Thermodynamics (Mechanical) 4

Gas Turbines and Brayton Cycle

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Overview for this Unit

CONTENT

- Simple gas turbine and its energy balance
- Turbine and compressor efficiency
- Options for improving Brayton cycle performance

AIM

- To provide you with underpinning knowledge needed to successfully analyse these cycles

Ref: Borgnakke and Sonntag (8th or 10th ed) Sections 10.1–10.4

Çengel (9th ed) Sections 7.4, 7.11, 7.12

Gas Turbine Power Plants

- Gas turbine power plants are more quickly constructed, less costly, and more compact than the vapour power plants.
- Gas turbines are suited for stationary power generation as well as for powering vehicles, including aircraft propulsion and marine power plants.

  ➢ Gas turbines are
  - increasingly used for large-scale power generation, and
  - for such applications fueled primarily by natural gas.



Image source: Allied Power Group

Simple Gas Turbine (Open Cycle)

- The *open* mode gas turbine is an *internal combustion* power plant.

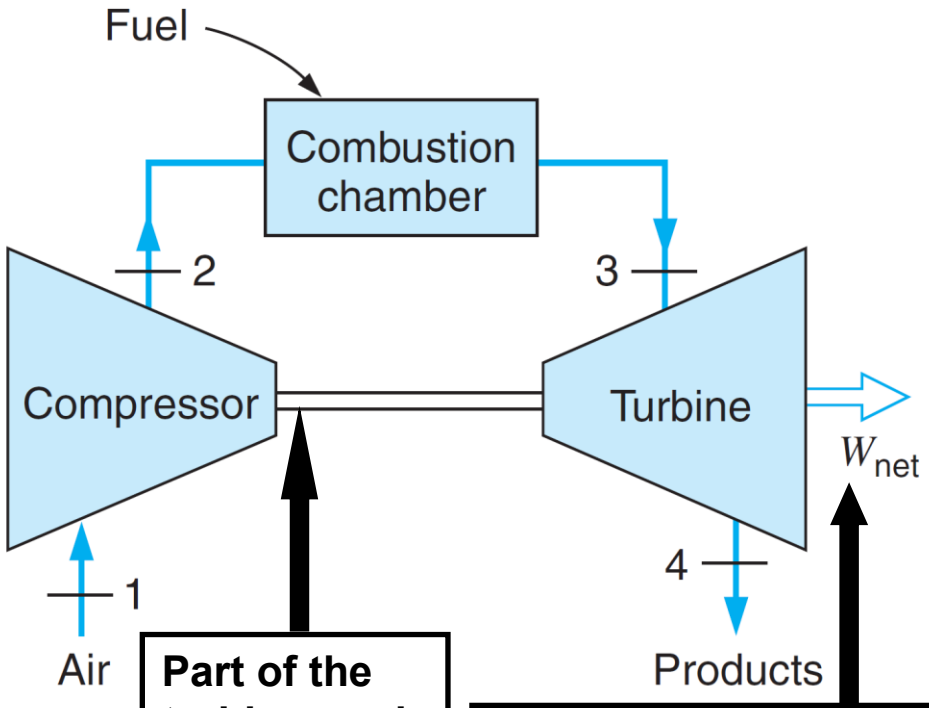
1) Fresh air at ambient conditions is continuously drawn into the compressor where it is compressed to a high pressure.

2) The high-P air proceeds into the combustion chamber, where it mixes with fuel and the mixture is burned at constant pressure.

3) The resulting high-T combustion products then enter the turbine where they expand to atmospheric pressure while producing power.

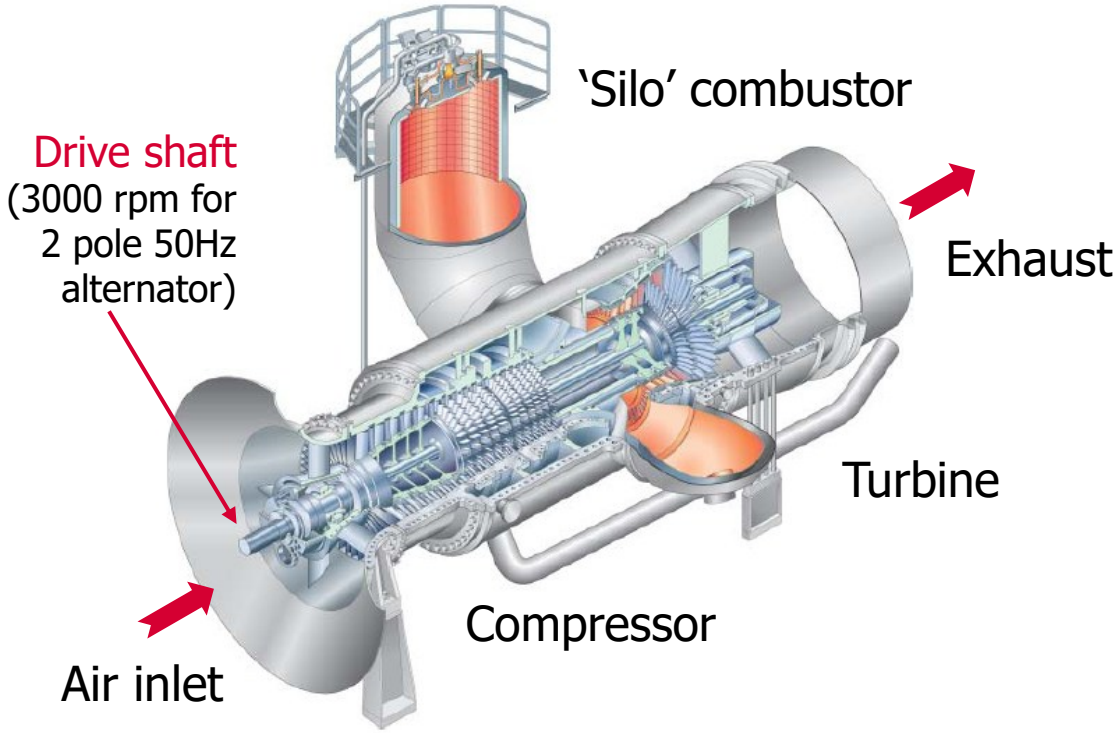
4) The exhaust gases (combustion products) leaving the turbine are discharged to the surroundings.

Note: Two functions of air: 1) it supplies the necessary **oxidant** for the combustion of the fuel, and 2) it serves as a coolant to keep the temperature of various components within safe limits.

