Thermodynamics (Mechanical) 4

Ideal Gas Mixtures & Psychrometry – Part 1

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Slides are based on Chapter 3: Properties of Pure Substances & Chapter 13: Gas Mixtures,
Thermodynamics: An Engineering Approach by Cengel and Boles.

Content

- The ideal gas equation of state
- Properties of ideal gas mixtures
- *P-v-T* behaviour of ideal gas mixture
- Common models for ideal gas mixtures

Objectives

- Develop rules for determining non-reacting gas mixture properties from knowledge of mixture composition and the properties of the individual components.
- Define the quantities used to describe the composition of a mixture, such as mass fraction, mole fraction and volume fraction.
- Apply the rules for determining mixture properties to ideal-gas mixtures and real-gas mixtures.
- Predict the *P*-*v*-*T* behaviour of gas mixtures based on Dalton's law of additive pressures and Amagat's law of additive volumes.

Ideal Gas Mixtures

- Many thermodynamic applications involve mixtures of ideal gases. That is, each of the gases in the mixture individually behaves as an ideal gas. In this section, we assume that the gases in the mixture do not react with one another to any significant degree.
- We restrict ourselves to a study of only ideal-gas mixtures. An ideal gas is one in which the equation of state can be assumed to be given by:

Ideal gas equation of state: $$PV = mRT \quad or \quad PV = NR_u T$$

Air is an example of an ideal gas mixture and has the following approximate composition.

Component	% by Volume*
N_2	78.09
O_2	20.95
Argon	0.93
CO_2 + trace elements	0.03

* Values from Technical Data on Fuel, 7th ed., Ed's J. W. Rose and J. R. Cooper, Scottish Academic Press, 1977.

Ideal Gas Mixtures

- A mixture of two or more gases of fixed chemical composition is called a *non-reacting gas mixture*.
- Consider a container having a volume V that is filled with a mixture of k different gases at a pressure P and a temperature T.

The properties of the mixture may be based on the mass of each component, called gravimetric analysis, or on the moles of each component, called molar analysis.

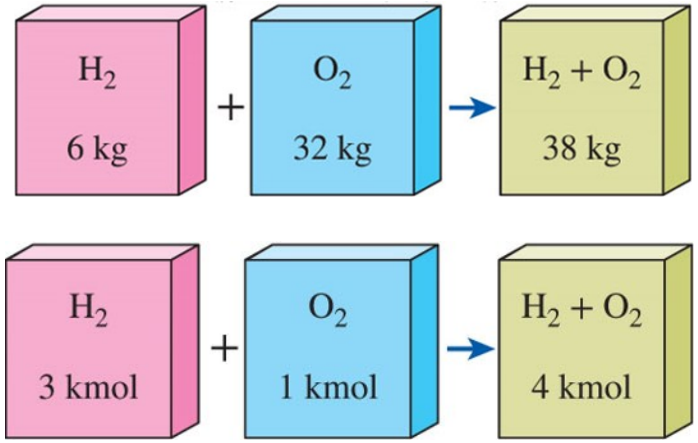
k gases

$T = T_m$ $\quad V = V_m$

$P = P_m$ $\quad m = m_m$

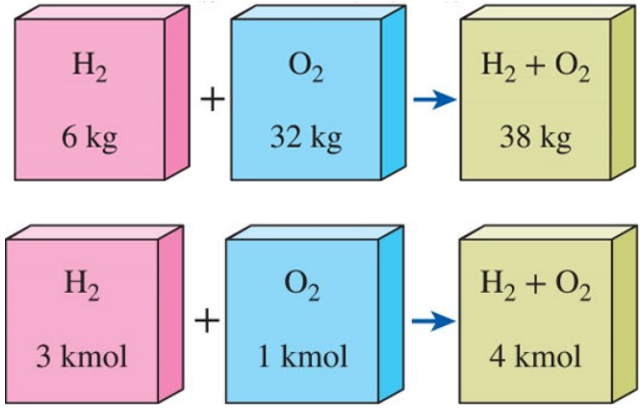
The total mass of the mixture (m_m) and the total moles of mixture (N_m) are defined as:

$$m_m = \sum_{i=1}^{k} m_i \quad \text{and} \quad N_m = \sum_{i=1}^{k} N_i$$



The **composition** of a gas mixture is described by specifying either the *mass fraction* (mf_i) or the *mole fraction* (y_i) of each component *i*

$$\mathrm{mf}_i = \frac{m_i}{m_m} \quad \text{and} \quad y_i = \frac{N_i}{N_m} \quad \text{where,} \quad \sum_{i=1}^{k} \mathrm{mf}_i = 1 \quad \text{and} \quad \sum_{i=1}^{k} y_i = 1$$



The mass (m_i) and mole number (N_i) for a given component are related through the molar mass (or molecular weight), M_i

$$m_i = N_i M_i \quad \text{or} \quad N_i = \frac{m_i}{M_i}$$

We know that the mass of the mixture, $m_m = \sum_{i=1}^{k} m_i = \sum_{i=1}^{k} N_i M_i = N_m M_m$

This will provide the apparent (or average) molar mass (M_m) of the mixture as,

$$M_m = \frac{m_m}{N_m} = \frac{\sum_{i=1}^{k} N_i M_i}{N_m} = \sum_{i=1}^{k} y_i M_i \qquad \text{(kg/kmol)}$$

The molar mass of a mixture can also be expressed as

$$M_m = \frac{m_m}{N_m} = \frac{m_m}{\sum m_i / M_i} = \frac{1}{\sum_{i=1}^{k} \frac{\mathrm{mf}_i}{M_i}} \qquad (\mathrm{kg/kmol}) \qquad \text{since} \quad N_i = \frac{m_i}{M_i}$$

The *apparent* (or average) *gas constant* of a mixture is expressed as

$$R_m = \frac{R_u}{M_m} \qquad (\mathrm{kJ/kg \cdot K})$$

R_u (kJ/kmol·K) is the molar, universal, or ideal gas constant

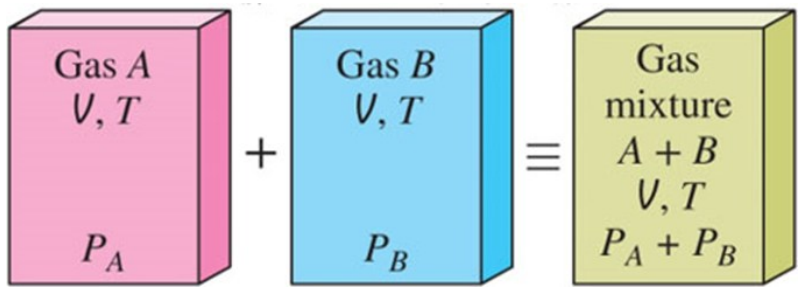
Mass and mole fractions of a mixture are related by:

$$\mathrm{mf}_i = \frac{m_i}{m_m} = \frac{N_i M_i}{N_m M_m} = y_i \frac{M_i}{M_m} = \frac{y_i M_i}{\sum_{i=1}^{k} y_i M_i}$$

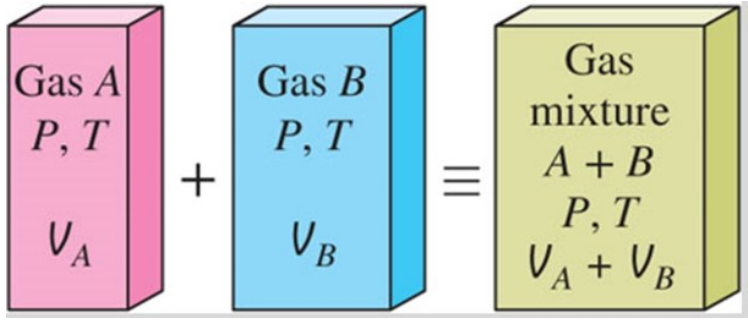
P-*v*-T Behaviour of Gas Mixtures

The prediction of the *P-v-T* behaviour of gas mixtures is usually based on two models: *Dalton's law of additive pressures* and *Amagat's law of additive volumes*.

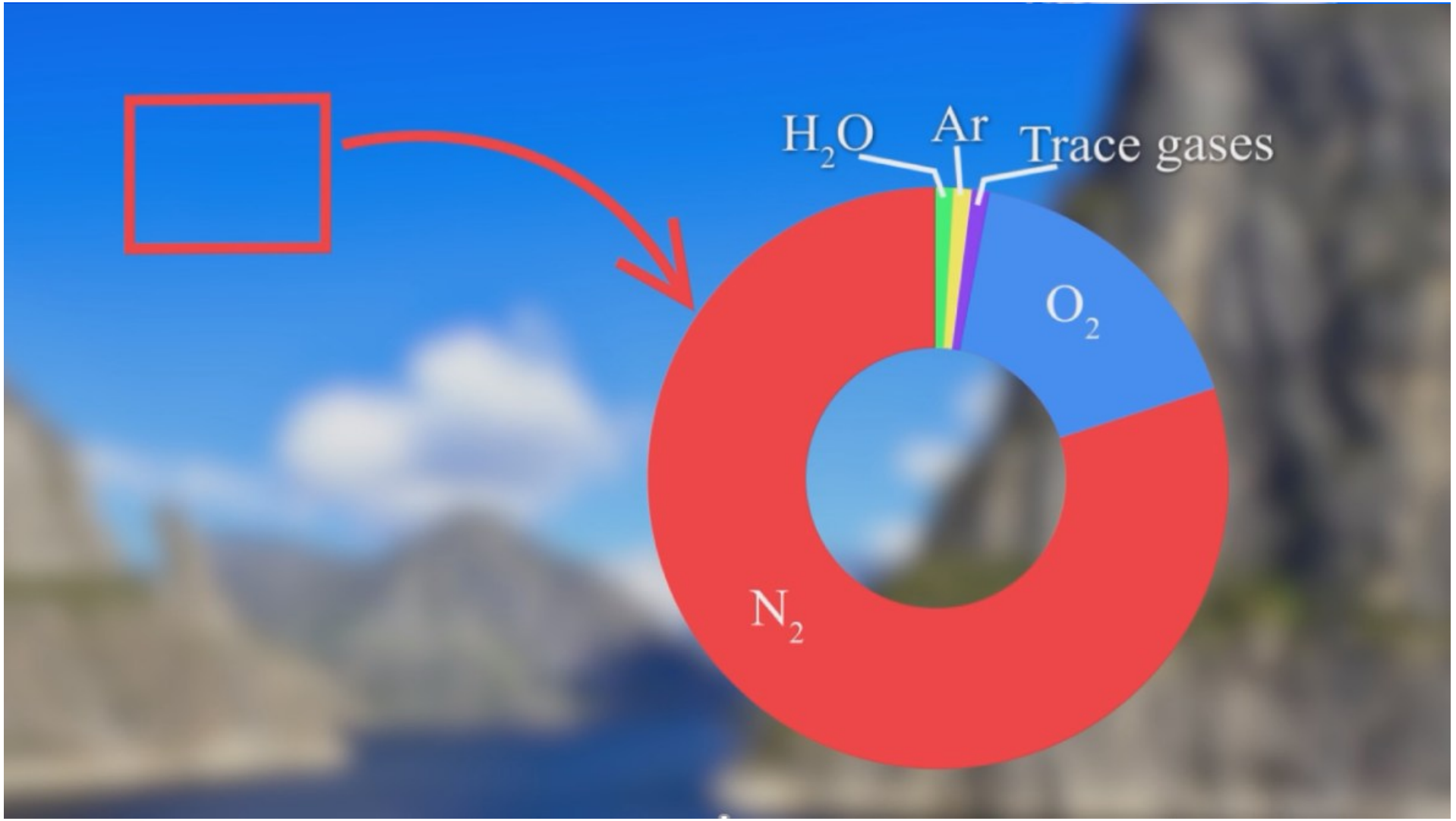
Dalton's law of additive pressures: The pressure of a gas mixture is equal to the sum of the pressures each gas would exert if it existed alone at the mixture temperature and volume.



Amagat's law of additive volumes: The volume of a gas mixture is equal to the sum of the volumes each gas would occupy if it existed alone at the mixture temperature and pressure.



P-*v*-T Behaviour of Gas Mixtures

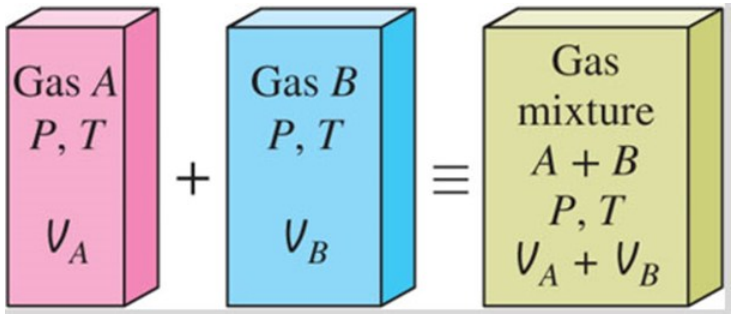


Volume Fraction (Amagat Model)

Amagat's law: $V_m = \sum_{i=1}^{k} V_i(T_m, P_m)$

Exact for ideal gases, approximate for real gases

Amagat's law of additive volumes for a mixture of two ideal gases



The volume fraction of the vf_i of any component is $vf_i = \frac{V_i(T_m, P_m)}{V_m}$ and $\sum_{i=1}^{k} vf_i = 1$

For an ideal gas mixture, $V_i = \frac{N_i R_u T_m}{P_m}$ and $V_m = \frac{N_m R_u T_m}{P_m}$

Taking the ratio of these two equations gives: $vf_i = \frac{V_i}{V_m} = \frac{N_i}{N_m} = y_i$

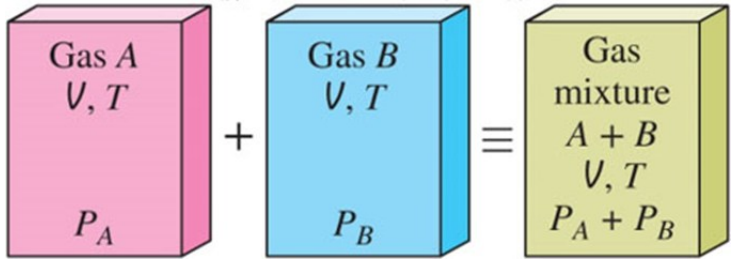
Volume fraction (vf_i) and mole fraction (y_i) of a component in an ideal gas mixture are the same.

Partial Pressure (Dalton Model)

Dalton's law: $$P_m = \sum_{i=1}^{k} P_i(T_m, V_m)$$

Exact for ideal gases, approximate for real gases

Dalton's law of additive pressures for a mixture of two ideal gases



The partial pressure of component *i* is defined as the product of the mole fraction (y_i) and the mixture pressure (P_m) according to Dalton's law. For the component *i*,

$$P_i = y_i P_m$$

For an ideal gas mixture, $$P_i' = \frac{N_i R_u T_m}{V_m} \quad \text{and} \quad P_m = \frac{N_m R_u T_m}{V_m}$$

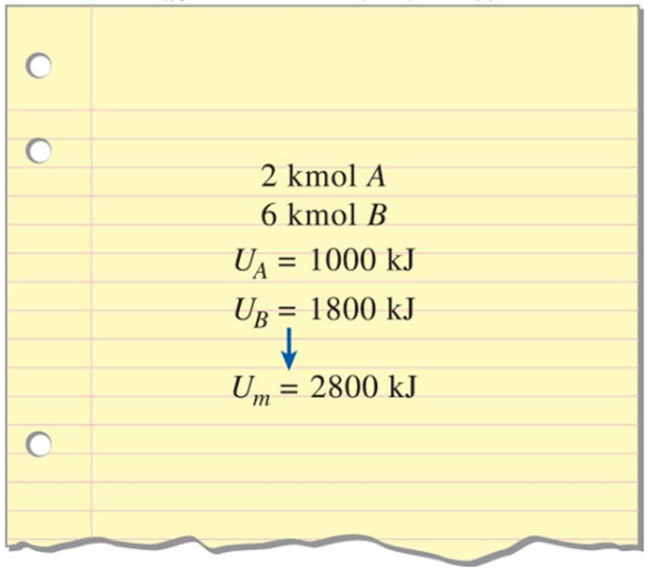
Taking the ratio of these two equations gives: $$\frac{P_i'}{P_m} = \frac{V_i}{V_m} = \frac{N_i}{N_m} = y_i \Rightarrow P_i' = y_i P_m = P_i$$

Partial pressure (P_i) and component pressure are the same and are equal to the product of the mole fraction and the mixture pressure.

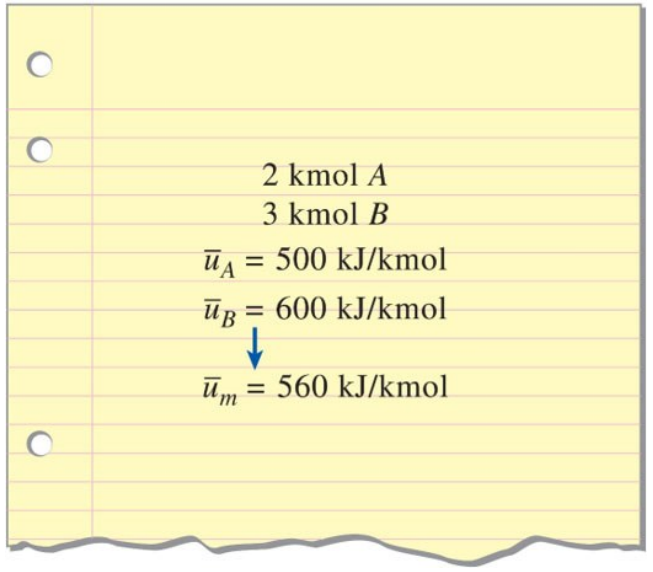
Properties of Ideal-gas Mixtures

- The extensive properties of a gas mixture (e.g. mass, volume, amount of energy), in general, can be determined by summing the contributions of each component of the mixture.

- The evaluation of intensive properties of a gas mixture (e.g. melting point, boiling point, density), however, involves averaging in terms of mass or mole fractions.



