{k-1}{k}}\right)$$

For fixed inlet conditions of the high-pressure turbine and fixed exit pressure (T_1, P_1 and P_4 are constant), the maximum work is found by setting the derivative of w_t with respect to P_2 to zero so that:

$$\frac{dw_{total}(P_2)}{dP_2} = 0$$

$$\dot{m}C_pT_1\left[\frac{1}{P_1^{\frac{k-1}{k}}} \times \frac{d}{dP_2}\left(P_2^{\frac{k-1}{k}}\right) + P_4^{\frac{k-1}{k}} \times \frac{d}{dP_2}\left(P_2^{-\frac{k-1}{k}}\right)\right] = 0$$

$$P_1^{-\frac{k-1}{k}} \times \frac{k-1}{k} \times P_2^{-\frac{1}{k}} + P_4^{\frac{k-1}{k}} \times -\frac{k-1}{k} \times P_2^{-2+\frac{1}{k}} = 0$$

$$P_1^{-\frac{k-1}{k}} P_2^{-\frac{1}{k}} = P_4^{\frac{k-1}{k}} P_2^{-2+\frac{1}{k}}$$

$$(P_1P_4)^{-\frac{k-1}{k}} = P_2^{-2+\frac{2}{k}} = P_2^{-2\left(1-\frac{1}{k}\right)} = P_2^{-2\left(\frac{k-1}{k}\right)}$$

$$P_1P_4 = P_2^2$$

And hence…

$$\frac{P_1}{P_2} = \frac{P_2}{P_4} = \frac{P_3}{P_4}$$

12) Assuming isobaric combustion and isentropic compression and expansion, demonstrate that the thermal efficiencies of the standard and regenerative Brayton cycles are:

$$\eta_{th} = 1 - \frac{1}{r_p^{\frac{k-1}{k}}}$$

$$\eta_{th,\ \text{regen}} = 1 - \frac{T_1}{T_a} r_p^{\frac{k-1}{k}}$$

where r_p is the pressure ratio across the compression stage, k is the specific heat ratio of the working fluid, T_1 is the compressor inlet temperature and T_a is the turbine inlet temperature.

See section 10.3 of Borgnakke and Sonntag (8th Ed.)

13) For a standard, ideal Brayton cycle, show that maximum work is achieved when $T_2 = (T_1 T_3)^{1/2}$. Then show that this corresponds to a pressure ratio r_p such that:

$$r_p = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$$

where T_1, T_2 and T_3 are the temperatures of the compressor inlet, compressor outlet and turbine inlet, respectively; r_p is the pressure ratio across the compression stage and k is the specific heat ratio of the working fluid.

Note: Start by writing the work output of the cycle as a function of T_2 and then derive it with respect to T_2.

First, write the net work of the cycle

$$w_{net} = w_{turbine} - w_{compressor} = (h_3 - h_4) - (h_2 - h_1) = C_p(T_3 - T_4 - T_2 + T_1)$$

Note that in 7 we have shown that $T_2/T_1 = T_3/T_4$...

$$w_{net} = C_p\left(T_3 - \frac{T_3 T_1}{T_2} - T_2 + T_1\right)$$

Now derive with respect to T_2...

$$\frac{dw(T_2)}{dT_2} = C_p\left(0 - T_3T_1\left(-\frac{1}{T_2^2}\right) - 1 + 0\right) = 0$$

$$T_3T_1 = T_2^2$$

$$T_2 = (T_3T_1)^{\frac{1}{2}}$$

Now just substitute into the pressure ratio expression...

$$r_p = \frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{(T_3T_1)^{\frac{1}{2}}}{T_1}\right)^{\frac{k}{k-1}} = \left(\left(\frac{T_3}{T_1}\right)^{\frac{1}{2}}\right)^{\frac{k}{k-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}$$

14) An intercooled gas turbine has ambient conditions of 1.013 bar, 15°C, compressor isentropic efficiency 87%, turbine isentropic efficiency 90%, compressor stage ratio 6.6, constant air flow of 200 kg/s, firing temperature 1400°C, temperature after intercooler 15 K above air inlet temperature, pressure drop in intercooler 0.15 bar, pressure drop in combustor 1% of inlet pressure. Effective air properties are C_p = 1.15 kJ/kg·K and k = 1.333.