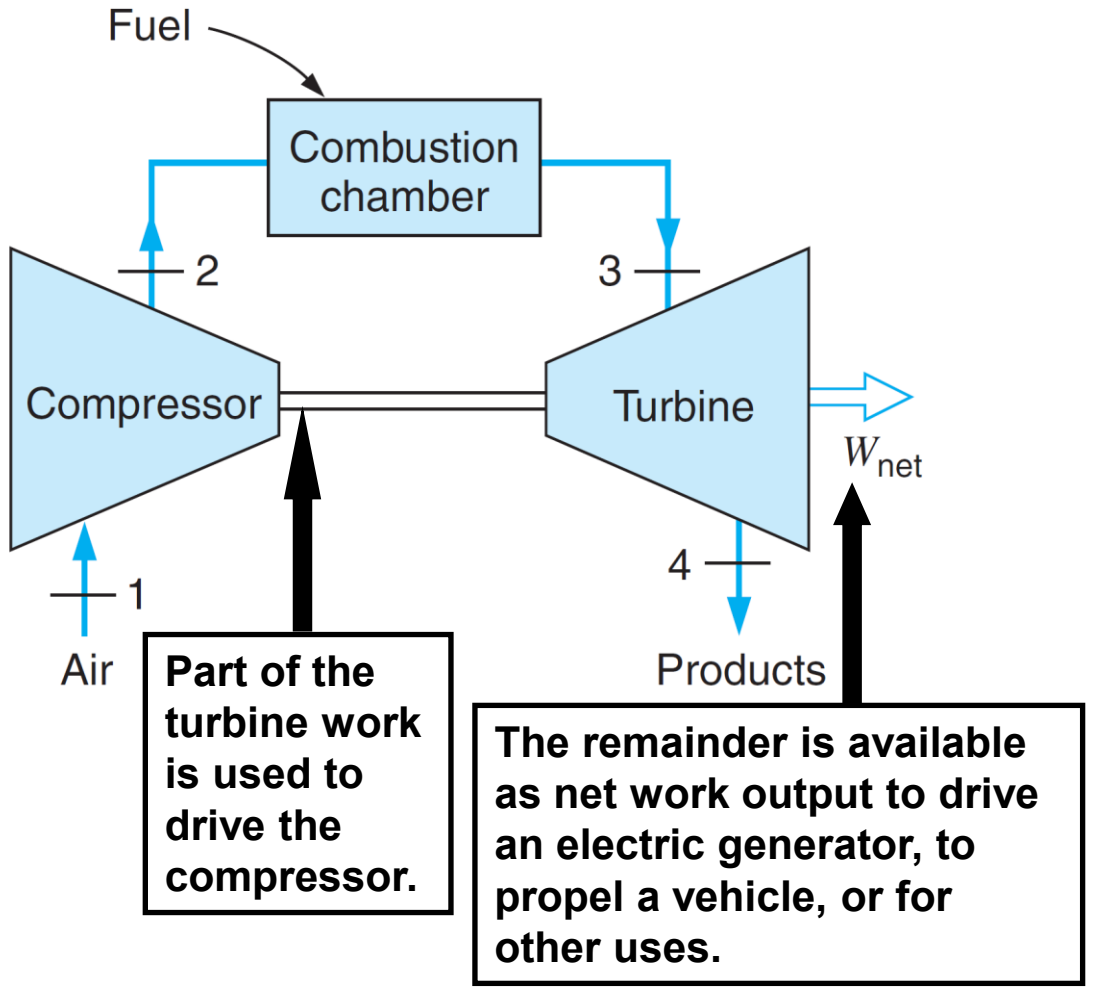


Simple Gas Turbine (Open Cycle)

- The *open* mode gas turbine is an *internal combustion* power plant.



Siemens SGT5-2000E
(Single-shaft, industrial gas turbine)



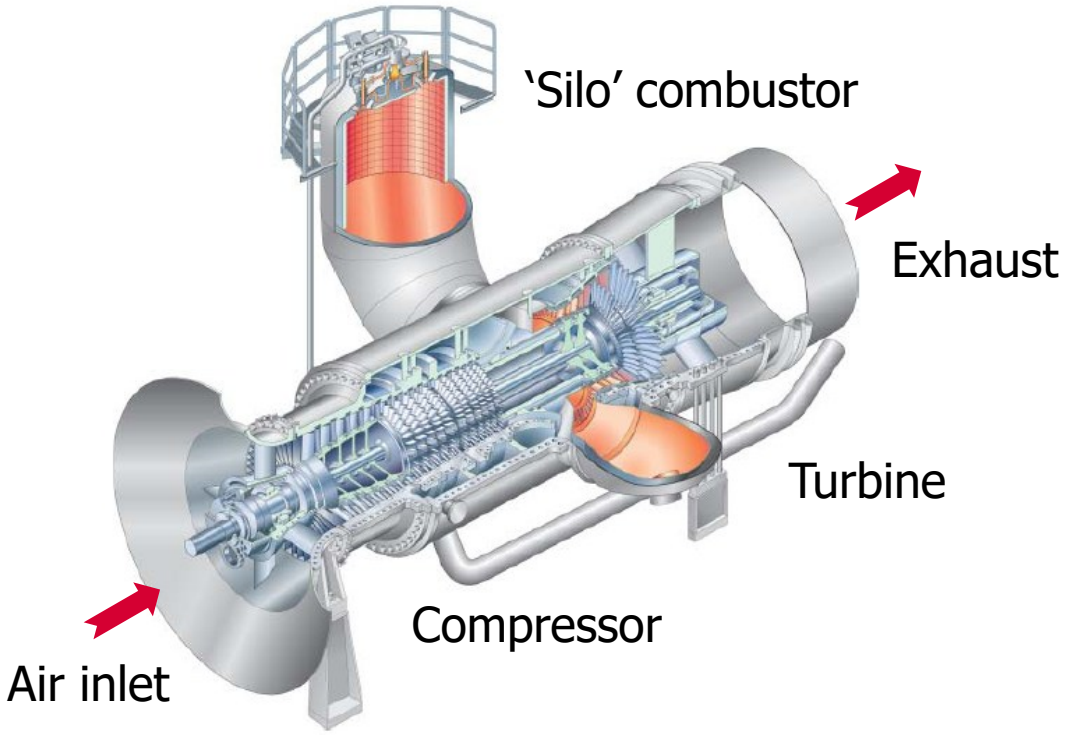
Simple Gas Turbine (Open Cycle)

- How a Gas Turbine Works

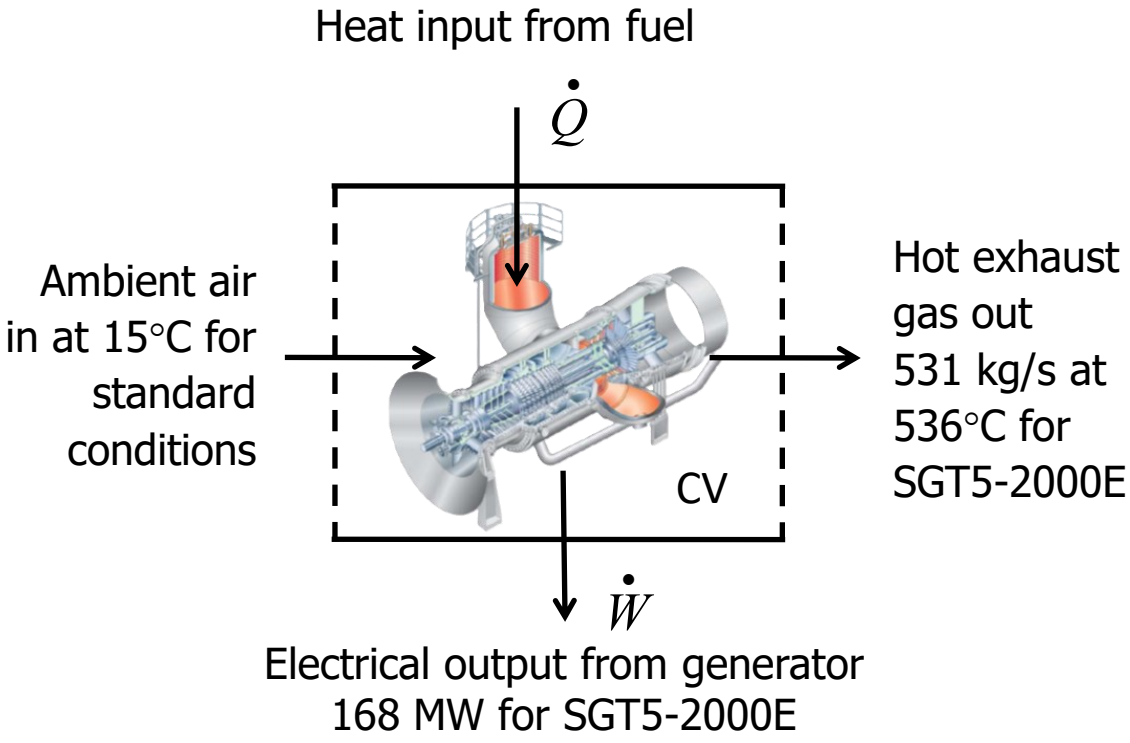


https://youtu.be/zcWkEKNvqCA

Example 3.1 (Simple Gas Turbine)