Extensive property



Intensive property

Properties of Ideal-gas Mixtures

Then the total internal energy, enthalpy, and entropy of a gas mixture can be expressed, respectively, as:

$$U_m = \sum_{i=1}^{k} U_i = \sum_{i=1}^{k} m_i u_i = \sum_{i=1}^{k} N_i \bar{u}_i \qquad \text{(kJ)}$$

$$H_m = \sum_{i=1}^{k} H_i = \sum_{i=1}^{k} m_i h_i = \sum_{i=1}^{k} N_i \bar{h}_i \qquad \text{(kJ)}$$

$$S_m = \sum_{i=1}^{k} S_i = \sum_{i=1}^{k} m_i s_i = \sum_{i=1}^{k} N_i \bar{s}_i \qquad \text{(kJ/K)}$$

Changes in properties of a gas mixture

$$\Delta U_m = \sum_{i=1}^{k} \Delta U_i = \sum_{i=1}^{k} m_i \Delta u_i = \sum_{i=1}^{k} N_i \Delta \bar{u}_1 \qquad \text{(kJ)}$$

$$\Delta H_m = \sum_{i=1}^{k} \Delta H_i = \sum_{i=1}^{k} m_i \Delta h_i = \sum_{i=1}^{k} N_i \Delta \bar{h}_1 \qquad \text{(kJ)}$$

$$\Delta S_m = \sum_{i=1}^{k} \Delta S_i = \sum_{i=1}^{k} m_i \Delta s_i = \sum_{i=1}^{k} N_i \Delta \bar{s}_i \qquad \text{(kJ/K)}$$

Properties of Ideal-gas Mixtures

The internal energy, enthalpy, and entropy of a mixture *per unit mass* or *per unit mole* of the mixture can be determined by dividing the previous equations by the mass or the mole number of the mixture,

$$u_m = \sum_{i=1}^{k} mf_i u_i \quad \text{and} \quad \overline{u}_m = \sum_{i=1}^{k} y_i \overline{u}_i \quad \text{(kJ/kg or kJ/kmol)}$$

$$h_m = \sum_{i=1}^{k} mf_i h_i \quad \text{and} \quad \overline{h}_m = \sum_{i=1}^{k} y_i \overline{h}_i \quad \text{(kJ/kg or kJ/kmol)}$$

$$s_m = \sum_{i=1}^{k} mf_i s_i \quad \text{and} \quad \overline{s}_m = \sum_{i=1}^{k} y_i \overline{s}_i \quad \text{(kJ/kg}\cdot\text{K or kJ/kmol}\cdot\text{K)}$$

$$C_{v,m} = \sum_{i=1}^{k} mf_i C_{v,i} \quad \text{and} \quad \overline{C}_{v,m} = \sum_{i=1}^{k} y_i \overline{C}_{v,i}$$

$$C_{p,m} = \sum_{i=1}^{k} mf_i C_{p,i} \quad \text{and} \quad \overline{C}_{p,m} = \sum_{i=1}^{k} y_i \overline{C}_{p,i}$$

Ratio of specific heats (k_m) is given by:

$$k_m = \frac{C_{p,m}}{C_{v,m}} = \frac{\overline{C}_{p,m}}{\overline{C}_{v,m}}$$

Entropy of Ideal Gas Mixtures

- Evaluation of Δu or Δh of the components of an ideal-gas mixture during a process is relatively easy since it *requires only a knowledge of the initial and final temperatures*. Care should be exercised, however, in evaluating the Δs of the components since the entropy of an ideal gas *depends on the pressure or volume of the component as well as on its temperature*.

- The entropy of a mixture of ideal gases is equal to the sum of the entropies of the component gases as they exist in the mixture. We employ the Gibbs-Dalton law that says each gas behaves as if it alone occupies the volume of the system at the mixture temperature. That is, the pressure of each component is the partial pressure.

- For constant specific heats, the entropy change of any component between states 1 and 2 is:

$$\Delta s_i = C_{p,i} \ln\left(\frac{T_2}{T_1}\right) - R_i \ln\left(\frac{P_{i,2}}{P_{i,1}}\right)$$

Entropy of Ideal Gas Mixtures (Thermo 2)

The starting point is from the combined first and second laws of thermodynamics for a reversible process

$$dw = Pdv \quad and \quad dq = Tds \qquad h \equiv u + Pv$$

For an ideal gas

$$du = C_v dT \qquad du = Tds - Pdv \qquad dh = Tds + vdP$$

$$dh = C_p dT$$

Using the equation of state for an ideal case, the entropy change can be expressed:

$$ds = C_v \frac{dT}{T} + \frac{P}{T} dv \qquad ds = C_p \frac{dT}{T} - \frac{v}{T} dP$$

$$ds = C_v \frac{dT}{T} + R \frac{dv}{v} \qquad ds = C_p \frac{dT}{T} - R \frac{dP}{P}$$

For a perfect gas with constant specific heats:

$$\Delta s = s_2 - s_1 = \int_{T1}^{T2} C_v \frac{dT}{T} + R \int_{v1}^{v2} \frac{dv}{v} \qquad \Delta s = s_2 - s_1 = \int_{T1}^{T2} C_p \frac{dT}{T} - R \int_{P1}^{P2} \frac{dP}{P}$$

$$\Delta s = s_2 - s_1 = C_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \qquad \Delta s = s_2 - s_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

Thermo 2 - Formula Sheet

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} \qquad \eta_{th} = 1 - \frac{Q_L}{Q_H} \qquad COP_R = \frac{Q_L}{W_{net,in}} \qquad COP_{HP} = \frac{Q_H}{W_{net,in}} = \frac{Q_H}{Q_H - Q_L}$$

$$\frac{Q_L}{Q_H} = \frac{T_L}{T_H} \qquad \oint \frac{\delta Q_{net}}{T} \leq 0 \qquad dS \geq \frac{\delta Q_{net}}{T} \qquad s_2 - s_1 = C_{av} ln \frac{T_2}{T_1}$$

$$s_2 - s_1 = C_v ln \frac{T_2}{T_1} + R ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = C_p ln \frac{T_2}{T_1} - R ln \frac{P_2}{P_1}$$

$$s(T_2, P_2) - s(T_1, P_1) = s^0(T_2) - s^0(T_1) - R ln \frac{P_2}{P_1}$$

$$\left(\frac{T_2}{T_1}\right)_{s=const.} = \left(\frac{v_1}{v_2}\right)^{k-1} \qquad \left(\frac{T_2}{T_1}\right)_{s=const.} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \qquad \left(\frac{P_2}{P_1}\right)_{s=const.} = \left(\frac{v_1}{v_2}\right)^{k}$$

$$S_{in} - S_{out} + S_{gen} = \Delta S_{system} \qquad \Delta S_{isolated} \geq 0 \qquad Q_{net} = \int_1^2 T dS$$

$$r = \frac{V_{max}}{V_{min}} = \frac{V_{BDC}}{V_{TDC}} \qquad MEP = \frac{W_{net}}{V_{max} - V_{min}} = \frac{w_{net}}{v_{max} - v_{min}} \qquad \frac{V_3}{V_2} = r_c$$

Entropy of Ideal Gas Mixtures

The entropy change of the mixture per mass of mixture is

$$\Delta s_m = \frac{\Delta S_m}{m_m} = \sum_{i=1}^{k} mf_i \Delta s_i$$

$$= \sum_{i=1}^{k} mf_i \left(C_{p,i} \ln\left(\frac{T_2}{T_1}\right) - R_i \ln\left(\frac{P_{i,2}}{P_{i,1}}\right)\right)$$

The entropy change of the mixture per mole of mixture is

$$\Delta \bar{s}_m = \frac{\Delta S_m}{N_m} = \sum_{i=1}^{k} y_i \Delta \bar{s}_i$$

$$= \sum_{i=1}^{k} y_i \left(\bar{C}_{p,i} \ln\left(\frac{T_2}{T_1}\right) - R_u \ln\left(\frac{P_{i,2}}{P_{i,1}}\right)\right)$$

In these two equations, recall that

$$P_{i,1} = y_{i,1} P_{m,1} \quad \text{and} \quad P_{i,2} = y_{i,2} P_{m,2}$$

Example 5.1

An ideal-gas mixture has the following volumetric analysis:

Component	% by Volume
N_2	60
CO_2	40

(a) Find the mass fractions.

(b) What is the mass of 1 m^3 of this gas when P = 1.5 MPa and T = 30°C?

(c) Find the average specific heats at 300 K, given values for the individual components as follows:

C_p N_2 = 1.039 kJ/kg·K and C_p CO_2 = 0.846 kJ/kg·K

(d) This gas is heated in a steady-flow process such that the temperature is increased by 120°C. Find the required heat transfer.

(e) This mixture undergoes an isentropic compression process from 0.1 MPa, 30°C, to 0.2 MPa. Find T_2.

(f) Find ΔS_m per kg of mixture when the mixture is compressed isothermally from 0.1 MPa to 0.2 MPa.

(g) Case where mixture is changing during the process: Both the N_2 and CO_2 are supplied in separate lines at 0.2 MPa and 300 K to a mixing chamber and are mixed adiabatically. The resulting mixture has the composition as given in part (a). Determine the entropy change due to the mixing process per unit mass of mixture.

Thermodynamics (Mechanical) 4

Ideal Gas Mixtures & Psychrometry – Part 2

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Slides are based on Chapter 3: Properties of Pure Substances & **Chapter 14: Gas-vapour Mixtures and Air-conditioning**, Thermodynamics: An Engineering Approach by Cengel and Boles.

Quick Summary of Previous Lecture

- Gas mixtures properties
  - Fairly straightforward, except entropy
- Common models for ideal gas mixtures
  - Amagat Model
  - Dalton Model
- Example problem

Next

- Gas-vapour mixtures
  - Of particular interest: water vapour in air

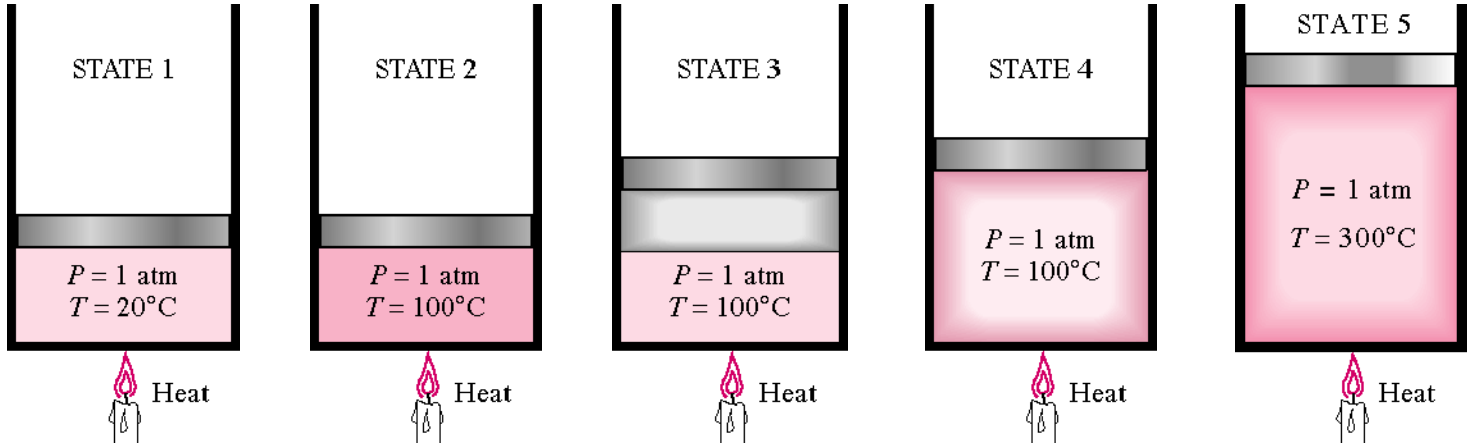
Contents

- Pure substances: Water/steam
  - Phase-change processes of pure substances
  - Property diagrams for phase-change process
- Gas-vapour mixtures
  - Definitions and properties
- The Psychrometric chart
  - Processes on psychrometric chart
  - Air conditioning processes (cooling, drying, re-heating, etc.)

Phase-change Processes of Pure Substances

Water/steam properties

Consider the results of heating liquid **water** from 20°C and 1 atm while keeping the pressure constant. We will follow a constant pressure process. First place liquid water in a piston-cylinder device where a fixed weight is placed on the piston to keep the pressure of the water constant at all times. As liquid water is heated while the pressure is held constant, the following events occur:

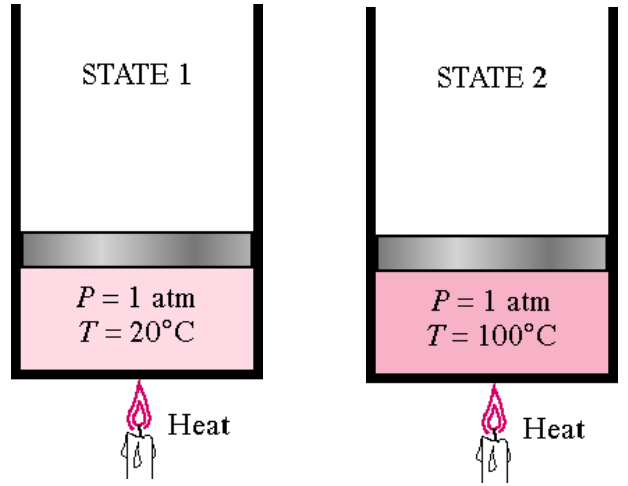


Phase-change Processes

Process 1-2:

The temperature and specific volume will increase from the compressed liquid, or subcooled liquid (state 1), to the saturated liquid (state 2).

In the compressed liquid region, the properties of the liquid are approximately equal to the properties of the saturated liquid state at the temperature.

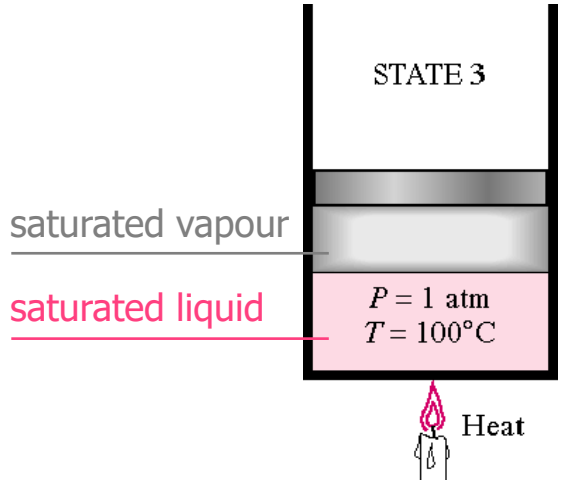


Process 2-3:

At state 2, the liquid has reached the temperature at which it begins to boil, called the saturation temperature (T_{sat}), and is said to exist as a saturated liquid. Properties at the saturated liquid state are noted by the subscript "f " and $v_2 = v_f$.

During the phase change both the temperature and pressure remain constant (according to the International Temperature Scale of 1990, ITS-90, water boils at 99.975°C ≅ 100°C when the pressure is 1 atm or 101.325 kPa).

At state 3, the liquid and vapour phase are in equilibrium and any point on the line between states 2 and 3 has the same temperature and pressure.



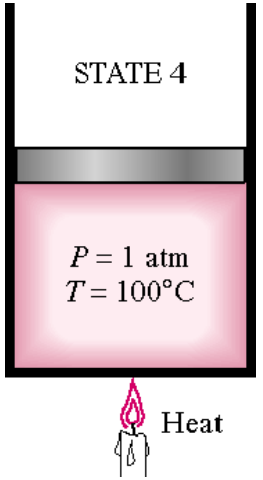
Phase-change Processes

Process 3-4:

At state 4, a saturated vapour exists and vapourisation is complete. The subscript "g " will always denote a saturated vapour state. Note $v_4 = v_g$.

Thermodynamic properties at the saturated liquid state and saturated vapour state are related. In the saturation region the temperature and pressure are dependent properties; i.e. if one is known then the other is automatically known.

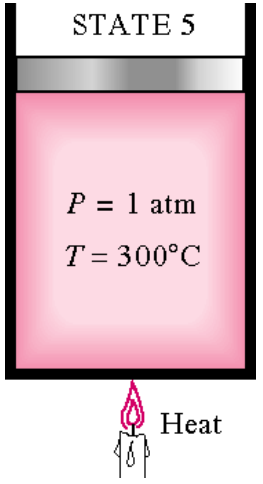
The saturation pressure (P_{sat}) is the pressure at which phase change will occur for a given temperature.



Process 4-5:

If the constant pressure heating is continued, the temperature will begin to increase above the saturation temperature, 100°C in this example, and the volume also increases.

State 5 is called a superheated state because T_5 is greater than the saturation temperature for the pressure and the vapour is not about to condense.



Property Diagrams

The variations of properties during phase-change processes are best studied and understood with the help of property diagrams, such as the **T-v, P-v,** and **P-T diagrams** for **pure substances**; i.e. The constant pressure heating process is illustrated in Figure 1.

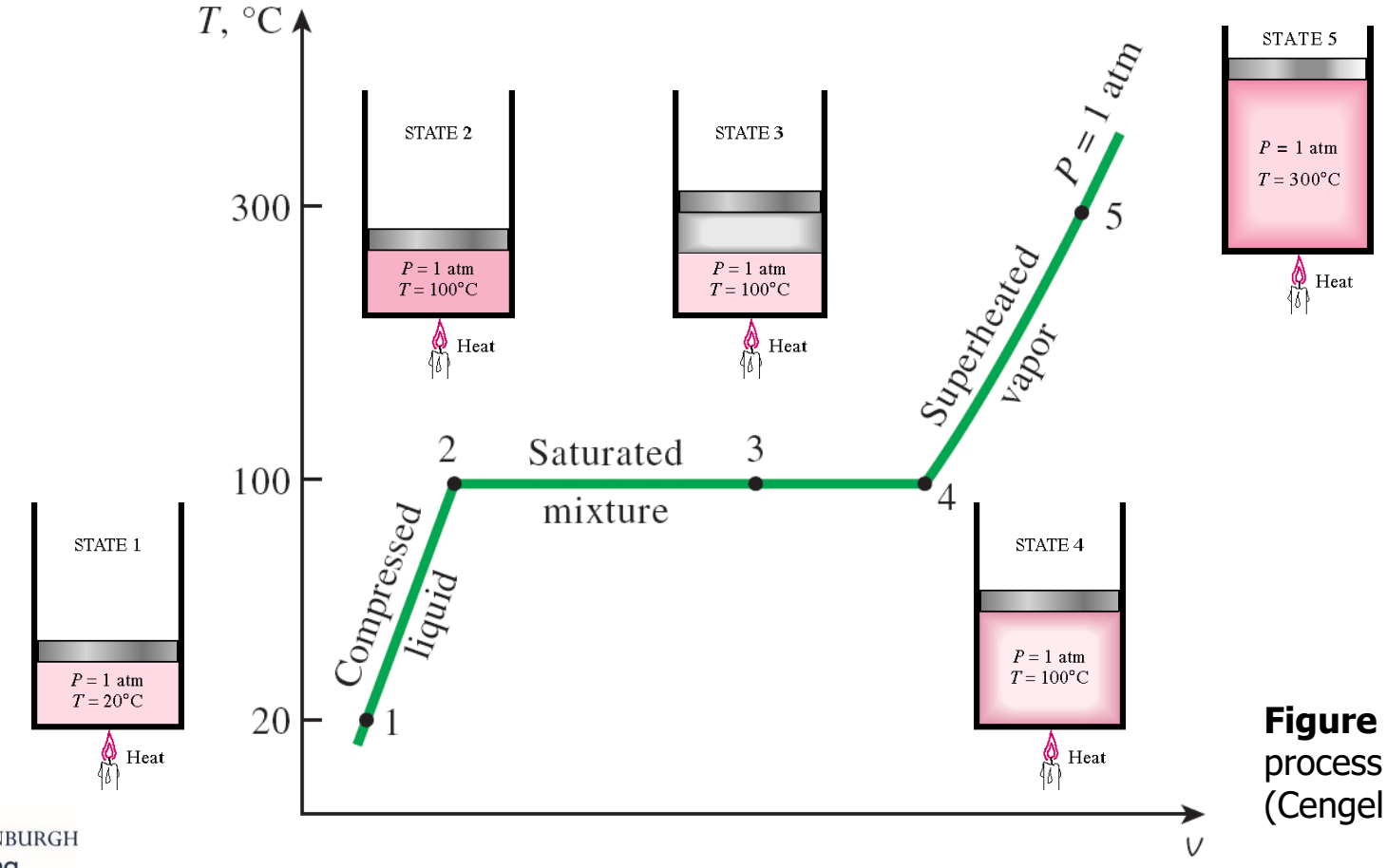


Figure 1. *T-v* diagram for the heating process water at constant pressure (Cengel and Boles, Ch3)

Property Diagrams

Consider repeating this process for other constant pressure lines leads to Figure 2.