What are the compressor work, turbine work, combustor heat input and efficiency for:

a) The above configuration?

$$\dot{W} = \dot{m}C_p\Delta T$$

For isentropic compression...

$$\Delta T = T_{2s} - T_1, \quad \frac{T_{2s}}{T_1} = r_p^{\frac{k-1}{k}}$$

Therefore, the work in the first compressor is...

$$\dot{W}_{comp1,s} = \dot{m}C_p(T_{2s} - T_1) = \dot{m}C_pT_1\left(\frac{T_{2s}}{T_1} - 1\right) = \dot{m}C_pT_1\left(r_p^{\frac{k-1}{k}} - 1\right)$$

Now we just substitute into the equation to get the isentropic work of the first compressor stage…

$$\dot{m} = 200\frac{kg}{s}$$

$$C_p = 1.15\frac{kJ}{kgK}$$

$$T_1 = 288\ K$$

$$k = 1.333$$

$$r_p = 6.6$$

$$\dot{W}_{comp1,s} = 39.9\ \text{MW}$$

Now to account for the isentropic efficiency of the compressor and calculate actual work…

$$\dot{W}_{comp1} = \frac{\dot{W}_{comp1,s}}{\eta_c} = \frac{39.9}{0.87} = 45.9\ \text{MW}$$

For the second compressor stage…

$$T_3 = 288 + 15 = 303\ K$$

$$\dot{W}_{comp2,s} = \dot{m}C_pT_3\left(r_p^{\frac{k-1}{k}} - 1\right) = 42\ \text{MW}$$

$$\dot{W}_{comp2} = \frac{42}{0.87} = 48.2\ \text{MW}$$

Now combining the two compressors…

$$\dot{W}_{comp} = 45.9 + 48.2 = 94.1\ \text{MW}$$

Now for the turbine… starting with the isentropic work of the turbine

$$\dot{W}_{T,s} = \dot{m}C_p\Delta T = \dot{m}C_p(T_{6,s} - T_5)$$

$$\frac{T_{6,s}}{T_5} = \left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = \left(\frac{1}{r_p}\right)^{\frac{k-1}{k}}$$

Where $r_p = 6.6$ *and* $P_6 = P_1 = P_{ambient} = 1.013$ *bar. We also know that* $P_5 = 99\%\ P_4$ *and that* $P_4 = r_p P_3$*. Hence…*

$$P_5 = 0.99r_p(P_3) = 0.99r_p(P_2 - \Delta P_{intercooler}) = 0.99r_p\left(r_p P_1 - \Delta P_{intercooler}\right)$$

By substituting everything into the above expression…

$$P_5 = 42.7\ bar$$

Now finding the pressure ratio of the turbine…

$$r_{pt} = \frac{P_5}{P_6} = \frac{42.7}{1.013} = 42.16$$

And the isentropic work of the turbine…

$$\dot{W}_{T,s} = \dot{m}C_p T_5\left(1 - \left(\frac{1}{r_{pt}}\right)^{\frac{k-1}{k}}\right) = 200 \times 1.15 \times 10^3 \times 1673\left(1 - \left(\frac{1}{42.16}\right)^{\frac{1.333-1}{1.333}}\right)$$

$$= 233994.7 \times 10^3\ \text{W} = \ 234\ \text{MW}$$

And accounting for the isentropic efficiency of the turbine to find actual turbine work…

$$\dot{W}_t = 0.9 \times 234 = 210\ \text{MW}$$

Finally, calculating the heat transfer for the combustor…

$$\dot{Q}_{in} = \dot{m}C_p\Delta T$$

Need to find T_4*, the temperature at the outlet of the 2nd compressor…*

$$T_4 = T_3 + \frac{\dot{W}_{comp2}}{\dot{m}C_p} = 513\ K$$

Therefore, the heat transfer into the combustor is…

$$\dot{Q}_{in} = \dot{m}C_p(T_5 - T_4) = \dot{m}C_p(1673 - 513) = 266800\ \text{kW} = \ 266.8\ \text{MW}$$

And finally, finding the efficiency η…

$$\eta = \frac{\dot{W}_t - \dot{W}_{comp}}{\dot{Q}_{in}} = \frac{210 - 94.1}{266.8} = 43.5\ \%$$

b) When a regenerator is added that heats the air to within 150 K of the maximum possible value?