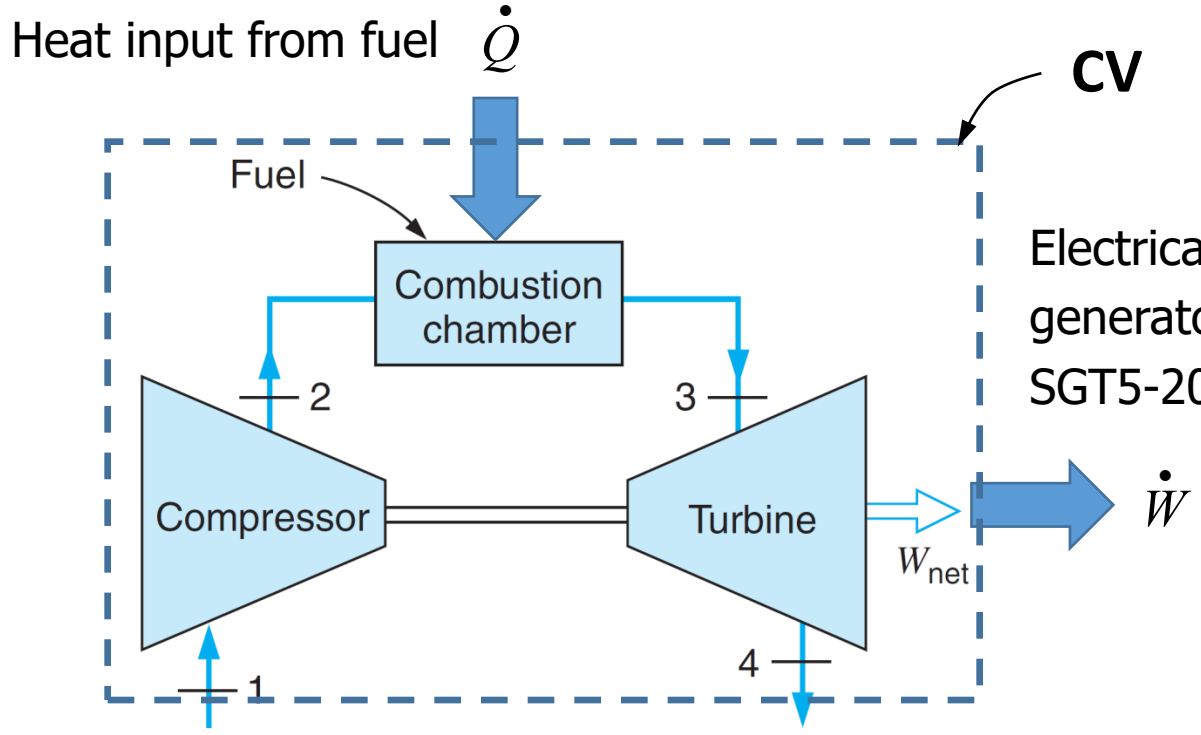
Siemens SGT5-2000E
(Single-shaft, industrial gas turbine)



Work out (gross)	MW	168
Gas flow rate	kg/s	531
Temperature in	°C	15
Temperature out	°C	536

- Find heat input and thermal efficiency.

Example 3.1 (Simple Gas Turbine)



Conservation of energy plus ideal gas properties can also be applied to the gas turbine

Specific heat (Cp)	kJ/kg·K	1.15
Work out (gross)	MW	168
Gas flow rate	kg/s	531
Temperature in	°C	15
Temperature out	°C	536
Heat input	MW	486.1
Efficiency	%	34.6

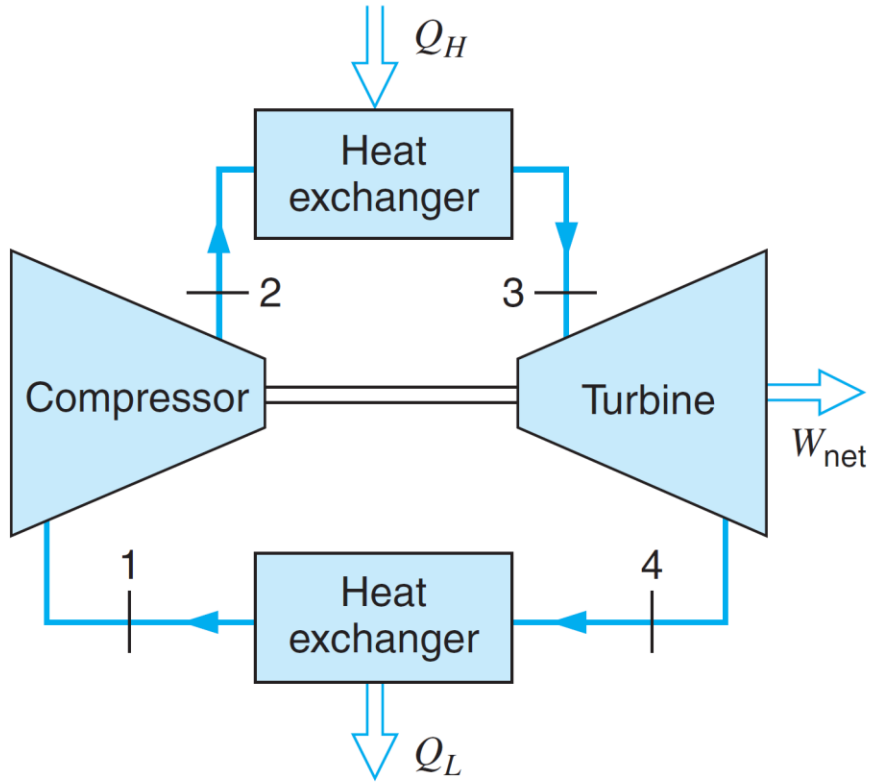
- Find heat input and thermal efficiency.

Closed Cycle

A *closed* gas turbine operates as follows:

- A gas circulates through four components: turbine, compressor, and two heat exchangers at higher and lower operating temperatures, respectively.
- The turbine and compressor play the same roles as in the open gas turbine.
- As the gas passes through the higher-temperature heat exchanger, it receives energy by heat transfer (Q_H) from an *external source*.
- The thermodynamic cycle is completed by heat transfer to the surroundings (Q_L) as the gas passes through the lower-temperature heat exchanger.

- Common working fluids for a closed Brayton cycle include air, N_2, helium, CO_2, and helium-xenon mixtures.



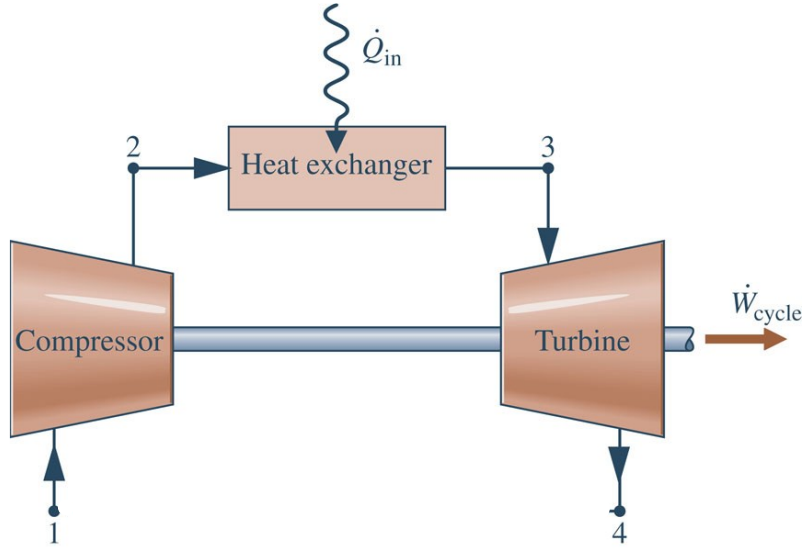
Process	Description
1-2	Compression (in a compressor)
2-3	Constant pressure heat addition (in a heat exchanger)
3-4	Expansion (in a turbine)
4-1	Constant pressure heat rejection (in a heat exchanger)

Air-Standard Brayton Cycle

- To conduct *elementary* analyses of gas turbine power plants, simplifications are required. Although highly idealised, an ***air-standard analysis*** can provide insights and qualitative information about actual performance.
- An ***air-standard analysis*** has the following elements:
  - The working fluid is air, which behaves as an ideal gas.
  - The temperature rise that would be brought about by combustion is accomplished by heat transfer from an external source.
  - With an air-standard analysis, we avoid the complexities of the combustion process and the change in composition during combustion, which simplifies the analysis considerably.
  - In a *cold* air-standard analysis, the specific heats are assumed constant at their ambient temperature values.

Air-Standard Brayton Cycle

- Air circulates through the components:
  - Air is drawn into the compressor from the surroundings and then compressed from state **1** to state **2**.
  - The temperature rise that would be achieved in the actual power plant with combustion is realised here by heat transfer, $\dot{Q}_{in}$
  - The high-pressure, high-temperature air expands through the turbine. The turbine drives the compressor and develops net power, $\dot{W}_{cycle}$
  - Air returns to the surroundings at state **4** with a temperature typically much greater than at state **1**.
    - After interacting with the surroundings, each unit of mass returns to the same condition as the air entering at state **1**, thereby completing a thermodynamic cycle.