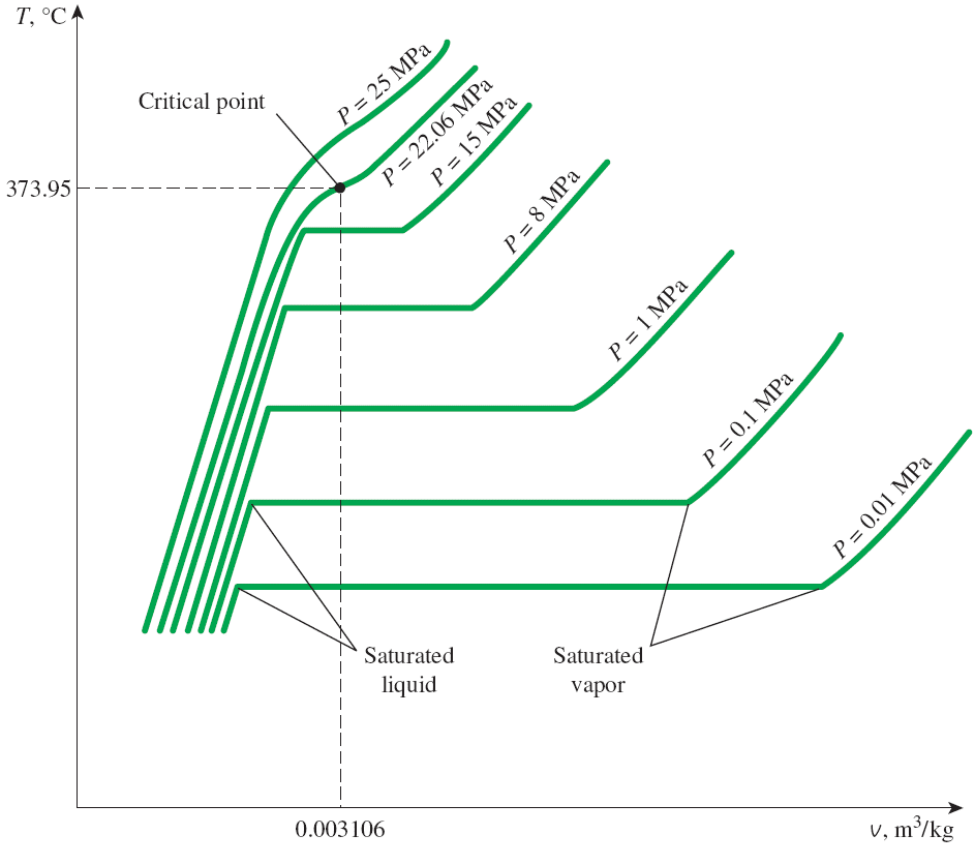
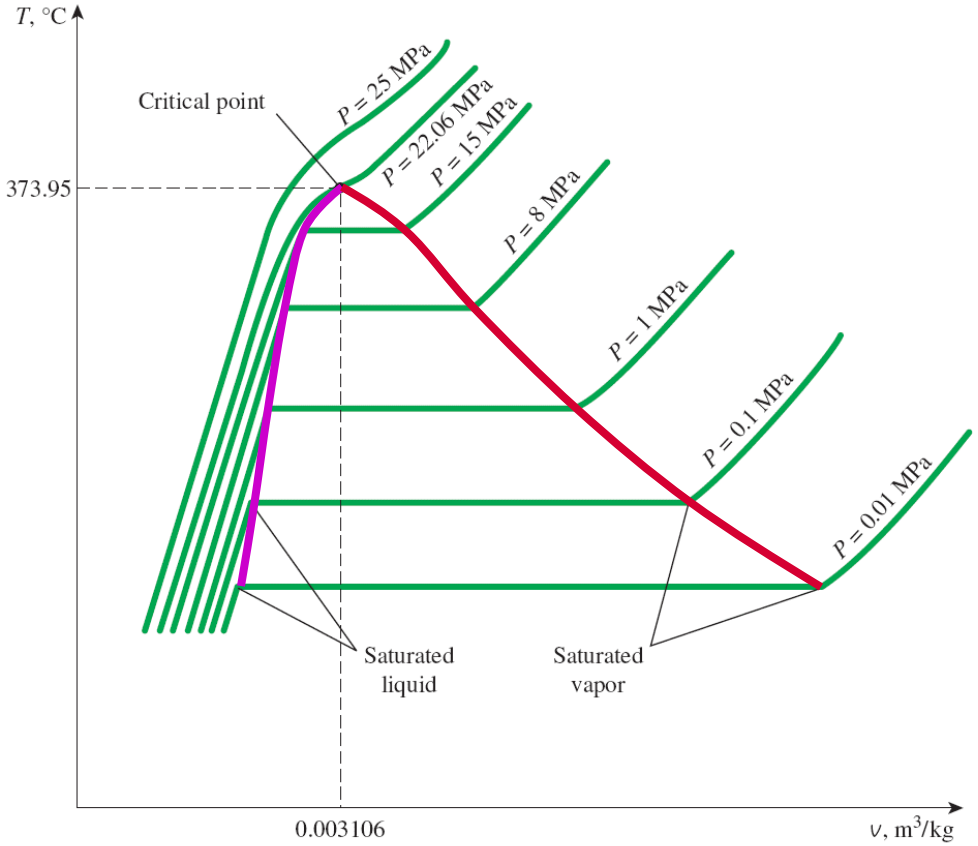
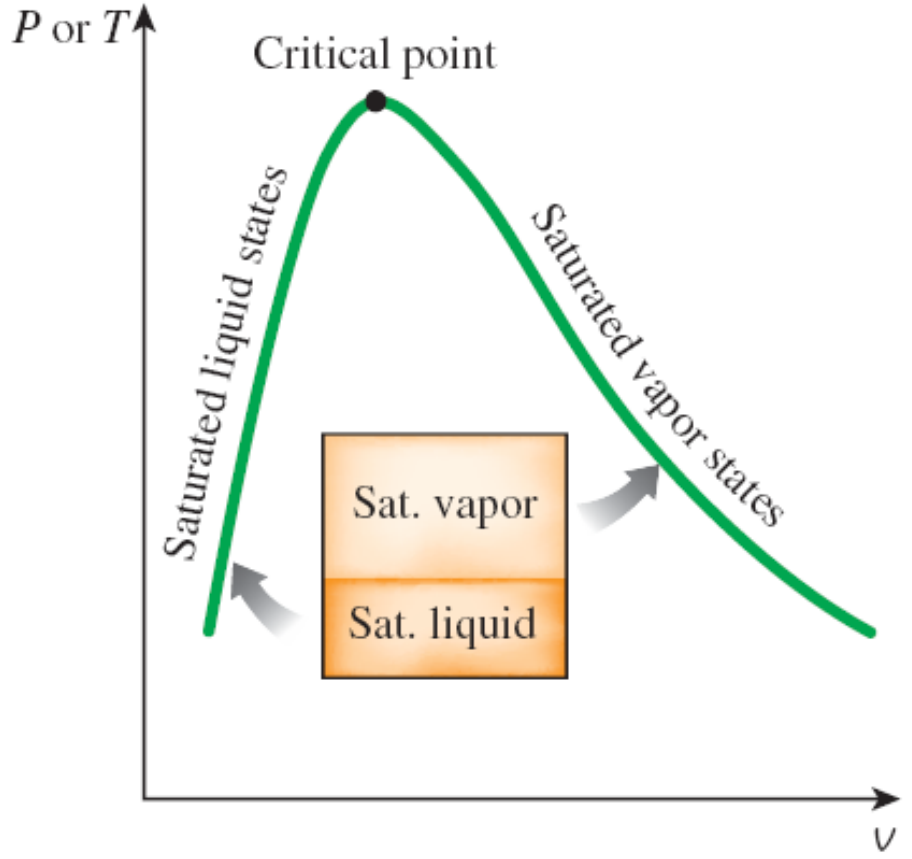


Figure 2. *T-v* diagram of constant-pressure phase-change processes of a pure substance at various pressures (numerical values are for water) (Cengel and Boles, Ch3)

Property Diagrams

If all of the saturated liquid states are connected, the saturated liquid line is established.
If all of the saturated vapour states are connected, the saturated vapour line is established.
These two lines intersect at the **critical point**.





T-v Diagram

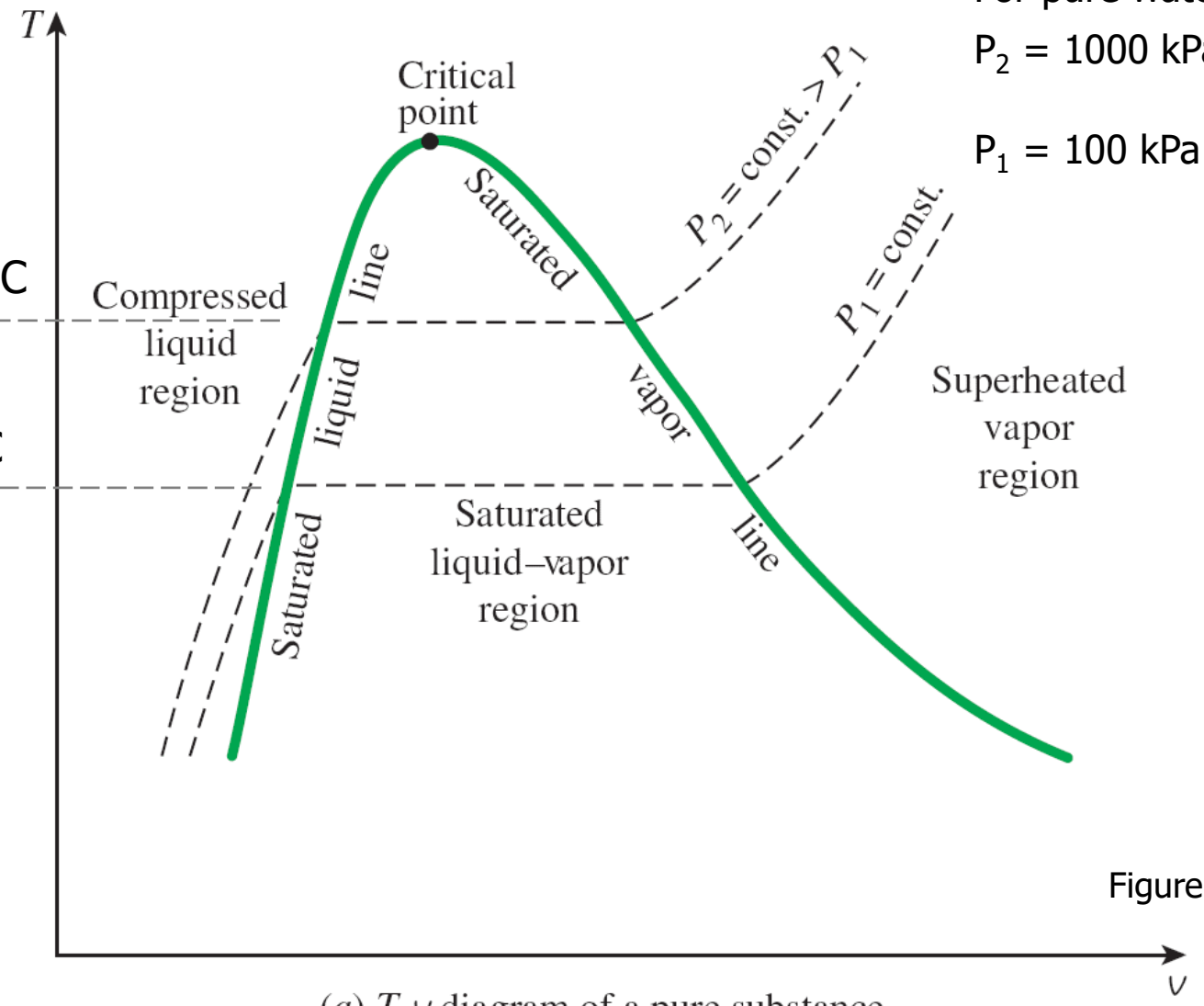
For pure water:

P_2 = 1000 kPa (10 bar)

P_1 = 100 kPa (1 bar)

T_{s2} = 179.88°C

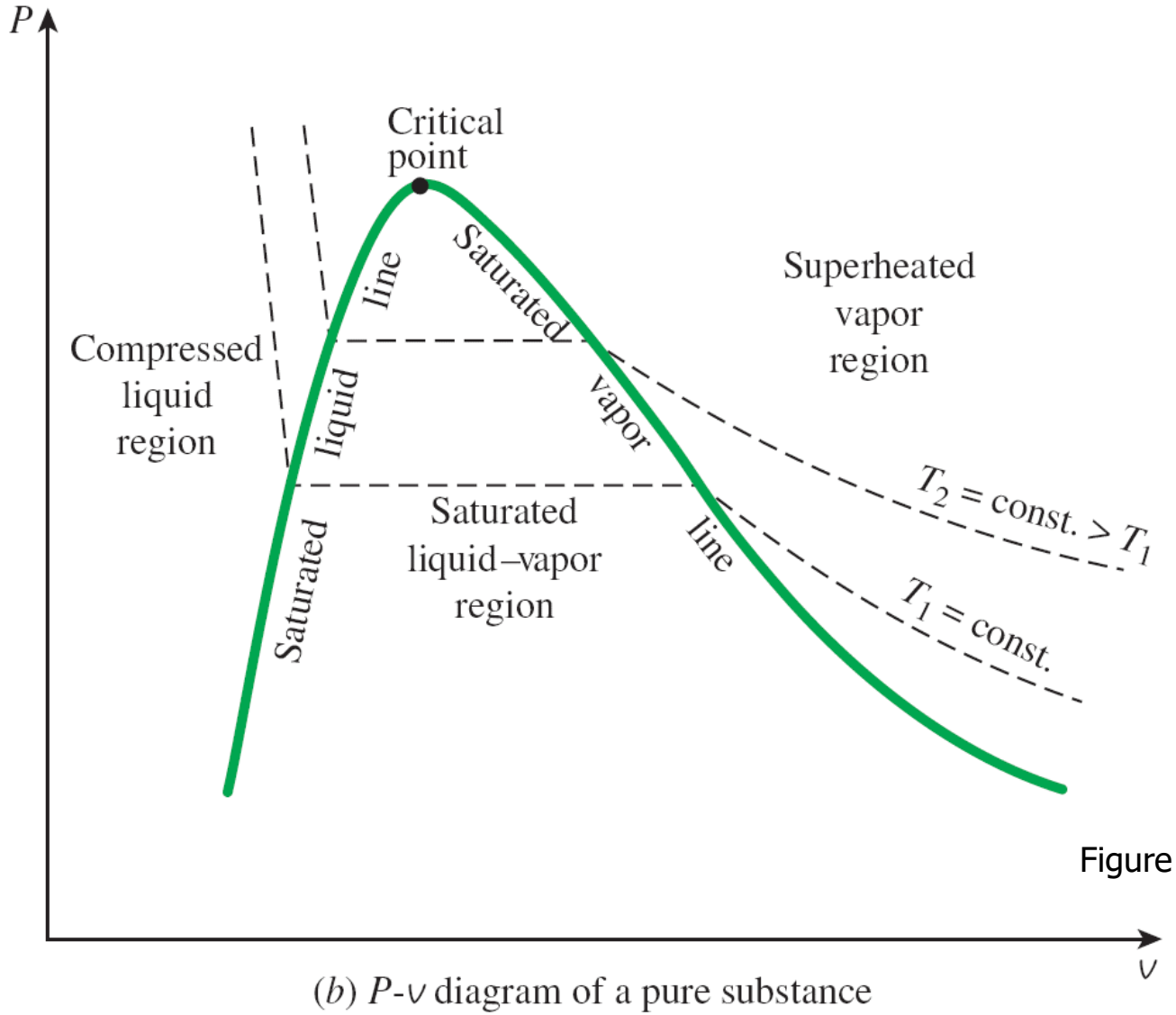
T_{s1} = 99.61°C



(a) T-v diagram of a pure substance

Figure source: Cengel and Boles, Ch3

P-v Diagram

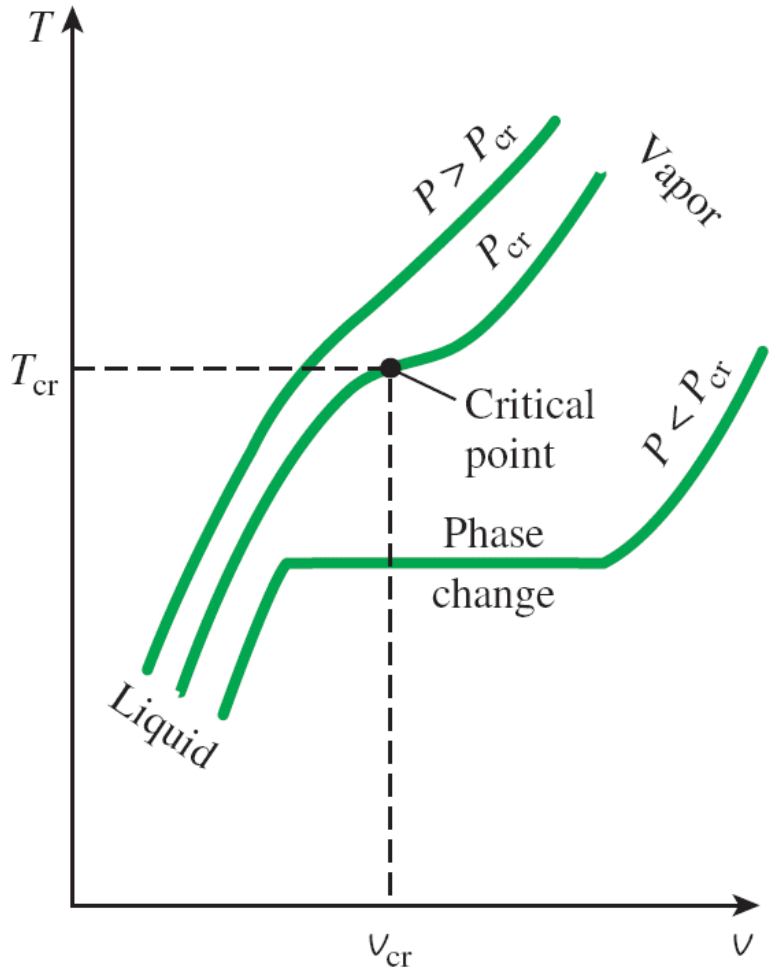


(b) P-v diagram of a pure substance

Figure source: Cengel and Boles, Ch3

Critical Point

At temperatures and pressures above the critical point, the phase transition from liquid to vapour is no longer discrete.