As in the previous case in a), the compressor and turbine powers are the same….

$$\dot{W}_{comp} = 94.1\ MW,\ \dot{W}_t = 210\ \text{MW}$$

The maximum possible air temperature after the regenerator T_7 is the turbine outlet temperature T_6. So we need to find T_6…

$$T_6 = T_5 - \frac{|\dot{W}_t|}{\dot{m}C_p} = 760\ \text{K}$$

The regenerator heats up to within 150 K *of the maximum possible value (T_6)…*

$$T_7 = T_6 - 150 = 610\ \text{K}$$

Then we calculate $\dot{Q}_{in} = \dot{m}C_p(T_5 - T_7) = 200 \times 1.15 \times (1673 - 610) = 245\ \text{MW}$

This value is lower than the heat provided by the combustor in a), because some of the heat is provided by the regenerator instead.

The new efficiency of the cycle is…

$$\eta = \frac{\dot{W}_t - \dot{W}_{comp}}{\dot{Q}_{in}} = \frac{210 - 94.1}{245} = 47.3\ \%$$

c) With a single-stage compressor with the same exit pressure and no regenerator?

For this case, we need to use a similar approach to the case in a), but there is only one compressor stage, so the r_p is changed…

$$r_p = 6.6^2 = 43.6$$

Then the new T_{2s} is found using isentropic compression calculations…

$$\frac{T_{2s}}{T_1} = r_p^{\frac{k-1}{k}}$$

$$T_{2s} = T_1 r_p^{\frac{k-1}{k}} = 288 \times 43.6^{\frac{1.333-1}{1.333}} = 739.5 \text{ K}$$

Then calculate the new compressor work…

$$\dot{W}_{comp,s} = \dot{m} C_p (T_{2s} - T_1) = 200 \times 1.15 \times (739.5 - 288) = 103.8 \text{ MW}$$

And to account for isentropic efficiency…

$$\dot{W}_{comp} = \frac{\dot{W}_{comp,s}}{\eta_c} = \frac{103.8}{0.87} = 119.3 \text{ MW}$$

Because the temperature of the compressor outlet is now much higher, the heat input from the combustor must be re-calculated…

Need to find the non-isentropic temperature of the compressor outlet…

$$T_2 = T_1 + \frac{\dot{W}_{comp}}{\dot{m} C_p} = 288 + \frac{119.3}{200 \times 1.15} = 806.7 \text{ K}$$

$$\dot{Q}_{in} = \dot{m} C_p (T_5 - T_2) = 200 \times 1.15 \times (1673 - 806.7) = 199.2 \text{ MW}$$

And then the new efficiency of the cycle is…

$$\eta = \frac{\dot{W}_t - \dot{W}_{comp}}{\dot{Q}_{in}} = \frac{210 - 119.3}{199.2} = 45.5 \text{ \%}$$

d) If the firing temperature of the gas turbine in c) is reduced to 1150°C?