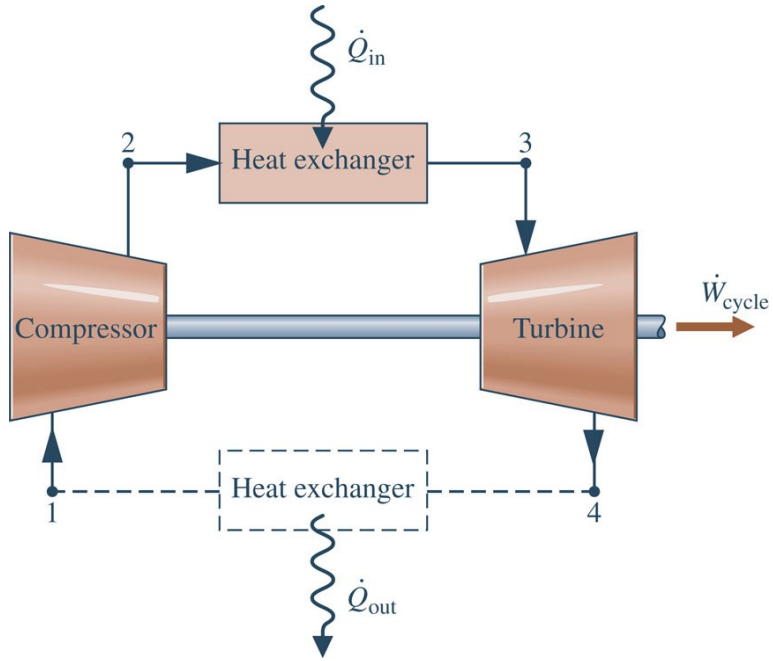


The schematic of a simple air-standard gas turbine power plant . The combustion process is replaced by a constant–pressure heat–addition process from an external source.

Cycle **1-2-3-4-1** is called the ***Brayton cycle***

Air-Standard Brayton Cycle

- Analysing each component as a control volume (CV) at steady state, assuming the compressor and turbine operate adiabatically (Q = 0), and neglecting kinetic and potential energy effects, we get the following expressions for the principal work and heat transfers (1st Law: **Q – W = ΔH + ΔKE + ΔPE**):



Turbine $$\frac{\dot{W}_t}{\dot{m}} = h_3 - h_4$$

Heat addition $$\frac{\dot{Q}_{in}}{\dot{m}} = h_3 - h_2$$

Compressor $$\frac{\dot{W}_c}{\dot{m}} = h_2 - h_1$$

Heat rejection $$\frac{\dot{Q}_{out}}{\dot{m}} = h_4 - h_1$$

Air-Standard Brayton Cycle

- The ***thermal efficiency*** is

$$\eta = \frac{\dot{W}_t/\dot{m} - \dot{W}_c/\dot{m}}{\dot{Q}_{in}/\dot{m}} = \frac{(h_3 - h_4) - (h_2 - h_1)}{h_3 - h_2}$$

- The ***back work ratio*** is

$$\text{bwr} = \frac{\dot{W}_c/\dot{m}}{\dot{W}_t/\dot{m}} = \frac{h_2 - h_1}{h_3 - h_4}$$

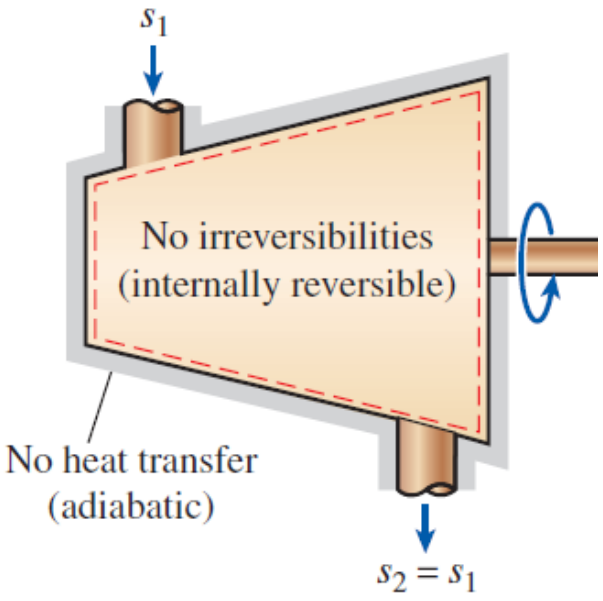
Note: A relatively large portion of the work developed by the turbine is required to drive the compressor. For **gas turbines**, back work ratios range from 20% to 80% compared to only 1-2% for vapour power plants.

- Since these equations have been developed from mass and energy balances, they apply equally when irreversibilities are present and in the absence of irreversibilities.

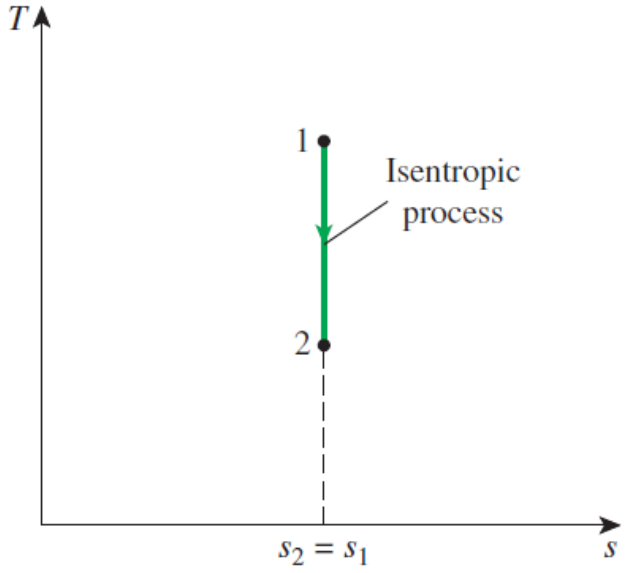
Ideal Brayton Cycle

Isentropic process: A process during which the entropy remains constant.

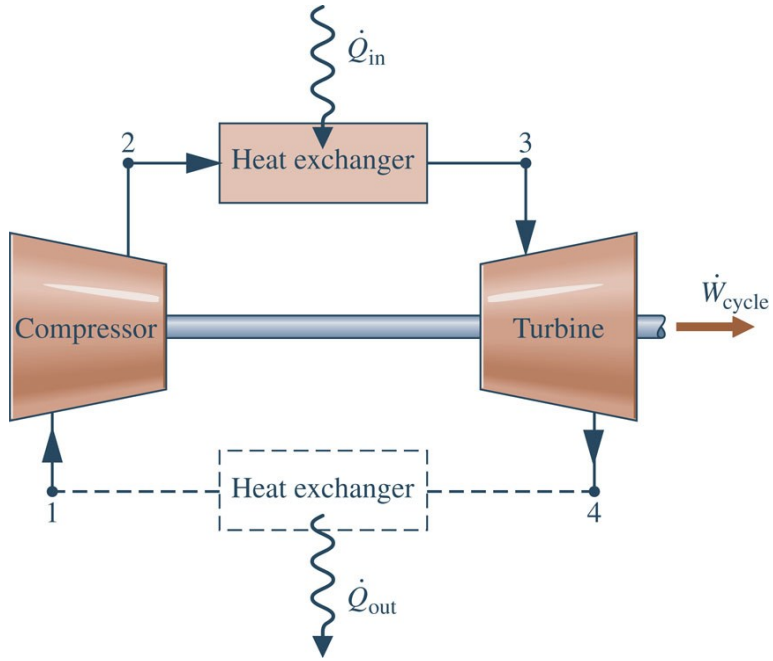
Isentropic process: $\Delta s = 0$ or $s_2 = s_1$ (kJ/kg·K)



During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.