Critical point: the point at which the saturated liquid and saturate vapor states are identical

Figure: At supercritical pressure ($P > P_{cr}$), there is no distinct phase-change (boiling) process (Cengel and Boles, Ch3)

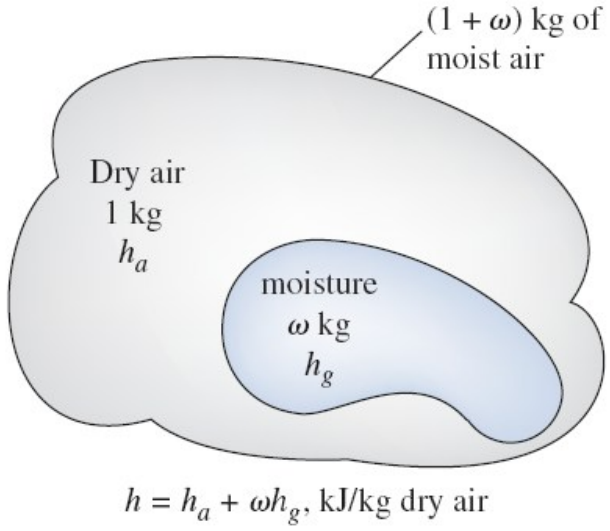
Gas-vapour Mixtures

At temperatures below the critical temperature, the gas phase of a substance is frequently referred to as a *vapour.*

The term ***vapour*** implies a gaseous state that is close to the saturation region of the substance, raising the possibility of condensation during a process.

Several gas–vapour mixtures can be encountered in engineering. In this lecture, we consider the *air–water vapour mixture,* which is the most commonly encountered gas-vapour mixture in practice.

- Air conditioning
- Humidifiers
- Wet cooling towers
- …

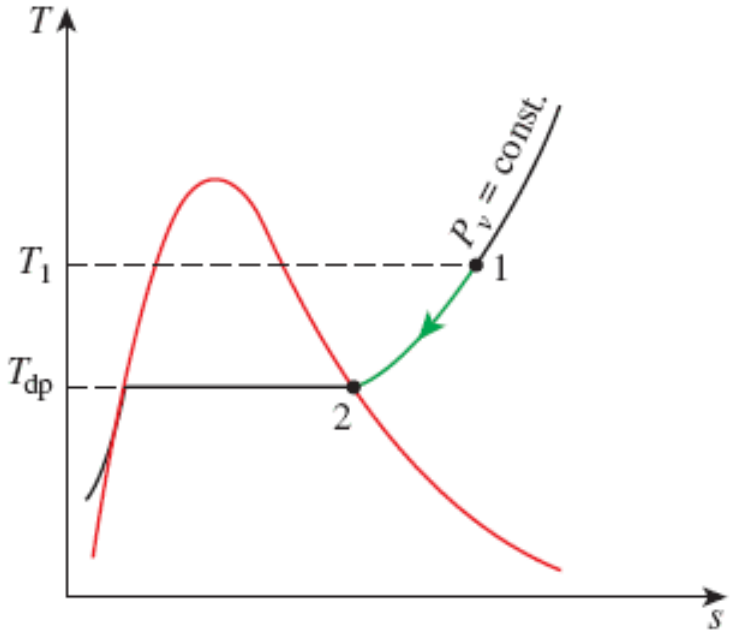


Enthalpy of moist (atmospheric) air is expressed per unit mass of dry air, not per unit mass of moist air (Cengel and Boles, Ch14).

Definitions for Gas-vapour Mixtures

Dew Point Temperature, T_{dp} or T_{dew}: The temperature at which vapour condensation begins when the air is cooled at constant pressure. It is the saturation temperature of water corresponding to the vapour pressure.

Consider cooling an air-water vapour mixture while the mixture total pressure is held constant. When the mixture is cooled to a temperature equal to the saturation temperature for the water-vapour partial pressure, condensation begins.



$$T_{dp} = T_s \; at \; P_v$$

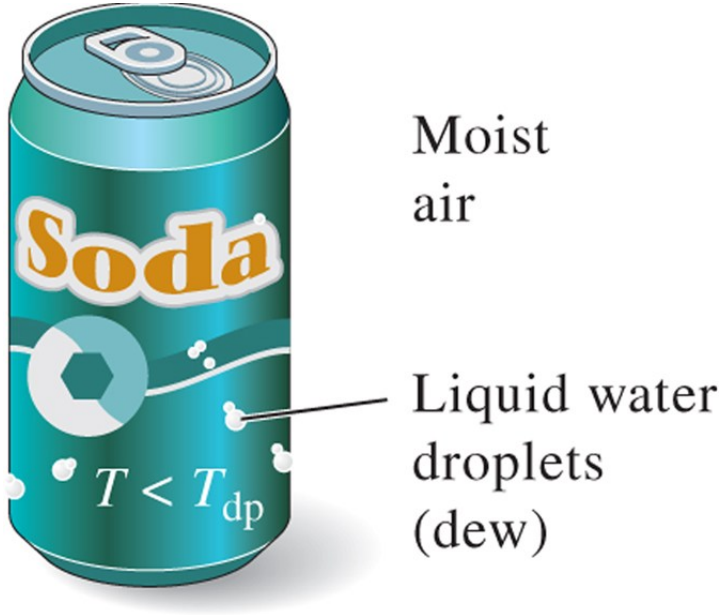
Figure:
Constant-pressure cooling of moist air and the dew-point temperature on the *T-s* diagram of water (Cengel and Boles, Ch14).

Definitions for Gas-vapour Mixtures

Dew Point Temperature, T_{dp} or T_{dew}: The temperature at which vapour condensation begins when the air is cooled at constant pressure. It is the saturation temperature of water corresponding to the vapour pressure.

Consider cooling an air-water vapour mixture while the mixture total pressure is held constant. When the mixture is cooled to a temperature equal to the saturation temperature for the water-vapour partial pressure, condensation begins.

Example: When the temperature of a cold drink is below the dew-point temperature of the surrounding air, it "sweats."



Definitions for Gas-vapour Mixtures

Relative Humidity, ϕ : The ratio of the amount of moisture the air holds (m_v) to the maximum amount of moisture the air can hold at the same temperature (m_g).

It can also be the ratio of the partial pressure of water vapour P_v to the equilibrium vapour pressure of water P_g at a given temperature. Relative humidity depends on temperature and the pressure of the system of interest.

$$\phi = \frac{m_v}{m_g} = \frac{P_v V / R_v T}{P_g V / R_v T} = \frac{P_v}{P_g}$$

$$P_g = P_{\text{sat}@T}$$ [saturation pressure of water at a given temperature]

What is the relative humidity of dry air and saturated air?

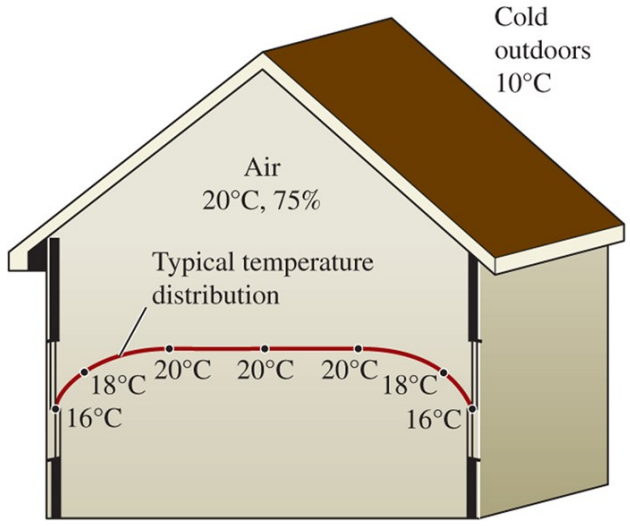
- The relative humidity ranges from 0 (for dry air) to 1 (for saturated air).

Definitions for Gas-vapour Mixtures

Example: Fogging of the Windows in a House

In cold weather, condensation frequently occurs on the inner surfaces of the windows due to the lower air temperatures near the window surface. Consider a house, shown in the figure that contains air at 20°C and 75 percent relative humidity (ϕ). At what window temperature will the moisture in the air start condensing on the inner surfaces of the windows?

[*The saturation pressure of water at 20°C is* $P_{sat} = 2.3392$ *kPa*]



$$T_{dp} = T_{\text{sat}@P_v}$$

$$P_v = \phi P_{g@20^\circ C} = (0.75)(2.3392\text{kPa}) = 1.754\text{kPa}$$

$$T_{dp} = T_{\text{sat}@1.754\text{ kPa}} = \mathbf{15.4°C}$$

Definitions for Gas-vapour Mixtures

Absolute humidity or specific humidity (humidity ratio), ω

$$\omega = \frac{\text{Mass of water vapour in air}}{\text{Mass of dry air}} = \frac{m_v}{m_a} = \frac{P_v V M_v / (R_u T)}{P_a V M_a / (R_u T)} = \frac{P_v M_v}{P_a M_a} = 0.622 \frac{P_v}{P_a} \Rightarrow \boxed{\omega = 0.622 \frac{P_v}{P - P_v}}$$

For molar masses:

M_v = 18.016 kg/kmol

M_a = 28.97 kg/kmol

$\Rightarrow M_v/M_a$ = 0.622; Also note, Pressure of mixture = Pressure of dry air + Pressure of vapour; $\boldsymbol{P}$ = total pressure

$$\text{Since, } P_v = \phi P_g, \quad \text{we get} \quad \omega = \frac{0.622 \phi P_g}{P - \phi P_g} \quad \text{(kg water vapour /kg dry air)}$$

[P_g is the saturation pressure (P_{sat}) of water at a given temperature]

Relative humidity (ϕ) can be expressed in terms of the absolute humidity, as:

$$\boxed{\phi = \frac{\omega P}{(0.622 + \omega) P_g}}$$

The terms absolute humidity or specific humidity are used synonymously with **humidity ratio**.
(Borgnakke and Sonntag, 8th Ed, Ch 11)

Definitions for Gas-vapour Mixtures

Volume of mixture per unit mass of dry air, v

$$v = \frac{V}{m_a} = \frac{m_m R_m T_m / P_m}{m_a}$$

But, recollect the Dalton mixture model, the components occupy the full volume at their partial pressure, so by definition:

$$v = \frac{V}{m_a} = v_a = \frac{R_a T_m}{P_a}$$

So the volume of the mixture per unit mass of dry air (v) is the specific volume of the dry air (v_a) calculated at the mixture temperature (T_m) and the partial pressure of the dry air (P_a).

Definitions for Gas-vapour Mixtures

Mass of mixture, m

$$m = m_a + m_v = m_a\left(1 + \frac{m_v}{m_a}\right) = m_a\left(1 + \omega\right)$$

Mass flow rate of dry air, $\dot{m}_a$

Based on the volume flow rate of mixture at a given state, the mass flow rate of dry air is

$$\dot{m}_a = \frac{\dot{V}}{v} \qquad \frac{\mathrm{m^3/s}}{\mathrm{m^3/kg_a}} = \frac{\mathrm{kg_a}}{\mathrm{s}}$$

Enthalpy of mixture per unit mass of dry air, h

$$h = \frac{H_m}{m_a} = \frac{H_a + H_v}{m_a} = \frac{m_a h_a + m_v h_v}{m_a} = h_a + \omega h_v$$

The total enthalpy (an extensive property) of atmospheric air H_m is the sum of the enthalpies of dry air H_a and the water vapour H_v.

Adiabatic Saturation Process

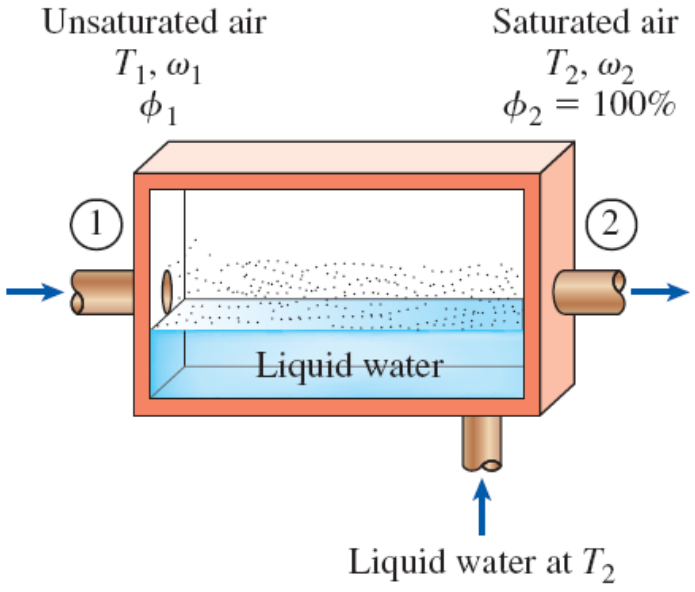
Relative humidity and specific (or absolute) humidity are frequently used in engineering and atmospheric sciences, and it is desirable to relate them to easily measurable quantities (such as temperature and pressure).

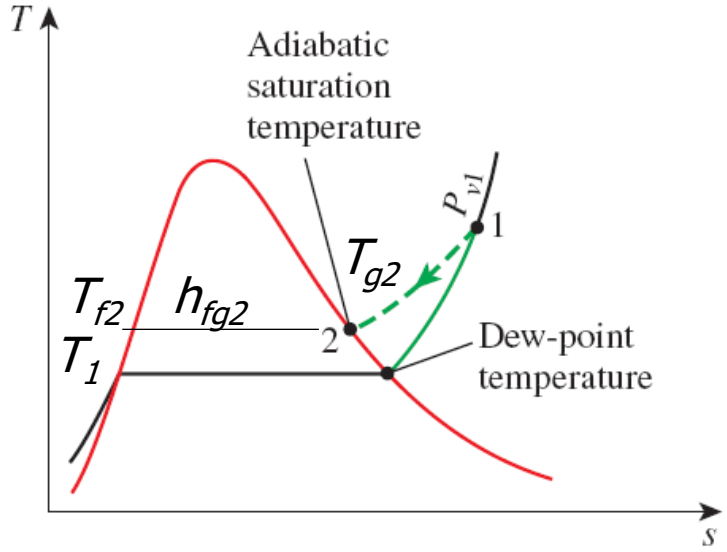
One way of determining the absolute or relative humidity is related to an *adiabatic saturation process*, shown schematically and on a *T-s* diagram.

Air having a relative humidity (ϕ_1) less than 100% flows over water contained in a well-insulated duct. Since ϕ_1 < 100%, some of the water evaporates and mixes with the air stream.

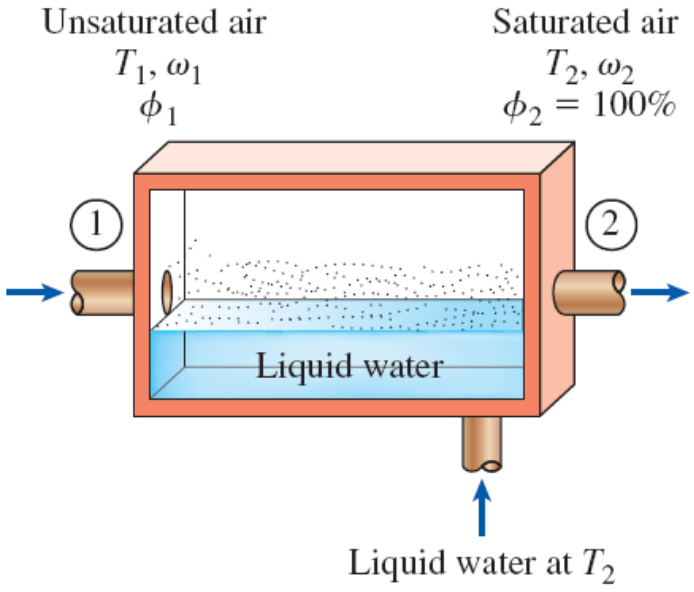
The moisture content of air increases and the temperature of the air-vapour mixture will decrease since part of the latent heat of vaporisation of the water that evaporates comes from the air.

If the channel is long enough, the airstream exits as saturated air (ϕ_2 = 100%) at temperature T_2, which is called the adiabatic saturation temperature.





If makeup water is supplied to the channel at the rate of evaporation at temperature T_2, the adiabatic saturation process can be analysed as a steady-flow process.



We assume that the total pressure is constant during the process.