Here, the work of the compressor is the same as in c), but we need to calculate the new work of the turbine and the heat transfer in the combustor…

$$\dot{W}_t = \dot{m}C_p\Delta T = \dot{m}C_p(T_{6s} - T_5)$$

$$\frac{T_{6s}}{T_5} = \left(\frac{P_6}{P_5}\right)^{\frac{k-1}{k}} = \left(\frac{1}{r_{pt}}\right)^{\frac{k-1}{k}}$$

$$P_6 = P_1 = P_{ambient} = 1.013 \text{ bar}$$

$$P_5 = 0.99 r_p P_1 = 0.99 \times 43.6 \times 1.013 = 43.7 \text{ bar}$$

$$r_{pt} = \frac{P_5}{P_6} = \frac{43.7}{1.013} = 43.1$$

Then the isentropic work of the turbine is…

$$\dot{W}_{t,s} = \dot{m}C_p T_5 \left(1 - \left(\frac{1}{r_{pt}}\right)^{\frac{k-1}{k}}\right) = 200 \times 1.15 \times 1423 \left(1 - \left(\frac{1}{43.1}\right)^{\frac{1.333-1}{1.333}}\right) = 199.5 \text{ MW}$$

And the actual work of the turbine is…

$$\dot{W}_t = \dot{W}_t \times 0.9 = 199.5 \times 0.9 = 179.6 \text{ MW}$$

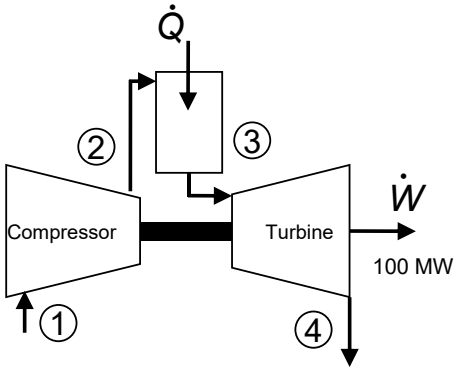
The new heat transfer in the combustor is…

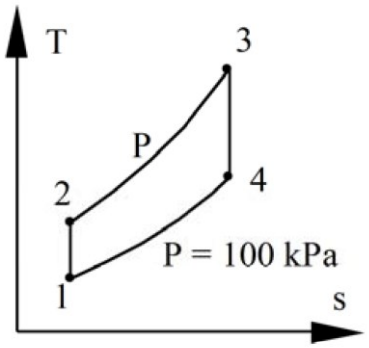
$$\dot{Q}_{in} = \dot{m}C_p(T_5 - T_2) = 200 \times 1.15 \times (1423 - 806.7) = 141.7 \text{ MW}$$

And finally, the new efficiency is…

$$\eta = \frac{\dot{W}_t - \dot{W}_{comp}}{\dot{Q}_{in}} = \frac{179.6 - 119.3}{141.7} = 42.6\ \%$$

15) A large stationary Brayton cycle gas-turbine power plant delivers a power output of 100 MW to an electric generator. The minimum temperature in the cycle is 300 K, and the exhaust temperature is 750 K. The minimum pressure in the cycle is 100 kPa, and the compressor pressure ratio is 18:1. Calculate the power output of the turbine. What fraction of the turbine output is required to drive the compressor? What is the thermal efficiency of the cycle?





This is an ideal Brayton cycle that has been covered in Thermo 2, and Tutorial 1 has several examples. Using that knowledge, find the turbine and compressor's specific work and the net specific work generated by the cycle. Then, you can calculate the mass flow rate for this cycle from the net power output and the net specific work as

$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{10{,}000\ \text{kW}}{580\ \text{kJ/kg}} = 172.4\ \text{kg/s}$$

The power output of the turbine:

$$\dot{W}_T = \dot{m} w_T = 172.4 \frac{\text{kg}}{\text{s}} * 966.6\ \frac{\text{kJ}}{\text{kg}} = \mathbf{166.64\ MW}$$