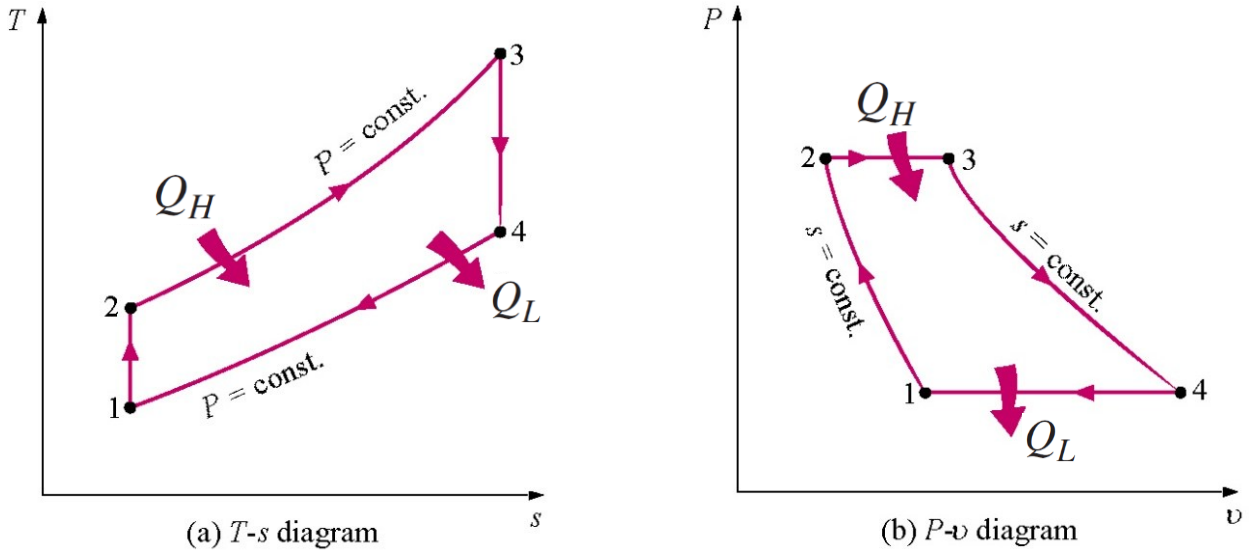


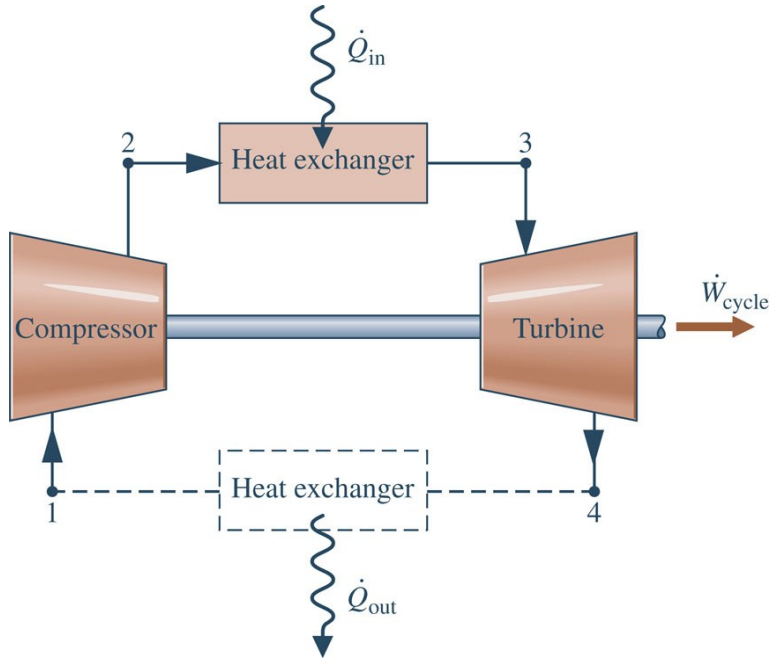
The isentropic process appears as a vertical line segment on a T-s diagram.



Process	Description
1-2	Isentropic compression ($s_1 = s_2$)
2-3	Constant pressure heat addition (in a heat exchanger)
3-4	Isentropic expansion ($s_3 = s_4$)
4-1	Constant pressure heat rejection (in a heat exchanger)

Ideal Brayton Cycle



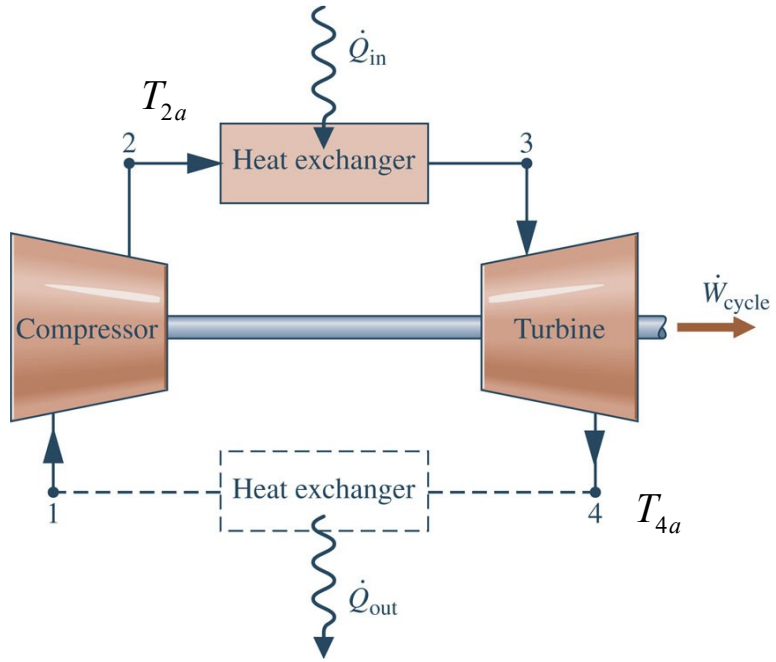


Processes 1–2 and 3–4 are isentropic, and $P_2 = P_3$ and $P_4 = P_1$. Thus,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_4}$$

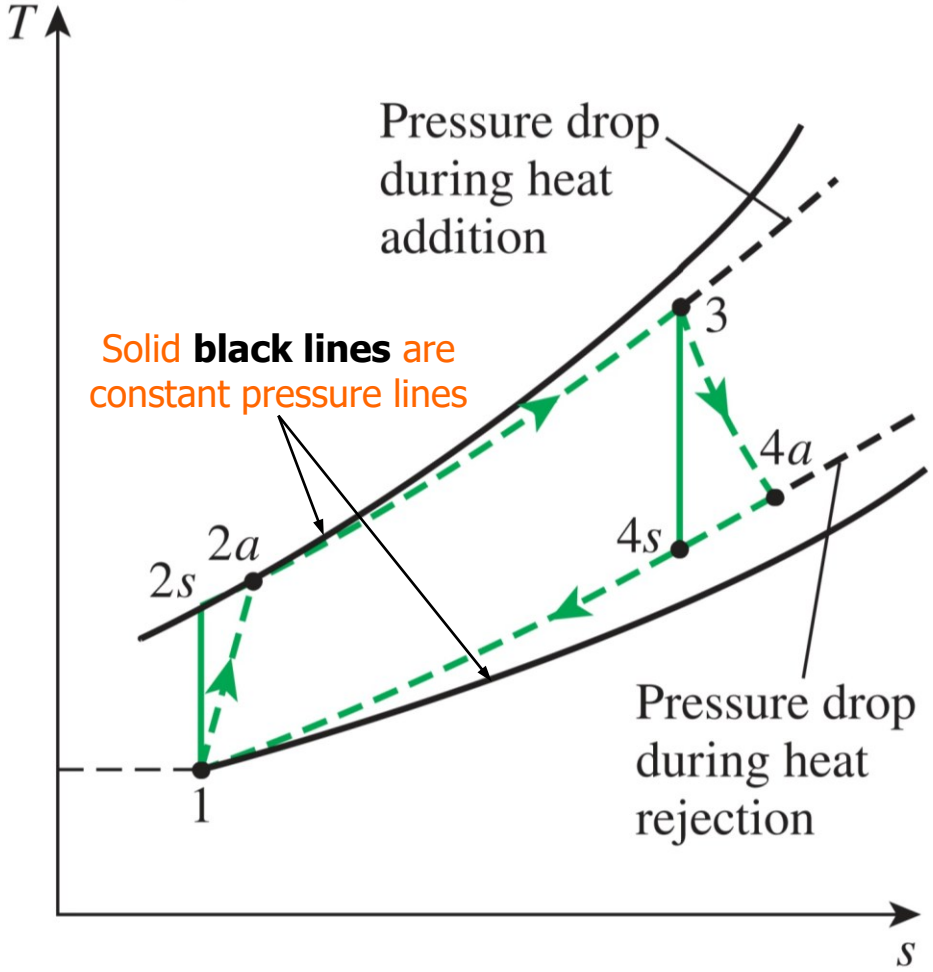
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Actual Brayton Cycle



Processes 1–2 and 3–4 are no-longer isentropic, and $P_2 \neq P_3$ and $P_4 \neq P_1$ (see T-s diagram).