Apply the conservation of energy to the steady-flow control volume

$$\dot{Q}_{net} + \sum_{inlets} \dot{m}_i \left(h + \frac{\vec{V}^2}{2} + gz \right)_i = \dot{W}_{net} + \sum_{exits} \dot{m}_e \left(h + \frac{\vec{V}^2}{2} + gz \right)_e$$

Neglecting the kinetic and potential energies and noting that the heat transfer and work are zero, we get

$$\dot{m}_1 h_1 + \dot{m}_{l2} h_{l2} = \dot{m}_2 h_2$$

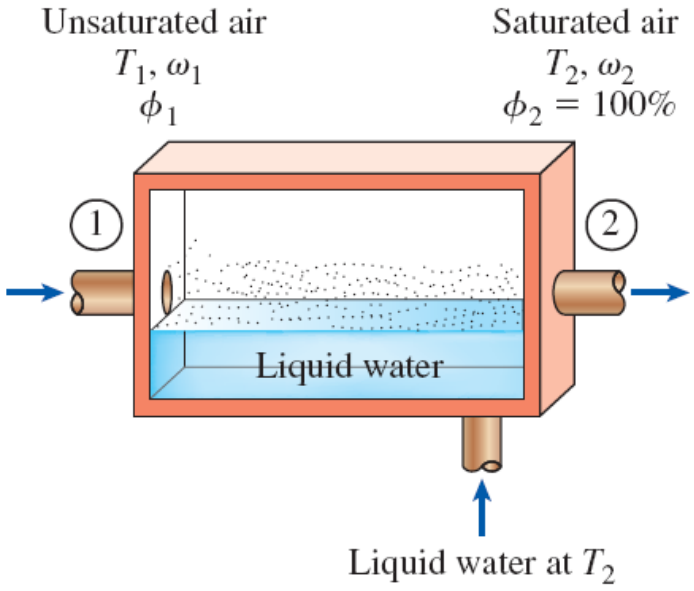
$$\dot{m}_{a1} h_{a1} + \dot{m}_{v1} h_{v1} + \dot{m}_{l2} h_{l2} = \dot{m}_{a2} h_{a2} + \dot{m}_{v2} h_{v2}$$

Conservation of mass for the steady-flow control volume is

$$\sum_{inlets} \dot{m}_i = \sum_{exits} \dot{m}_e$$

Dry air:[1] $\dot{m}_{a1} = \dot{m}_{a2} = \dot{m}_a$

Water vapour:[2] $\dot{m}_{v1} + \dot{m}_{l2} = \dot{m}_{v2}$



The mass flow rate water that must be supplied to maintain steady-flow is,

$$\dot{m}_{l2} = \dot{m}_{v2} - \dot{m}_{v1} = \dot{m}_a(\omega_2 - \omega_1)$$ where absolute humidity, $\omega = \dot{m}_v/\dot{m}_a$

Divide the conservation of energy equation $(\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{l2}h_{l2} = \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2})$ by $\dot{m}_a$, then

$$h_{a1} + \omega_1 h_{v1} + (\omega_2 - \omega_1)h_{l2} = h_{a2} + \omega_2 h_{v2}$$

What are the knowns and unknowns in this equation?

[1] The mass flow rate of dry air remains constant.

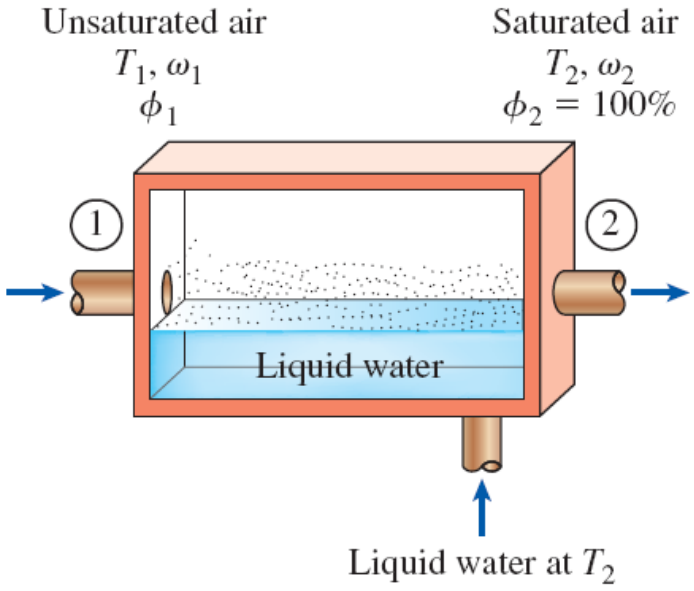
[2] The mass flow rate of vapour in the air increases by an amount equal to the rate of evaporation.

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Divide the conservation of energy equation $(\dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} + \dot{m}_{l2}h_{l2} = \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2})$ by $\dot{m}_a$, then

$$h_{a1} + \omega_1 h_{v1} + (\omega_2 - \omega_1)h_{l2} = h_{a2} + \omega_2 h_{v2}$$

What are the knowns and unknowns in this equation?

[1] The mass flow rate of dry air remains constant.

[2] The mass flow rate of vapour in the air increases by an amount equal to the rate of evaporation.

Solving for ω_1 as a function of specific enthalpies (h_a, h_v):

$$\omega_1 = \frac{h_{a2} - h_{a1} + \omega_2(h_{v2} - h_{l2})}{h_{v1} - h_{l2}}$$

Or using C_{pa}, measured temperatures T_2 and T_1 *and* latent heat of vaporisation (h_{fg2}) at T_2 :

$$= \frac{C_{pa}(T_2 - T_1) + \omega_2(h_{fg2})}{h_{g1} - h_{f2}}$$

Since ω_1 is also defined by the total pressure P_1 and the partial pressure of water vapour P_{v1} *as*

$$\omega_1 = 0.622 \frac{P_{v1}}{P_1 - P_{v1}}$$

We can solve for P_{v1} as

$$P_{v1} = \frac{\omega_1 P_1}{0.622 + \omega_1}$$

Then, the relative humidity at state 1 from P_{v1} is

$$\boxed{\phi_1 = \frac{P_{v1}}{P_{g1}}}$$

Solving for ω_1 as a function of specific enthalpies (h_a, h_v):

$$\omega_1 = \frac{h_{a2} - h_{a1} + \omega_2(h_{v2} - h_{l2})}{h_{v1} - h_{l2}}$$

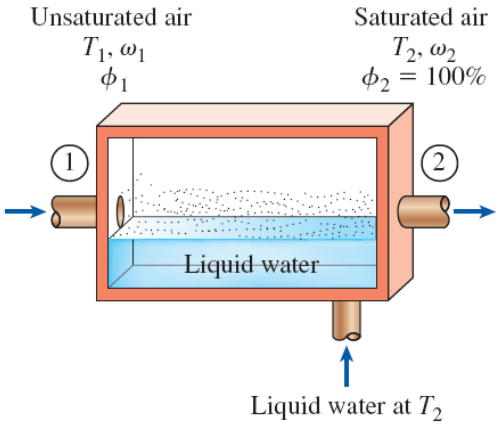
Or using C_{pa}, measured temperatures T_2 and T_1 *and* latent heat of vaporisation (h_{fg2}) at T_2 :

$$= \frac{C_{pa}(T_2 - T_1) + \omega_2(h_{fg2})}{h_{g1} - h_{f2}}$$

$$\omega_2 = 0.622 \frac{P_{v2}}{P_2 - P_{v2}}$$

$$P_{v2} = \frac{\omega_2 P_2}{0.622 + \omega_2}$$

$$\phi_2 = \frac{P_{v2}}{P_{g2}} = 100\%$$



Thus we conclude that the specific humidity (and relative humidity) of air can be determined from by measuring the pressure and temperature of air at the inlet and the exit of an adiabatic saturator.

Wet-Bulb and Dry-Bulb Temperatures

The adiabatic saturation process provides a means of determining the absolute or relative humidity of air, but it requires a long channel or a spray mechanism to achieve saturation conditions at the exit.

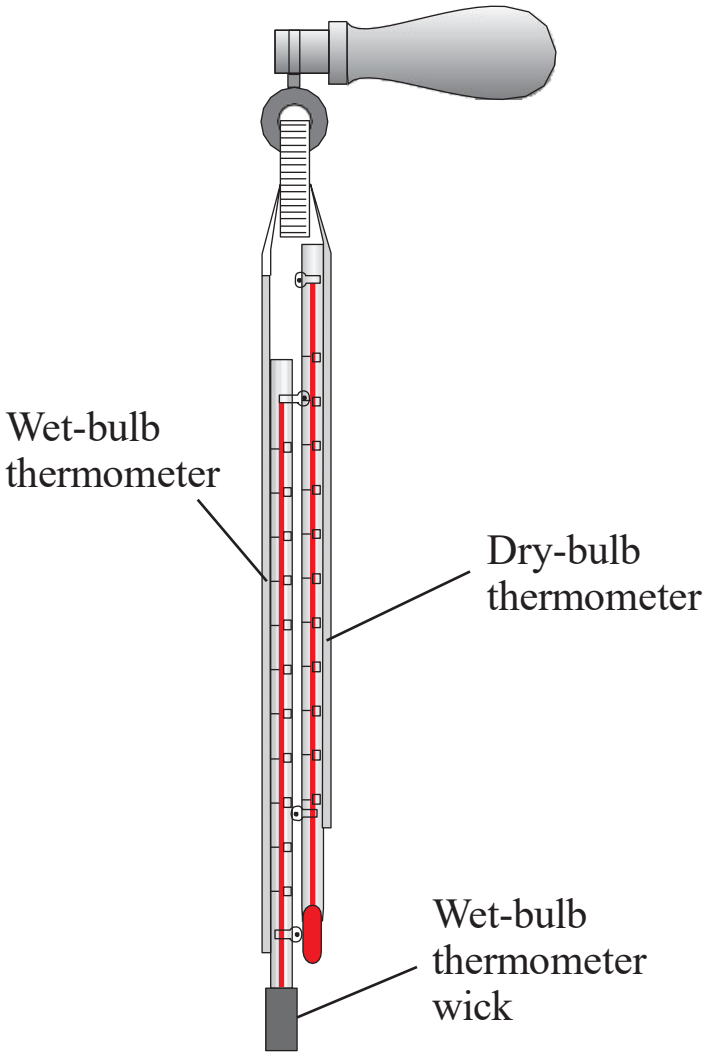
In normal practice, the state of atmospheric air is specified by determining the wet-bulb and dry-bulb temperatures, T_{wb} and T_{db}.

These temperatures are measured by using a device called a psychrometer.

https://www.youtube.com/watch?v=7Lib0dI_tmc

The psychrometer is composed of two thermometers mounted on a rotating frame.

- One thermometer is fitted with a wet gauze (or cotton wick saturated with water) and reads the wet-bulb temperature.
- The other thermometer reads the dry-bulb or ordinary temperature.



Sling psychrometer

Wet-Bulb and Dry-Bulb Temperatures

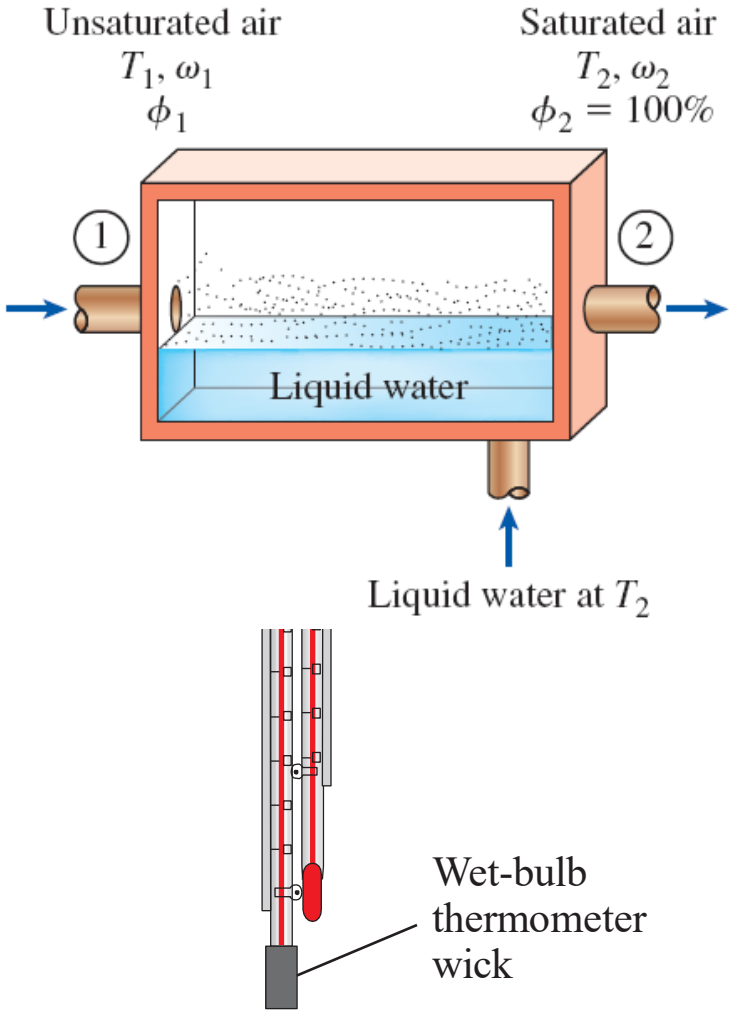
As the psychrometer is swung through the air, water vapourises from the wet gauze, resulting in a lower temperature to be registered by the thermometer.

The dryer the atmospheric air, the lower the wet-bulb temperature will be.

When the relative humidity of the air is near 100%, there will be little difference between the wet-bulb and dry-bulb temperatures.

The wet-bulb temperature is approximately equal to the adiabatic saturation temperature. Therefore, the wet-bulb temperature T_{wb} can be used in place of T_2 (exit temperature of saturated air) to determine the specific humidity of air.

The wet-bulb and dry-bulb temperatures and the atmospheric pressure uniquely determine the state of the atmospheric air.



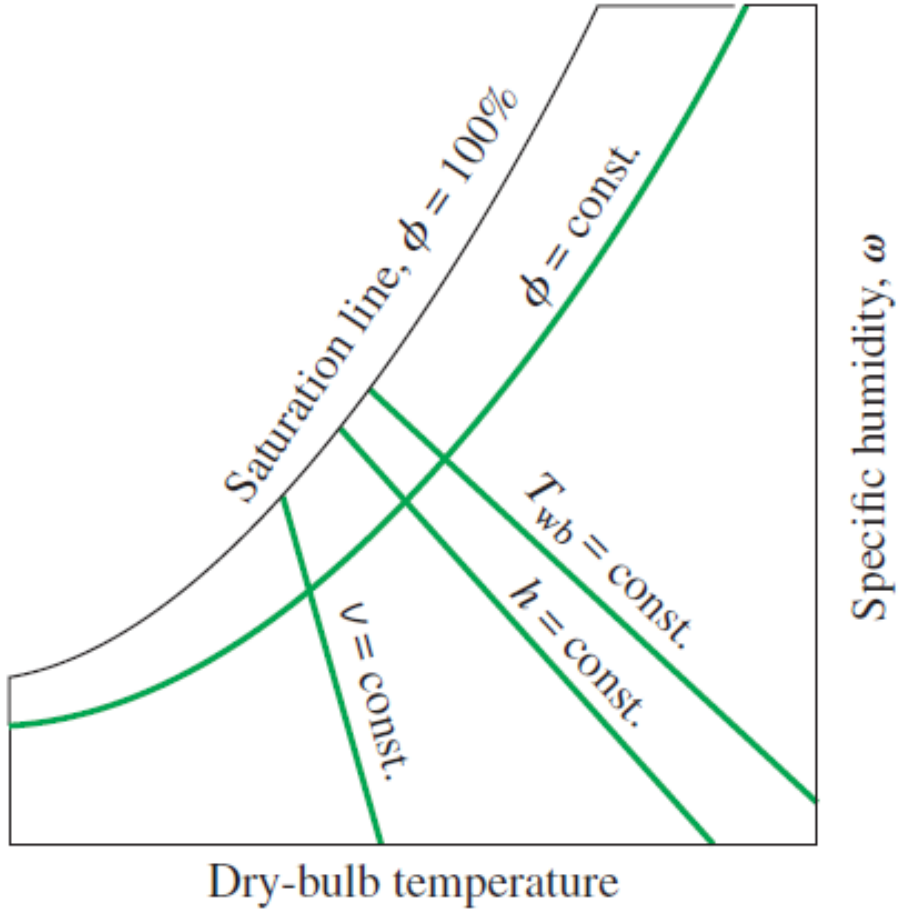
Sling psychrometer

The Psychrometric Chart

Psychrometric chart: A graphical representation of the physical and thermal properties of atmospheric air for a particular barometric pressure.

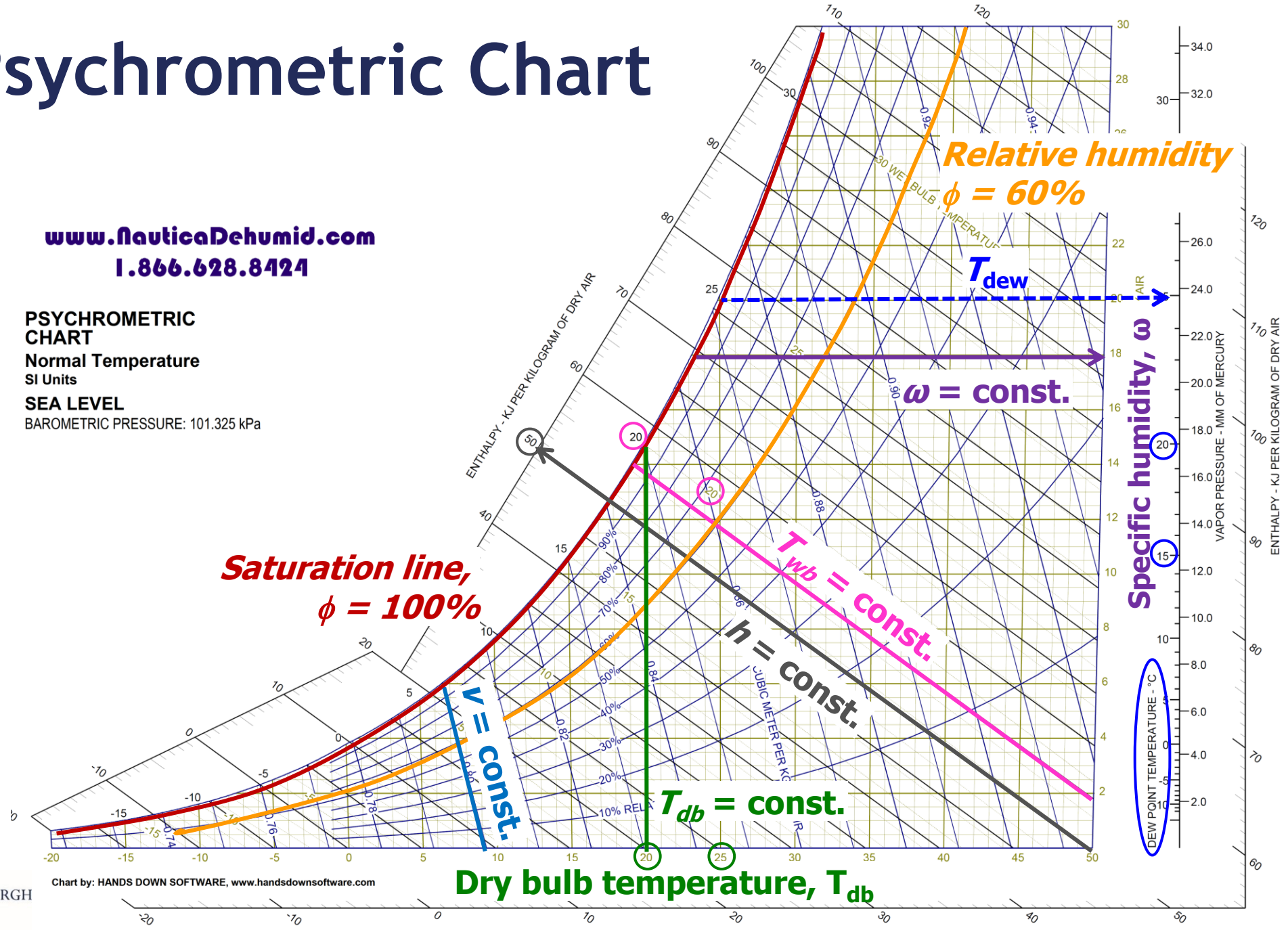
Presents moist air properties in a convenient form. They are used extensively in air-conditioning (A-C) applications. The psychrometric chart serves as a valuable aid in visualising the A-C processes such as heating, cooling, and humidification.