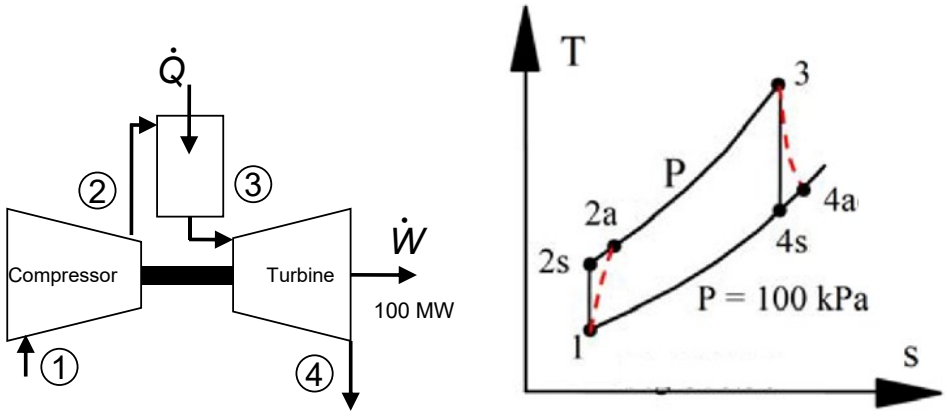
The fraction of the turbine output is required to drive the compressor:

$$\frac{w_c}{w_T} = \frac{386.6\ \frac{\text{kJ}}{\text{kg}}}{966.6\ \frac{\text{kJ}}{\text{kg}}} = \mathbf{0.4}$$

Cycle efficiency:

$$\eta = \frac{w_{net}}{q_H} = \frac{w_T - w_c}{q_H} = \frac{580\ \text{kJ/kg}}{1031.7\ \text{kJ/kg}} = \mathbf{56.2\%}$$

16) Repeat the previous problem, but assume the compressor has an isentropic efficiency of 85% and the turbine has an isentropic efficiency of 88%.



The original cycle was the reversible Brayton cycle, so this means:

Minimum T: $T_1 = 300$ K

Exhaust T: $T_{4s} = 750$ K

Pressure ratio: $P_2/P_1 = 18$

For compressor:

$$T_{2s} = T_1(P_2/P_1)^{\frac{k-1}{k}} = 300\text{ K}\ (18)^{0.2857} = 685.1\text{ K}$$

$$w_{Cs} = h_2 - h_1 = C_{P0}(T_2 - T_1) = 1.004\ (685.1 - 300) = 386.6\text{ kJ/kg}$$

$$\Rightarrow\ w_C = w_{SC}/\eta_{SC} = 386.6/0.85 = 454.82\text{ kJ/kg} = C_{P0}(T_2 - T_1)$$

$$\Rightarrow\ T_2 = T_1 + w_c/C_{P0} = 300 + 454.82/1.004 = 753.0\text{ K}$$

For turbine:

$$T_3 = T_{4s}\ (P_3/P_{4s})^{\frac{k-1}{k}} = 750\text{ K}\ (18/1)^{0.2857} = 1712.7\text{ K}$$

$$w_{Ts} = h_3 - h_4 = C_{P0}(T_3 - T_4) = 1.004\text{ kJ/kgK}\ (1712.7 - 750)\text{ K} = 966.6\text{ kJ/kg}$$

$$\Rightarrow w_T = \eta_{ST}\ \ w_{ST} = 0.88 \times 966.6 = 850.6\text{ kJ/kg} = C_{P0}(T_3 - T_4)$$

$$\Rightarrow T_4 = T_3 - w_T/C_{P0} = 1712.7\text{ K} - 850.6\text{ kJ/kg}\ /1.004\text{ kJ/kgK} = 865.5\text{ K}$$

Net cycle work and efficiency:

$$w_{NET} = w_T - w_C = 850.6 - 454.82 = 395.8 \text{ kJ/kg}$$

$$\dot{m} = \dot{W}_{NET}/w_{NET} = 100\,000 \text{ kW} / 395.8 \text{ kJ/kg} = 252.6 \text{ kg/s}$$

$$\dot{W}_T = \dot{m} w_T = 252.6 \text{ kg/s} \times 850.6 \text{ kJ/kg} = \mathbf{214.9\ MW}$$

$$w_C/w_T = 454.82 / 850.6 = \mathbf{0.535}$$

$$q_H = C_{P0}(T_3 - T_2) = 1.004 \text{ kJ/kg-K } (1712.7 - 753.0) \text{ K} = 963.5 \text{ kJ/kg}$$

$$\eta_{TH} = w_{NET}/q_H = 395.8 / 963.5 = \mathbf{0.41}$$