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} \quad ; \quad \frac{T_3}{T_{4s}} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k}$$

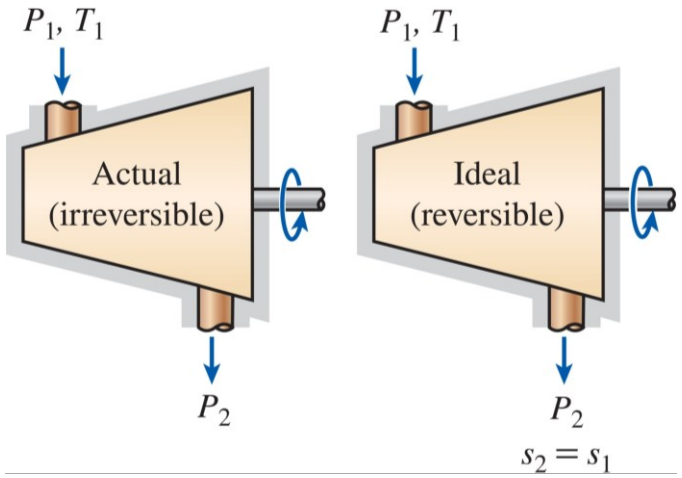


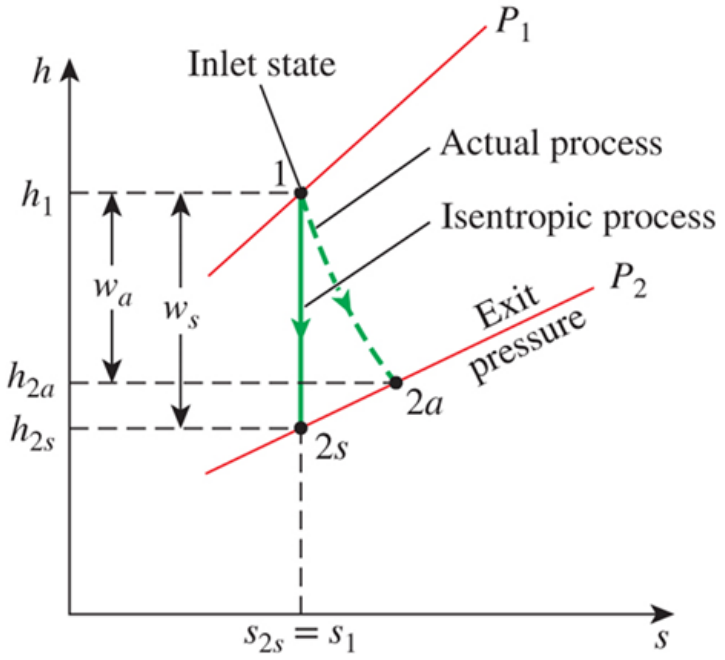
Deviation of an actual gas-turbine cycle from the ideal Brayton cycle as a result of irreversibilities.
Irreversibilities in turbine and compressors and pressure drops and heat losses in heat exchangers.

Isentropic Efficiencies

- Most steady-flow devices operate under adiabatic conditions, and the ideal process for these devices is the isentropic process. The parameter that describes how a device approximates a corresponding isentropic device is called the isentropic (or adiabatic) efficiency. It is defined for turbines, compressors, and nozzles as follows:

Turbine:





The *h*–*s* diagram for the actual and isentropic processes of an adiabatic turbine.

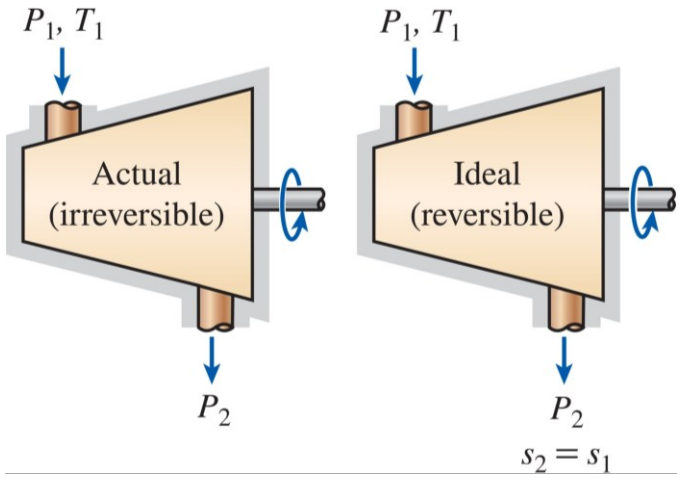
The isentropic work is the maximum possible work output that the adiabatic turbine can produce; therefore, the actual work (w_a) is less than the isentropic work (w_s).

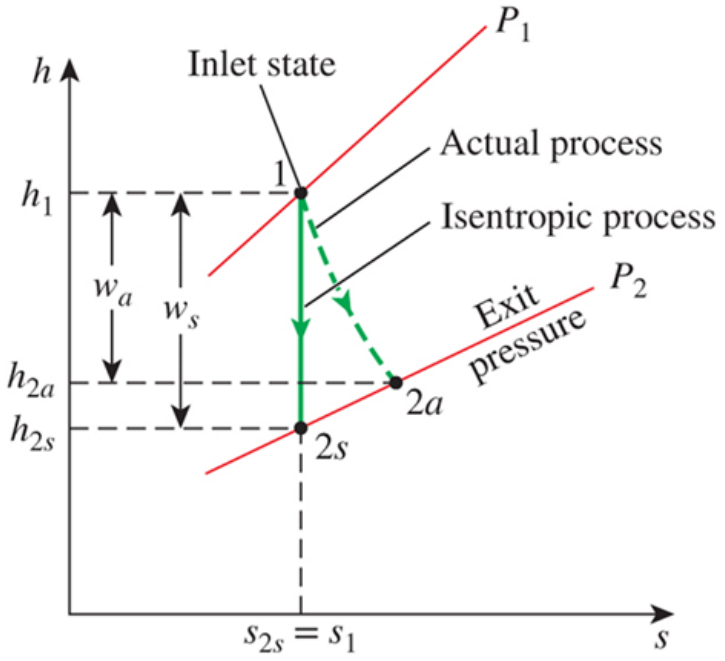
The ratio between actual work and isentropic work is known as turbine's isentropic efficiency.

Isentropic Efficiencies

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Turbine:





The *h*–*s* diagram for the actual and isentropic processes of an adiabatic turbine.

$$\eta_T = \frac{Actual\ turbine\ work}{Isentropic\ turbine\ work} = \frac{w_a}{w_s}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

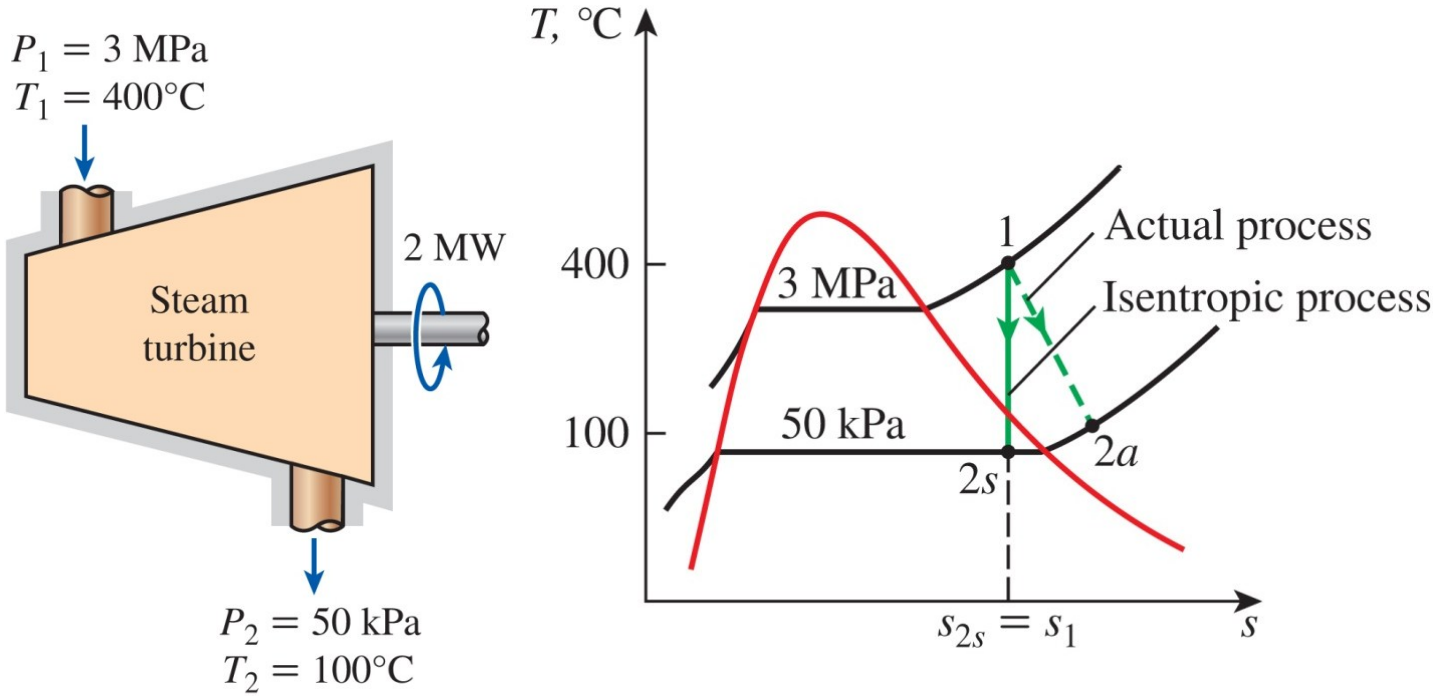
Well-designed large turbines may have isentropic efficiencies above 90%. Small turbines may have isentropic efficiencies below 70%.