- Dry-bulb temperature is shown on the horizontal axis
- Specific humidity is shown on the vertical axis
- Curved lines are saturation lines, give relative humidity
- …



Schematic of a Psychrometric Chart

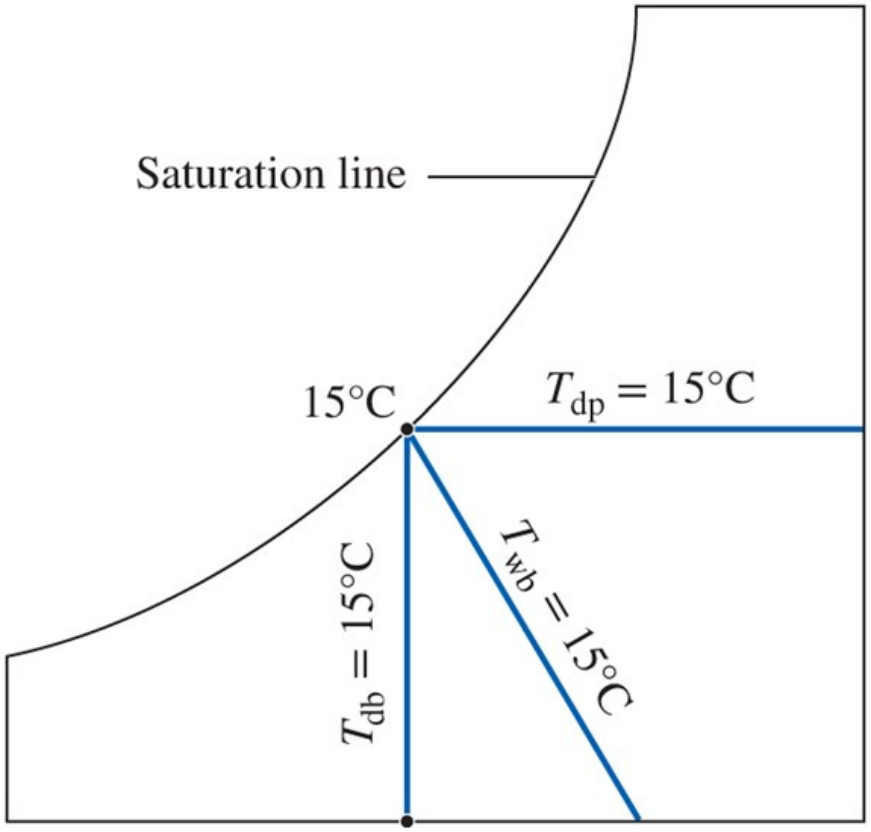
The Psychrometric Chart



The Psychrometric Chart

- Dry-bulb temperature, T_{db}
- Humidity ratio, ω
- Dew point temperature, T_{dp}?

Since the dew point is the state where moist air becomes saturated when cooled at constant pressure, the dew point for a given state is determined from the chart by following a line of constant ω (constant p_v) to the saturation line where $\phi = 100\%$.



For saturated air ($\phi = 100\%$), the dry-bulb, wet-bulb, and dew-point temperatures are identical.

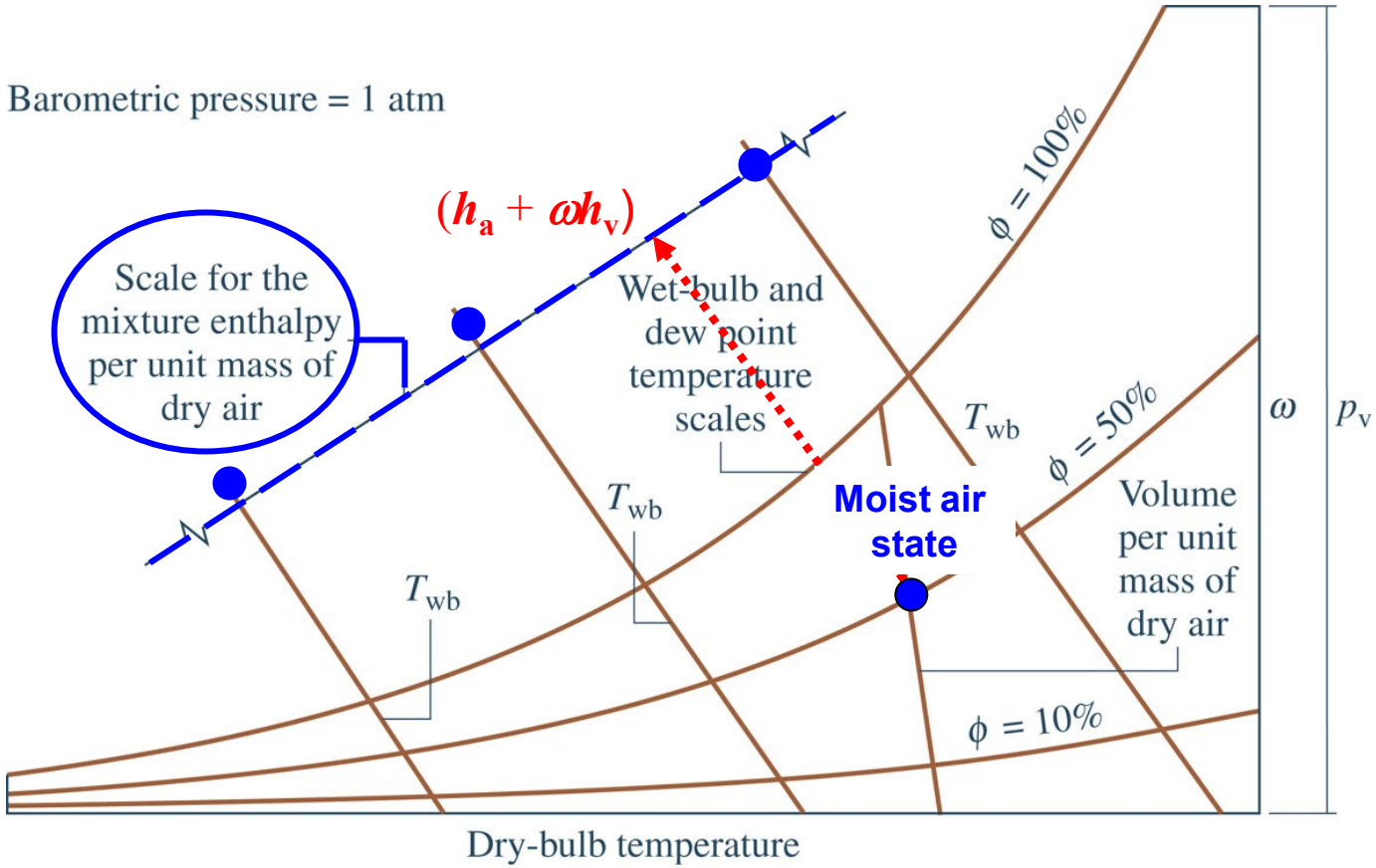
The Psychrometric Chart

- Mixture enthalpy per unit mass of dry air, $(h_a + \omega h_v)$

The value of $(h_a + \omega h_v)$ is calculated using

$$h_a = C_{pa} T$$

T in °C, $C_{pa} = 1.005$ kJ/kg·K



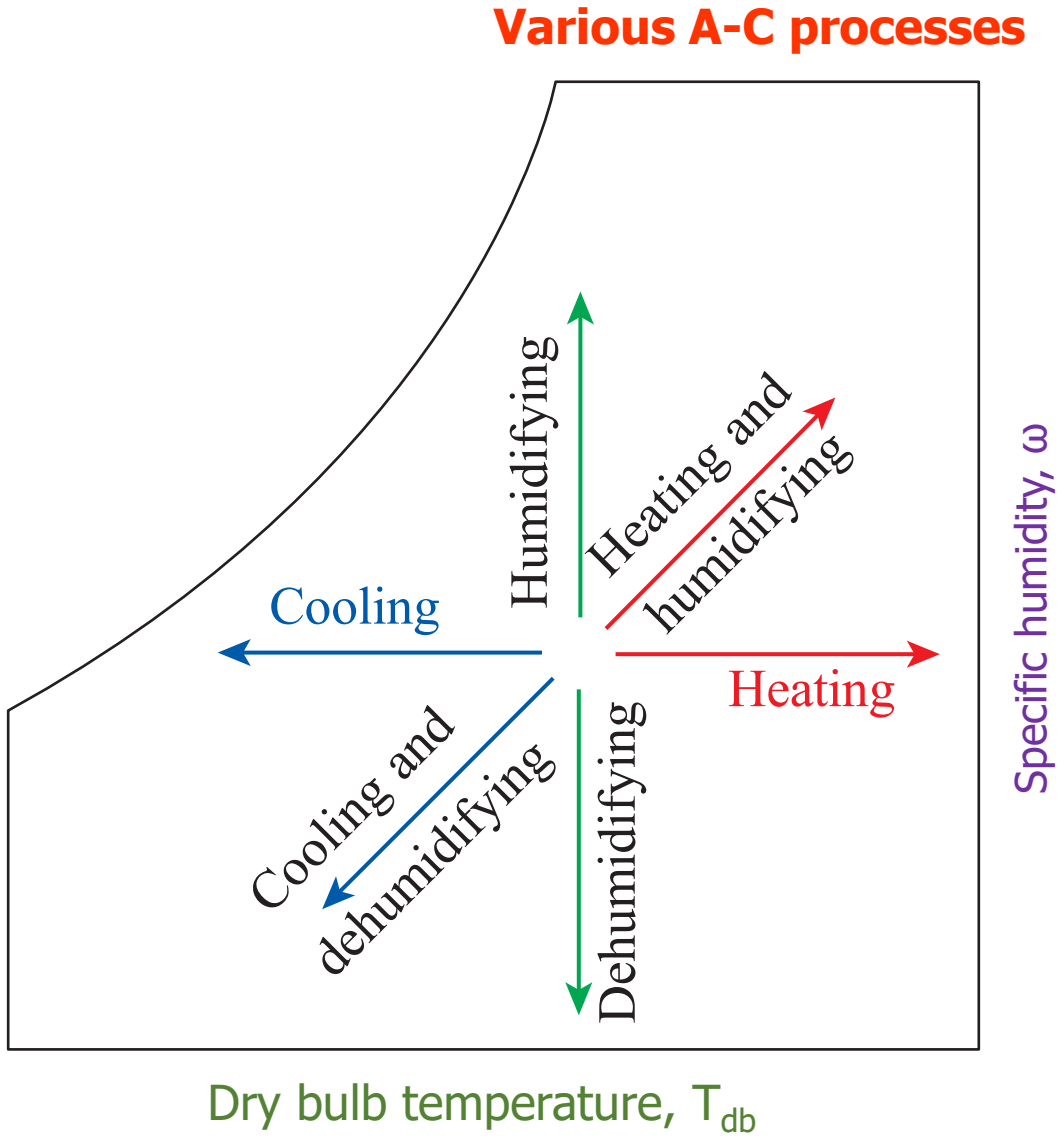
Air-Conditioning Processes

Maintaining a living space or an industrial facility at the desired temperature and humidity requires some processes called air-conditioning processes.

These processes include
simple heating (raising the temperature),
simple cooling (lowering the temperature),
humidifying (adding moisture), and
dehumidifying (removing moisture).

Sometimes two or more of these processes are needed to bring the air to a desired temperature and humidity level.

Air is commonly heated and humidified in winter and cooled and dehumidified in summer.



Air-Conditioning Processes

Most air-conditioning processes can be modelled as steady-flow processes with the following general mass and energy balances:

- Mass balance

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}}$$

- Mass balance for dry air:

$$\sum_{\text{in}} \dot{m}_a = \sum_{\text{out}} \dot{m}_a \quad (\text{kg/s})$$

- Mass balance for water:

$$\sum_{\text{in}} \dot{m}_w = \sum_{\text{out}} \dot{m}_w \text{ or } \sum_{\text{in}} \dot{m}_a \omega = \sum_{\text{out}} \dot{m}_a \omega$$

- Energy balance

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

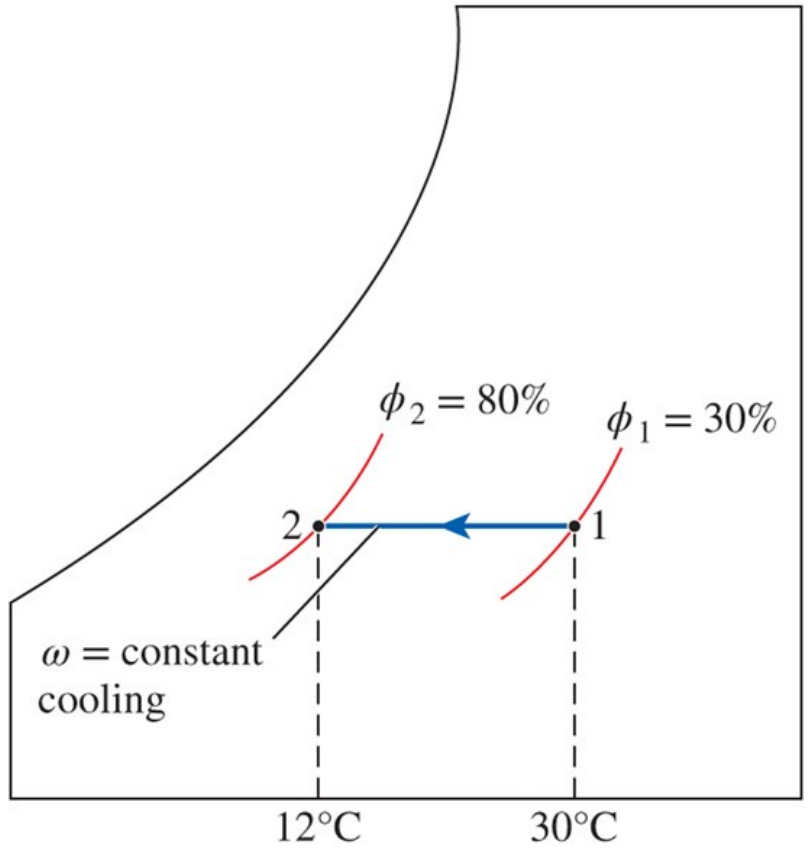
$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum_{\text{in}} \dot{m}h = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum_{\text{out}} \dot{m}h$$

The work term usually consists of the *fan work input*, which is small relative to the other terms in the energy balance relation.

Air-Conditioning Processes

Simple Heating and Cooling (ω = constant)

- Many residential heating systems consist of a stove, a heat pump, or an electric resistance heater. The air in these systems is heated by circulating it through a duct that contains the tubing for the hot gases or the electric resistance wires.
- Cooling can be accomplished by passing the air over some coils through which a refrigerant or chilled water flows.
- Heating and cooling appear as a horizontal line since no moisture is added to or removed from the air.



- During simple cooling, specific humidity remains constant, but relative humidity increases.