Ref: Chapter 7, Thermodynamics: An Engineering Approach by Yunus A. Çengel and Michael A. Boles

Isentropic Efficiencies

- Most steady-flow devices operate under adiabatic conditions, and the ideal process for these devices is the isentropic process. The parameter that describes how a device approximates a corresponding isentropic device is called the isentropic (or adiabatic) efficiency. It is defined for turbines, compressors, and nozzles as follows:

Turbine example:



Find the isentropic efficiency of the steam turbine and mass flow rate of the steam.

Hints:

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

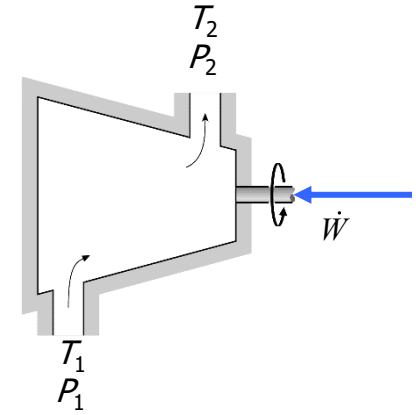
$$\dot{W}_{a,\text{out}} = \dot{m}(h_1 - h_{2a})$$

Use steam table for finding *h*

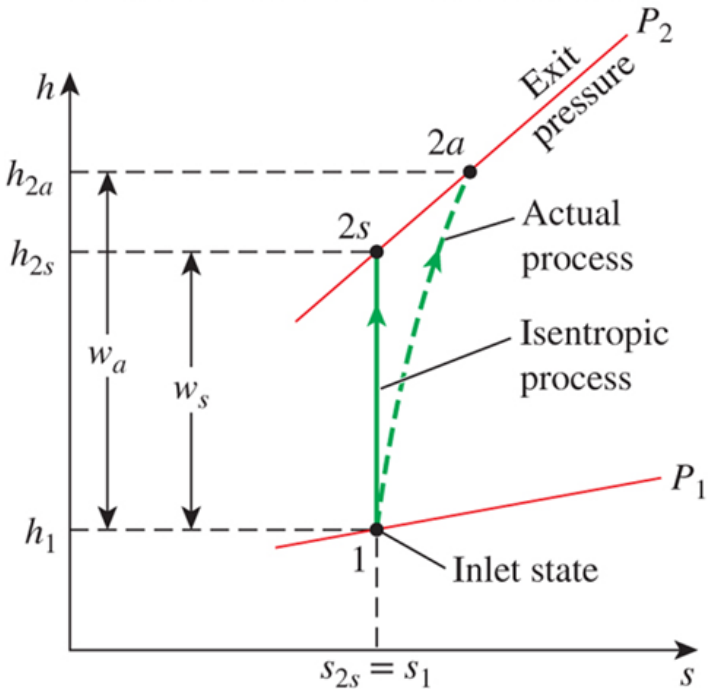
Isentropic Efficiencies

Compressor and Pump:

- The isentropic work is the minimum possible work that the adiabatic compressor requires; therefore, the actual work is greater than the isentropic work. Since efficiencies are defined to be less than 1, the compressor isentropic efficiency is defined as



Compressor or pump



The *h*–*s* diagram of the actual and isentropic processes of an adiabatic compressor.

$$\eta_C = \frac{Isentropic\ compressor\ work}{Actual\ compressor\ work} = \frac{w_s}{w_a}$$

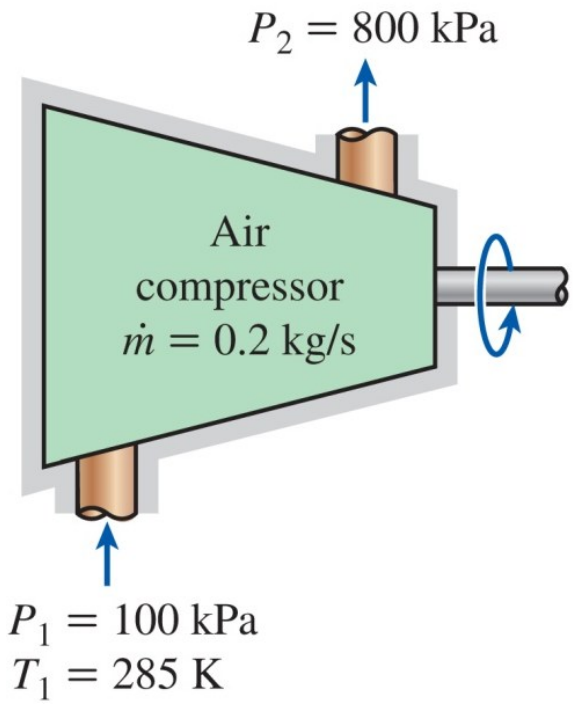
$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

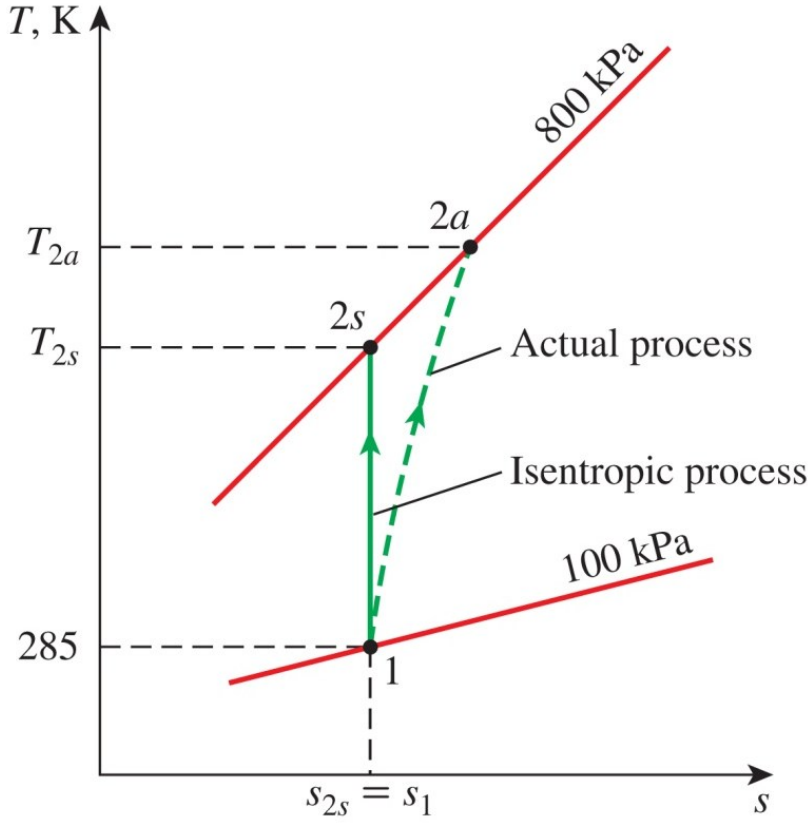
Well-designed compressors have isentropic efficiencies in the range from 75 to 85%.

Review the efficiency of a pump and an isothermal compressor on your own.

Isentropic Efficiencies

Compressor example:





If the isentropic efficiency of the compressor is 80%, determine the exit temperature of air and power input to the compressor.