Air-Conditioning Processes

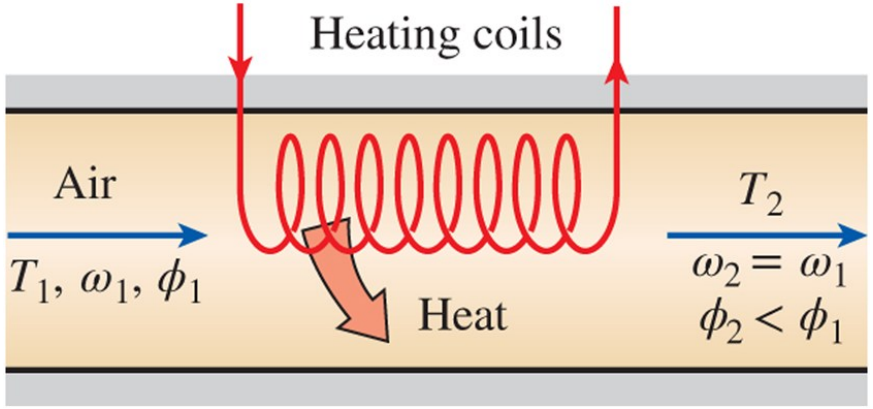
Simple Heating and Cooling (ω = constant)

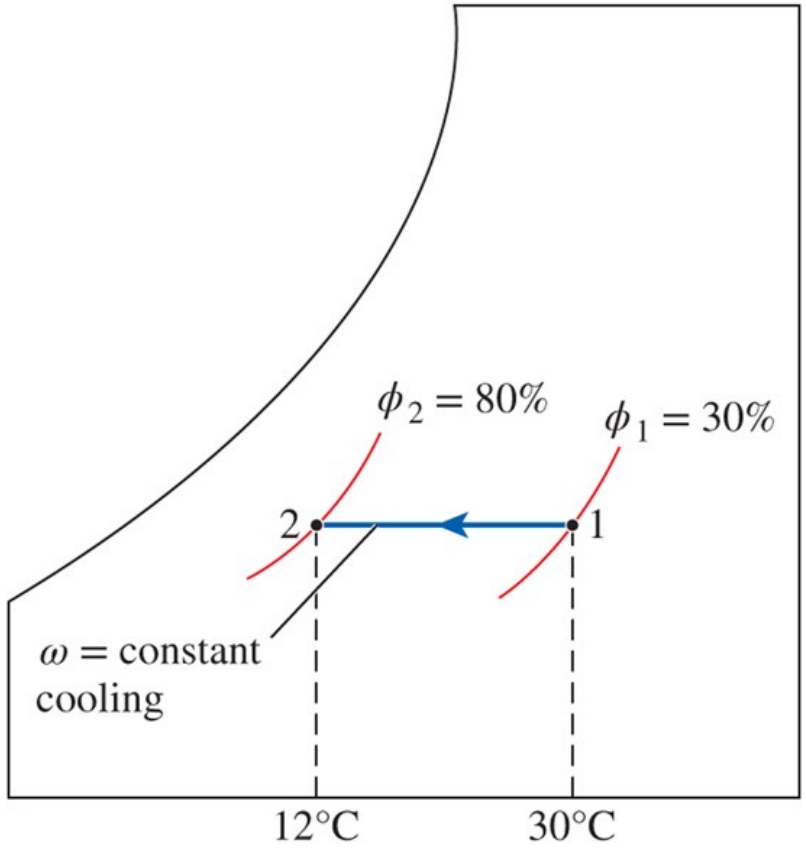
$$\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$$ • Dry air mass balance

$$\omega_1 = \omega_2$$ • Water mass balance

$$\dot{Q} = \dot{m}_a \left(h_2 - h_1 \right)$$ • Energy balance

$$q = h_2 - h_1$$



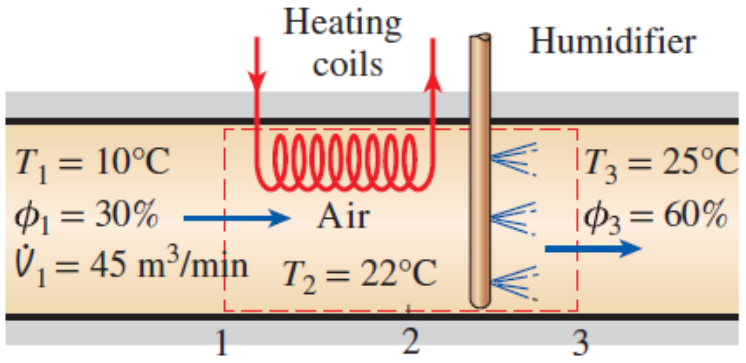


- During simple heating, specific humidity (ω) remains constant, but relative humidity (ϕ) decreases.
- During simple cooling, specific humidity remains constant, but relative humidity increases.

Air-Conditioning Processes

Heating with Humidification

Problems with the low relative humidity resulting from simple heating can be eliminated by humidifying the heated air. The air is first passed through a heating section and then through a humidifying section.



- Dry air mass balance:

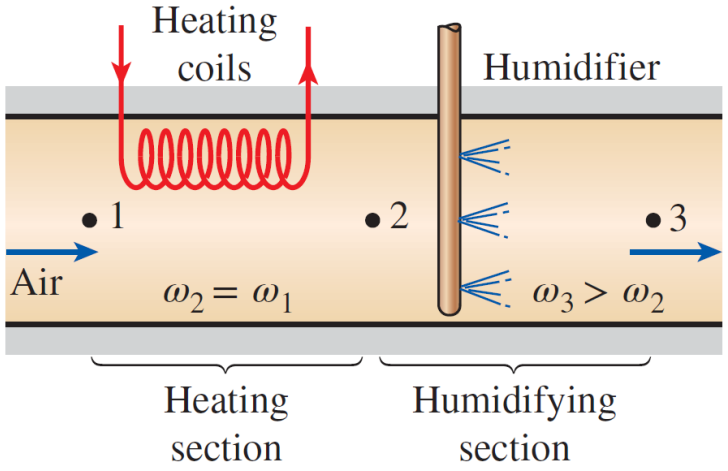
$$\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$$

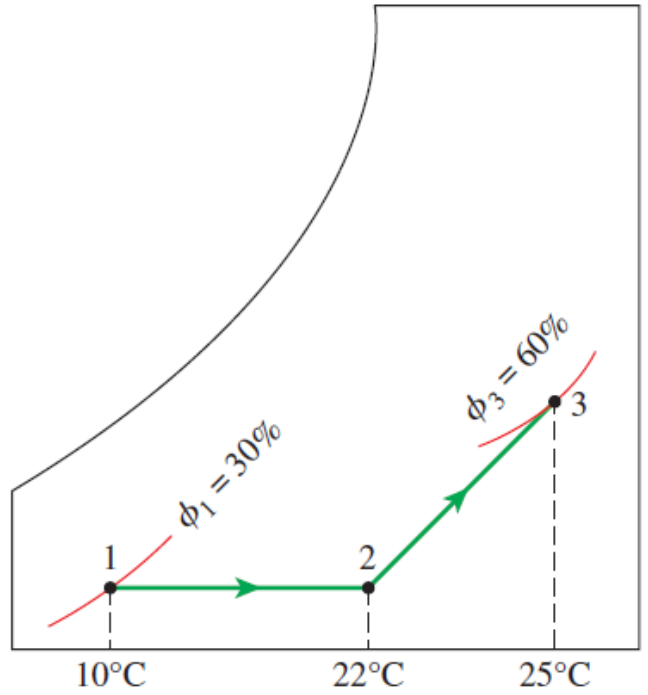
- Water mass balance:

$$\dot{m}_{a_1}\omega_1 = \dot{m}_{a_2}\omega_2 \rightarrow \omega_1 = \omega_2$$

$$\dot{m}_{a_2}\omega_2 + \dot{m}_w = \dot{m}_{a_3}\omega_3 \rightarrow \dot{m}_w = \dot{m}_a\left(\omega_3 - \omega_2\right)$$

- Energy balance: $\dot{Q}_{\text{in}} + \dot{m}_a h_1 = \dot{m}_a h_2 \rightarrow \dot{Q}_{in} = \dot{m}_a\left(h_2 - h_1\right)$



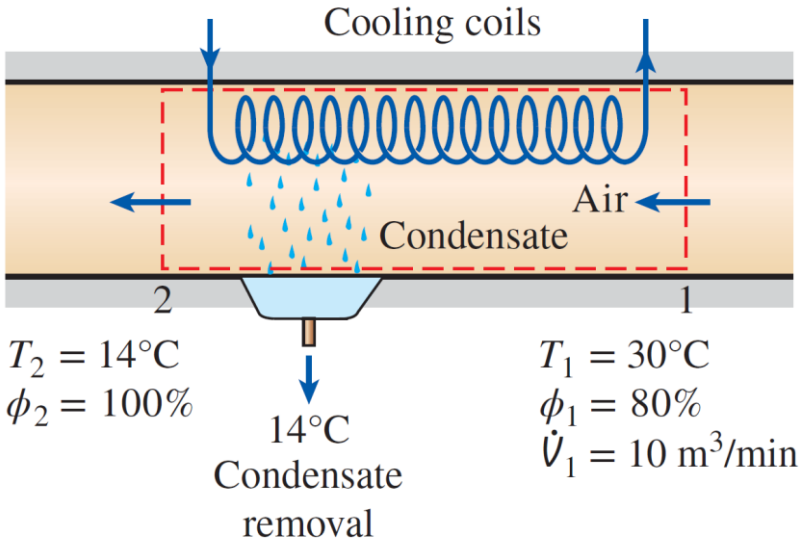


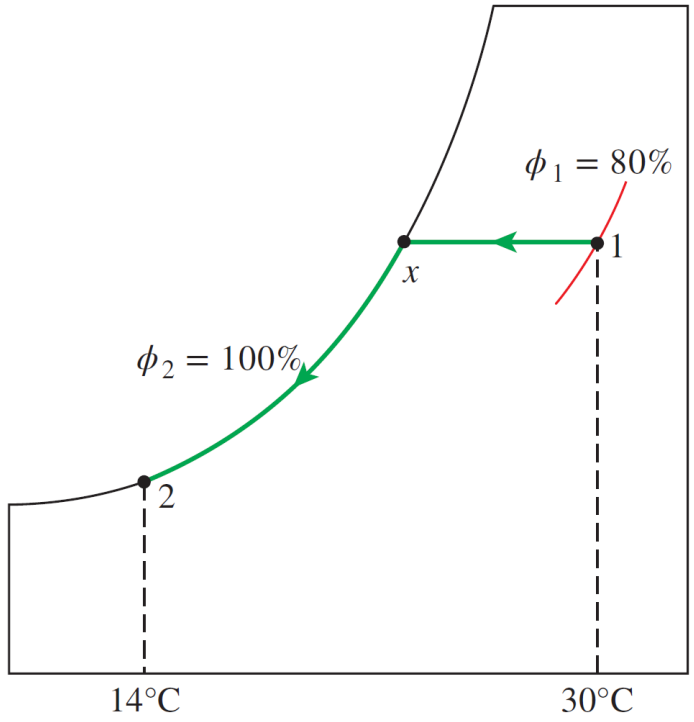
Air-Conditioning Processes

Cooling with Dehumidification

The specific humidity of air remains constant during a simple cooling process, but its relative humidity increases. If the relative humidity reaches undesirably high levels, it may be necessary to remove some moisture from the air, that is, to dehumidify it. This requires cooling the air below its dew-point temperature.

- Dry air mass balance: $\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$
- Water mass balance: $\dot{m}_{a_1}\omega_1 = \dot{m}_{a_2}\omega_2 + \dot{m}_w \rightarrow \dot{m}_w = \dot{m}_a(\omega_1 - \omega_2)$
- Energy balance: $\sum_{\text{in}} \dot{m}h = \dot{Q}_{\text{out}} + \sum_{\text{out}} \dot{m}h \rightarrow \dot{Q}_{\text{out}} = \dot{m}(h_1 - h_2) - \dot{m}_w h_w$





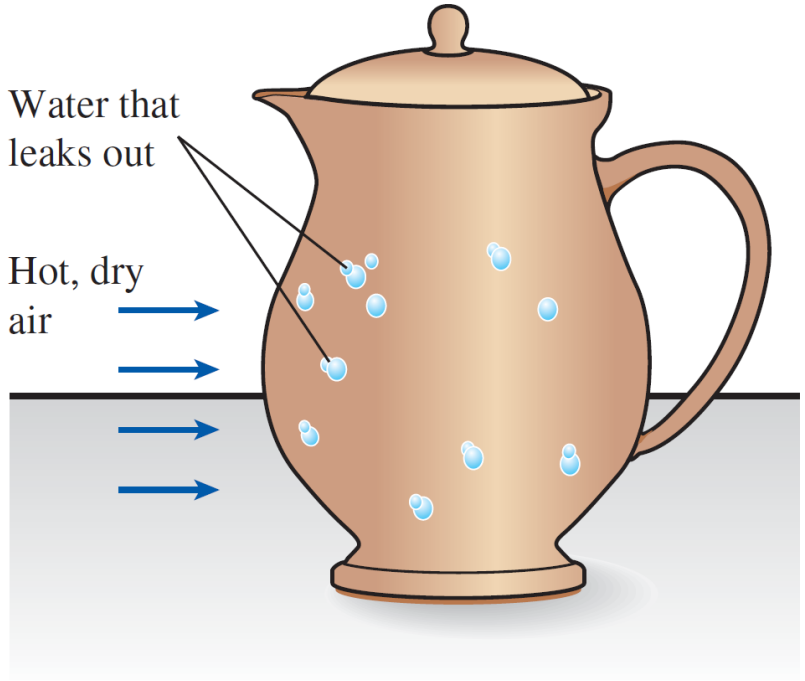
Air-Conditioning Processes

Evaporative Cooling

In desert (*hot and dry*) climates, we can avoid the high cost of conventional cooling by using *evaporative coolers*, also known as *swamp coolers*.

As water evaporates, the latent heat of vaporisation is absorbed from the water body and the surrounding air. As a result, both the water and the air are cooled during the process.

Evaporative cooling process is essentially identical to adiabatic saturation process.



Water in a porous jug left in an open, breezy area cools as a result of evaporative cooling.

Example 5.2

Air conditioning (cooling, drying and re-heating)

Air at 1 atm, 35°C and with a relative humidity of 80% is cooled to 18°C in an air conditioning unit.

a) What is the specific humidity at 35°C?

b) What is the relative humidity and specific humidity at 18°C?

c) What temperature would the air have to be raised to after cooling to 18°C to get a RH of 50%, and what is its specific humidity at that point?

d) How much heat has to be removed per kg/s of dry air to cool the vapour/air mixture to 18°C?

e) How much heat has to be added per kg/s of dry air to subsequently heat the vapour/air mixture to get a RH of 50%?

Quick Summary of THIS Part

- Relative humidity
- Absolute humidity
- Adiabatic saturation temperature
- Wet-bulb and dry-bulb temperatures
- A useful tool: the Psychrometric chart
- Air-conditioning processes
- Applications and an example problem

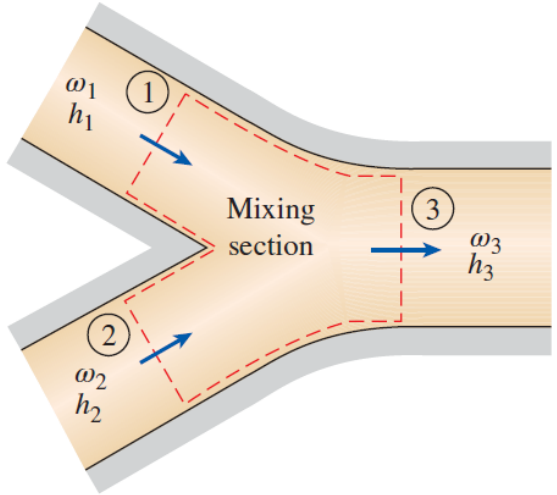
Next

- Adiabatic mixing of airstreams and wet cooling towers

Air-Conditioning Processes

Adiabatic Mixing of Airstreams

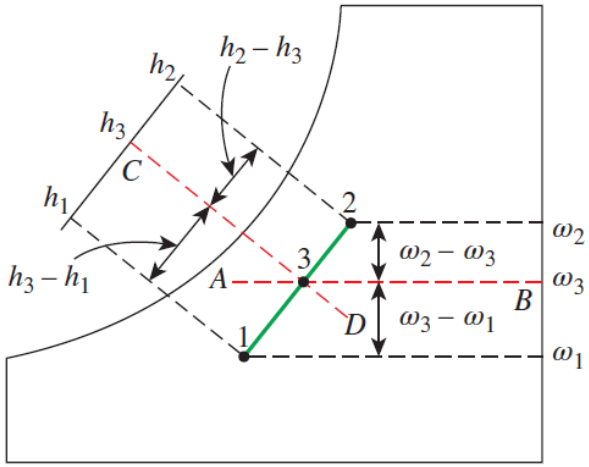
Many A-C applications require the mixing of two airstreams. This is particularly true for large buildings, most production and process plants, and hospitals, which require that the conditioned air be mixed with a certain fraction of fresh outside air before it is routed into the living space.



- Mass of dry air: $\dot{m}_{a_1} + \dot{m}_{a_2} = \dot{m}_{a3}$
- Mass of water vapour: $\omega_1 \dot{m}_{a_1} + \omega_2 \dot{m}_{a_2} = \omega_3 \dot{m}_{a_3}$
- Energy balance: $\dot{m}_{a_1} h_1 + \dot{m}_{a_2} h_2 = \dot{m}_{a_3} h_3$

Eliminating m_{a3} from the preceding relations,

$$\frac{\dot{m}_{a_1}}{\dot{m}_{a_2}} = \frac{\omega_2 - \omega_3}{\omega_3 - \omega_1} = \frac{h_2 - h_3}{h_3 - h_1}$$



When two airstreams at states 1 and 2 are mixed adiabatically, the state of the mixture lies on the straight line connecting the two states.

Example 5.3

Mixing of Two Airstreams

Two airstreams are mixed steadily and adiabatically. The first stream enters at 35°C and 30 percent relative humidity at a rate of 15 m^3/min, while the second stream enters at 12°C and 90 percent relative humidity at a rate of 25 m^3/min.

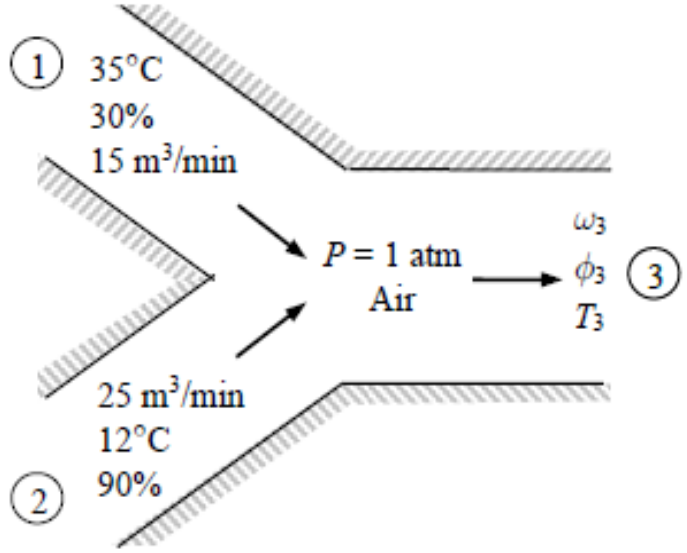
Assuming that the mixing process occurs at a pressure of 1 atm, determine:

the specific humidity,

the relative humidity,

the dry-bulb temperature, and

the volume flow rate of the mixture.



Air-Conditioning Processes

Wet Cooling Towers

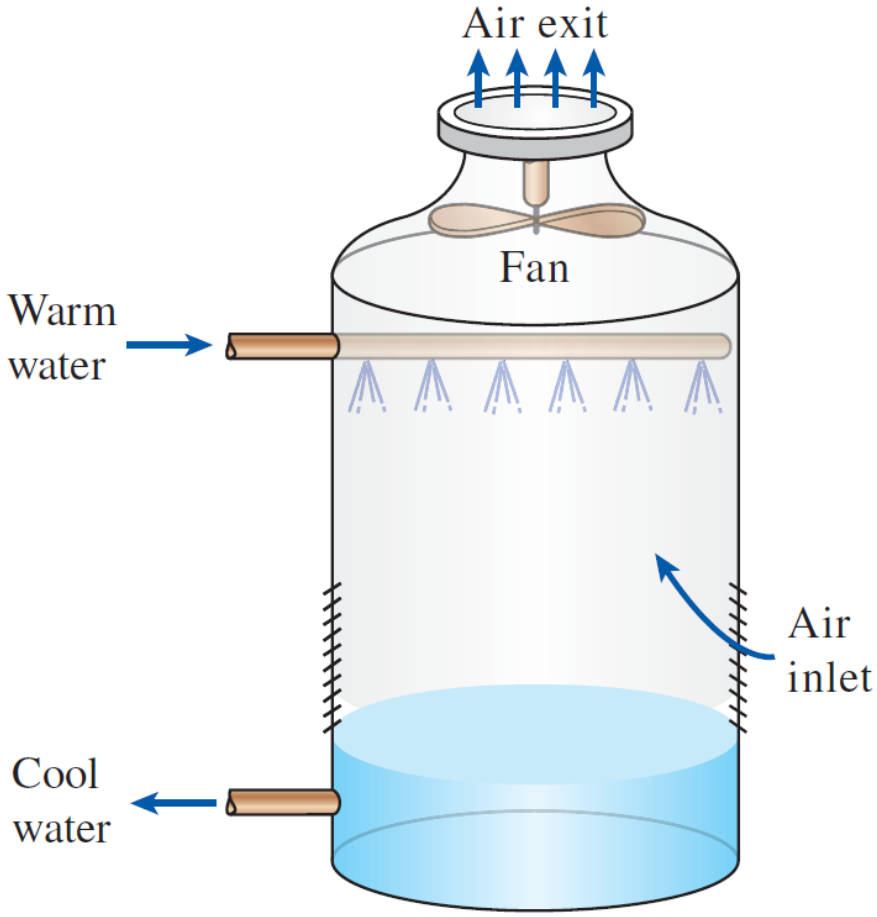
Power plants, large air-conditioning systems, and some industries generate large quantities of waste heat that is often rejected to cooling water from nearby lakes or rivers.

In some cases, however, the cooling water supply is limited or thermal pollution is a serious concern.

In such cases, the waste heat must be rejected to the atmosphere, with cooling water recirculating and serving as a transport medium for heat transfer between the source and the sink (the atmosphere).

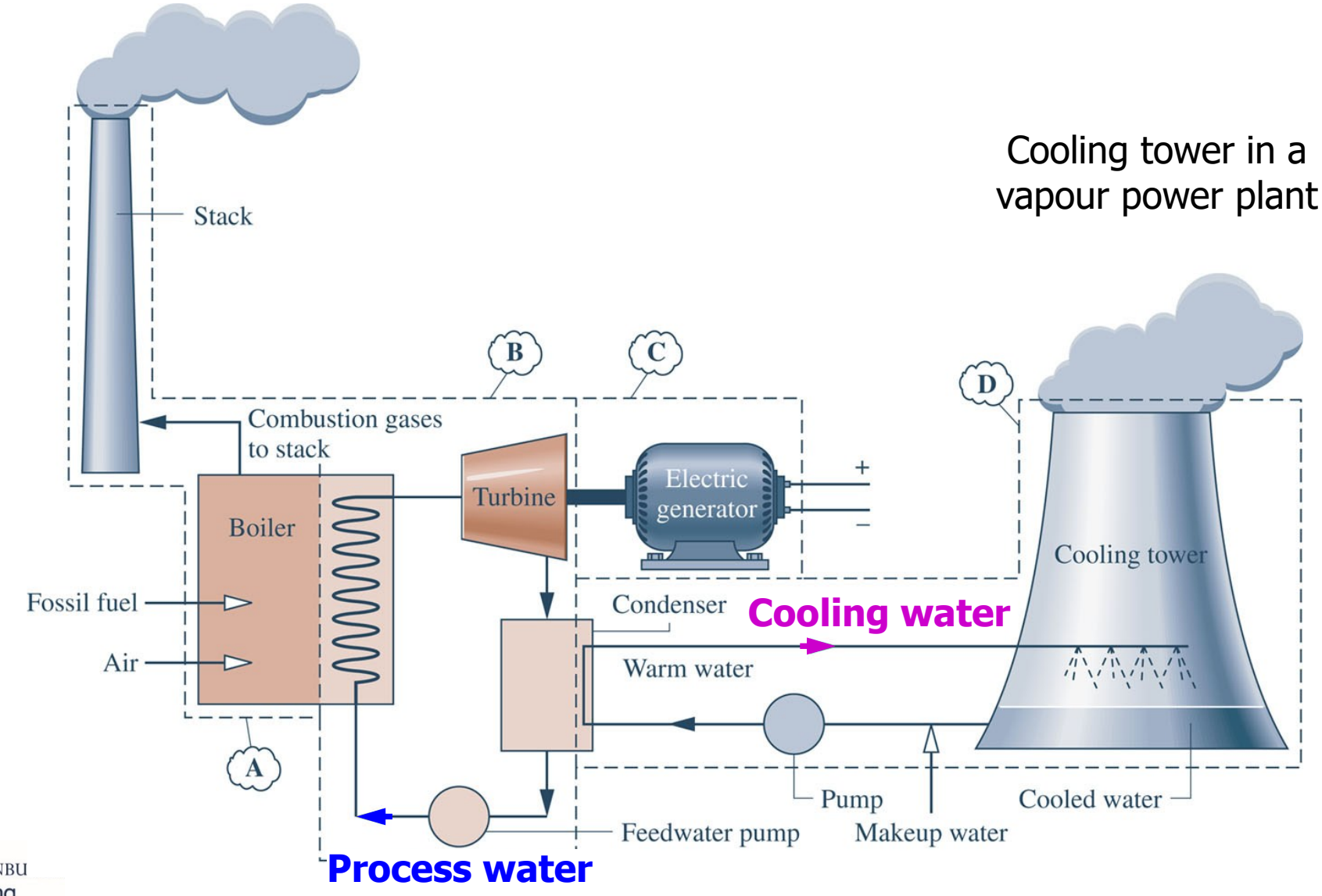
One way of achieving this is through the use of wet cooling towers.

A wet cooling tower is essentially a semi-enclosed evaporative cooler.



An induced-draft counterflow cooling tower.

Cooling Towers



Cooling tower in a vapour power plant

Cooling Towers

Cooling towers are a special type of heat exchangers in which air and water are brought into direct contact with each other in order to reduce the temperature of the hot water. Small volumes of water evaporate, lowering the amount of the water that is being circulated through the cooling tower.



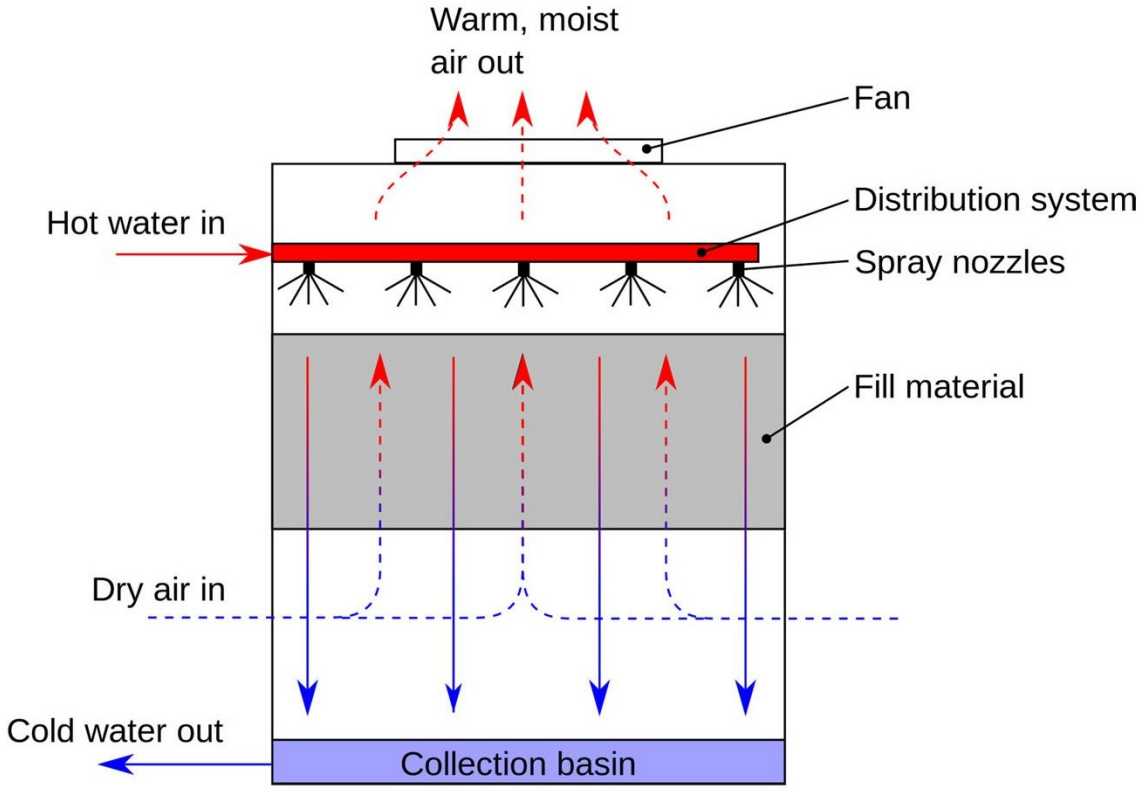
http://spxcooling.com/en/products/list/cooling/cooling-towers/



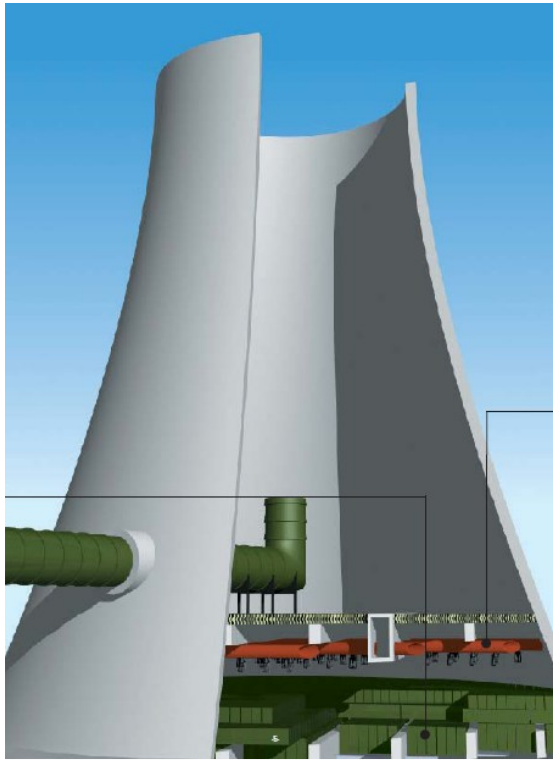
Cooling tower simultaneously used to discharge treated flue gases from the 550 MW power generation unit of a coal-fired power station

How Cooling Towers Work

- There are many different types of cooling towers but the cooling tower working principles stay pretty much the same. Most cooling towers work based on the principle of "evaporative cooling".



https://www.coolingtowerproducts.com/blog/how-cooling-towers-work-diagram-pictures-2015.htm

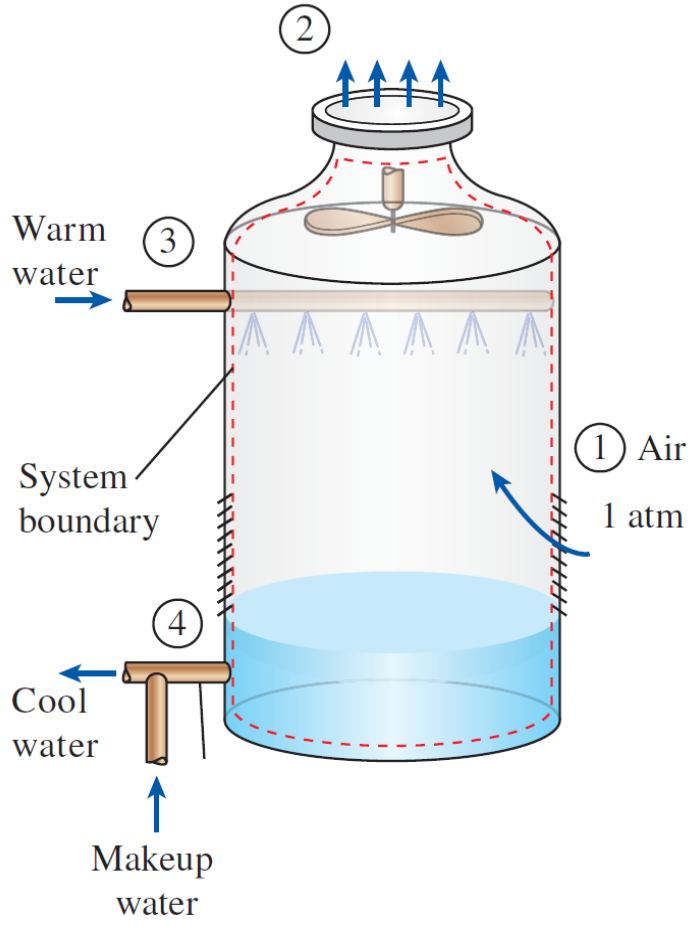


http://spxcooling.com/en/products/list/cooling/cooling-towers/

Analysis of a Wet Cooling Tower

Assumptions

1. Steady operating conditions exist and thus mass flow rate of dry air remains constant during the entire process.
2. Dry air and water vapour are ideal gases.
3. The kinetic and potential energy changes are negligible.
4. The cooling tower is adiabatic.



Dry air mass balance: $\dot{m}_{a_1} = \dot{m}_{a_2} = \dot{m}_a$

Water mass balance: $\dot{m}_3 + \dot{m}_{a_1}\omega_1 = \dot{m}_4 + \dot{m}_{a_2}\omega_2$

$$\dot{m}_3 - \dot{m}_4 = \dot{m}_a\left(\omega_2 - \omega_1\right) = \dot{m}_{\text{makeup}}$$

Energy balance: $\sum_{\text{in}} \dot{m}h = \sum_{\text{out}} \dot{m}h \rightarrow \dot{m}_{a_1}h_1 + \dot{m}_3 h_3 = \dot{m}_{a_2}h_2 + \dot{m}_4 h_4$

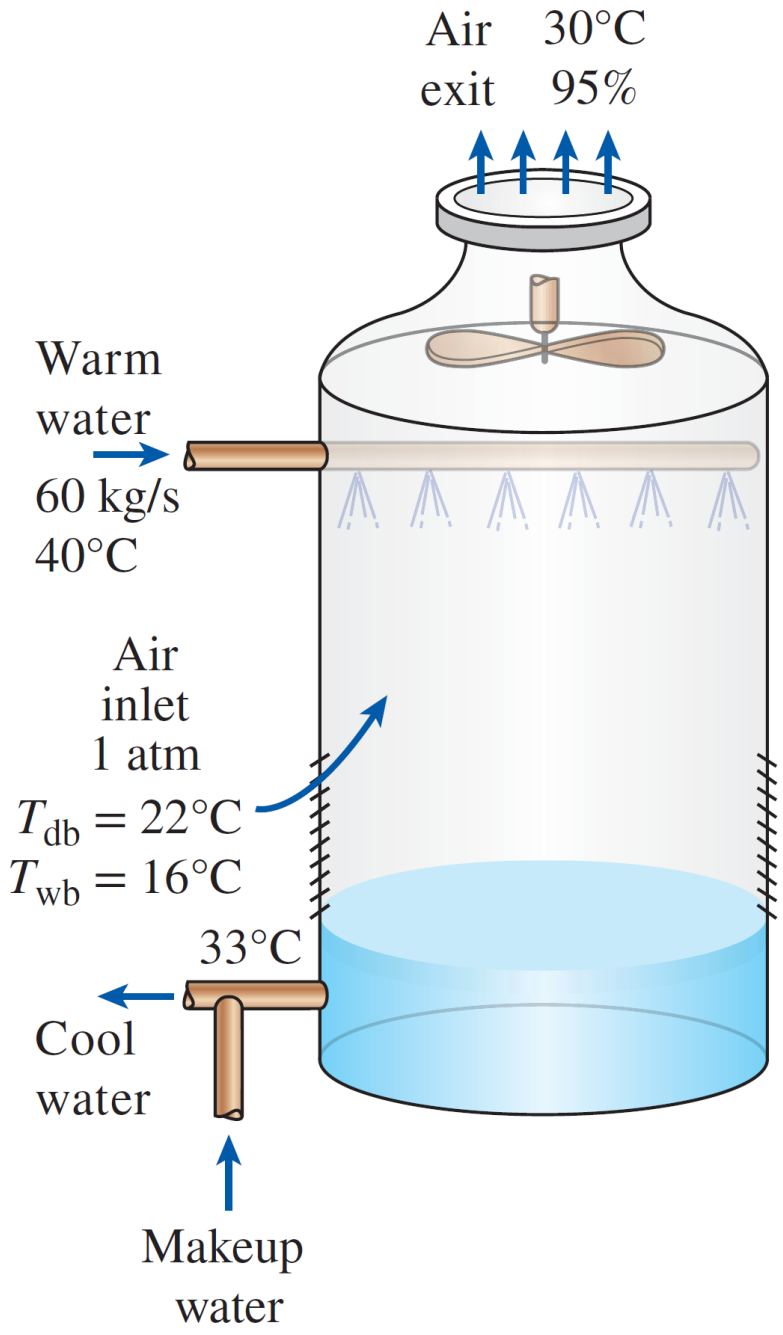
$$\dot{m}_3 h_3 = \dot{m}_a\left(h_2 - h_1\right) + \left(\dot{m}_3 - \dot{m}_{\text{makeup}}\right)h_4 \quad \Rightarrow \dot{m}_a = \frac{\dot{m}_3\left(h_3 - h_4\right)}{\left(h_2 - h_1\right) - \left(\omega_2 - \omega_1\right)h_4}$$

Example 5.4

A wet cooling tower is to cool 60 kg/s of water from 40 to 33°C. Atmospheric air enters the tower at 1 atm with dry and wet-bulb temperatures of 22 and 16°C, respectively, and leaves at 30°C with a relative humidity of 95 percent. Using the psychrometric chart, determine

(a) the volume flow rate of air into the cooling tower and

(b) the mass flow rate of the required makeup water.



Thanks for listening

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