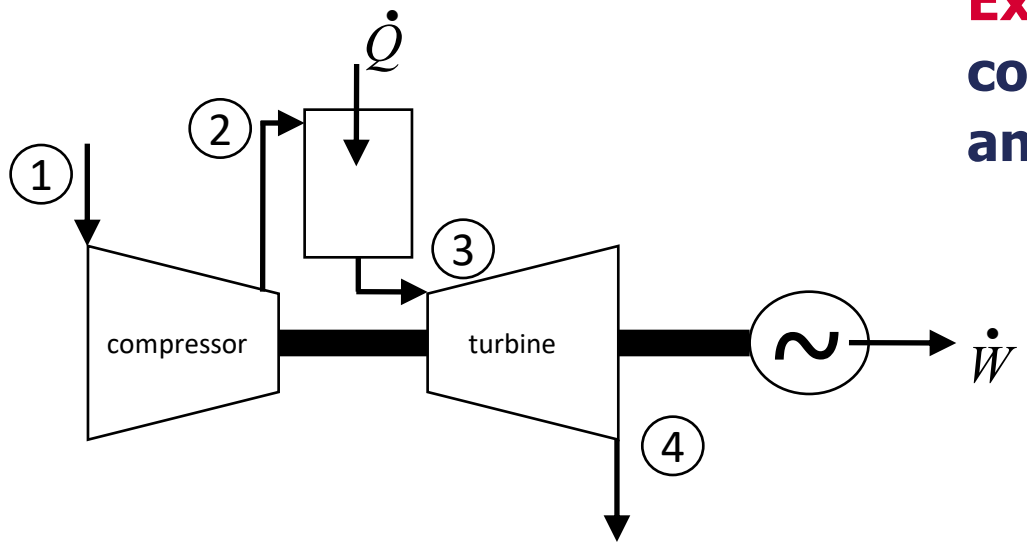
Hints:

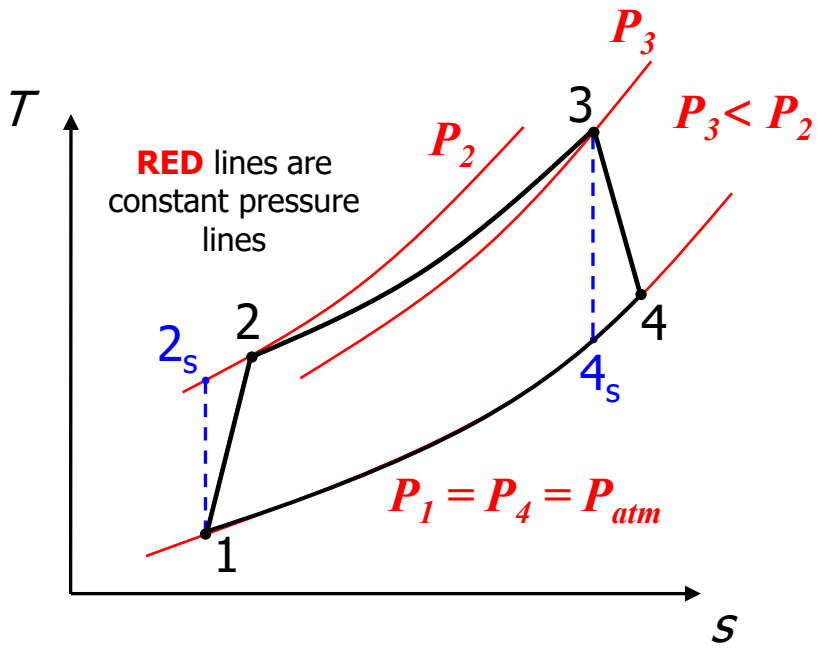
$$\eta_C \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

$$\dot{W}_{a,\text{in}} = \dot{m}(h_{2a} - h_1)$$

Consider air is an ideal gas

Example 3.2: Analysis of gas turbine using compressor and turbine isentropic efficiencies and combustor pressure drop

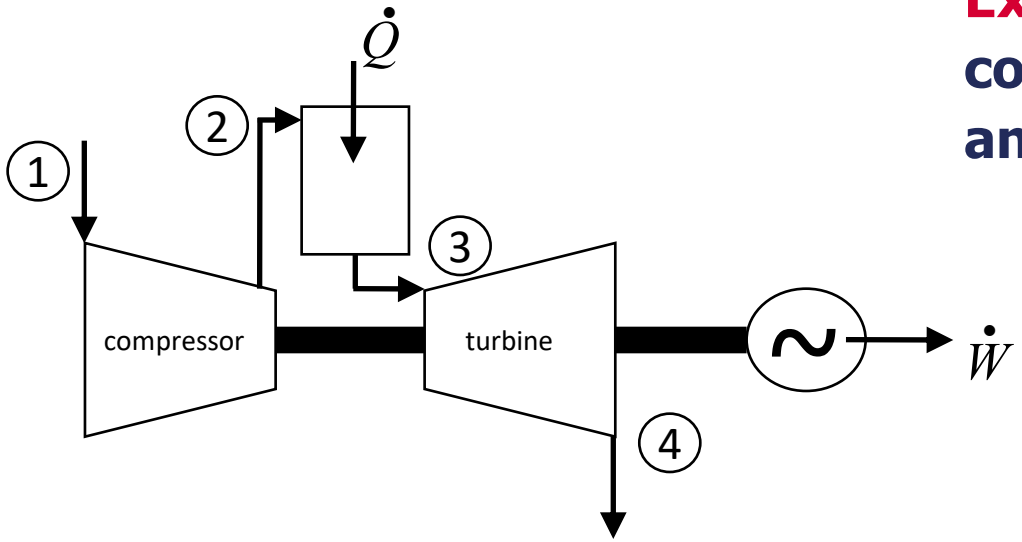


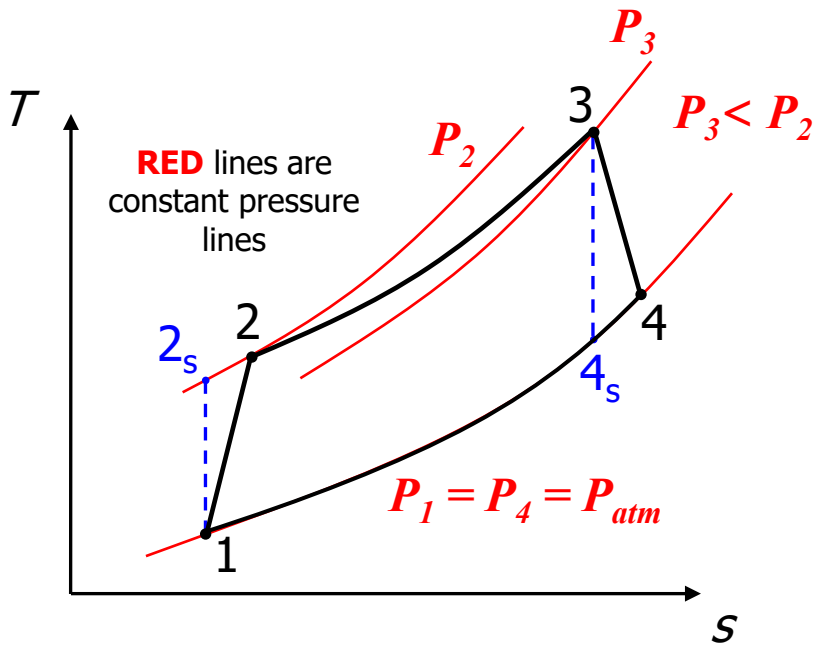


Compressor		Actual	Ideal
R for air	kJ/kg.K	0.287	0.287
Cp	kJ/kg.K	1.15	1.15
Cv	kJ/kg.K	0.863	0.863
k		1.333	1.333
Pressure ratio	P_2/P_1	11.7	11.7
Inlet temperature	K	288.15	288.15
Isentropic exit temperature	K	532.4	532.4
Isentropic temperature rise	K	244.2	244.2
Isentropic power input	MW	149.1	149.1
Isentropic efficiency		85%	100%
Actual temperature rise	K	287.3	244.2
Actual exit temperature	K	575.4	532.4
Gas flow rate	kg/s	531	531
Power input	MW	175.4	149.1
Extra power due to irreversibilities	MW	26.3	0.0

Note:
Given quantities or intermediate calculations
To be determined

Example 3.2: Analysis of gas turbine using compressor and turbine isentropic efficiencies and combustor pressure drop

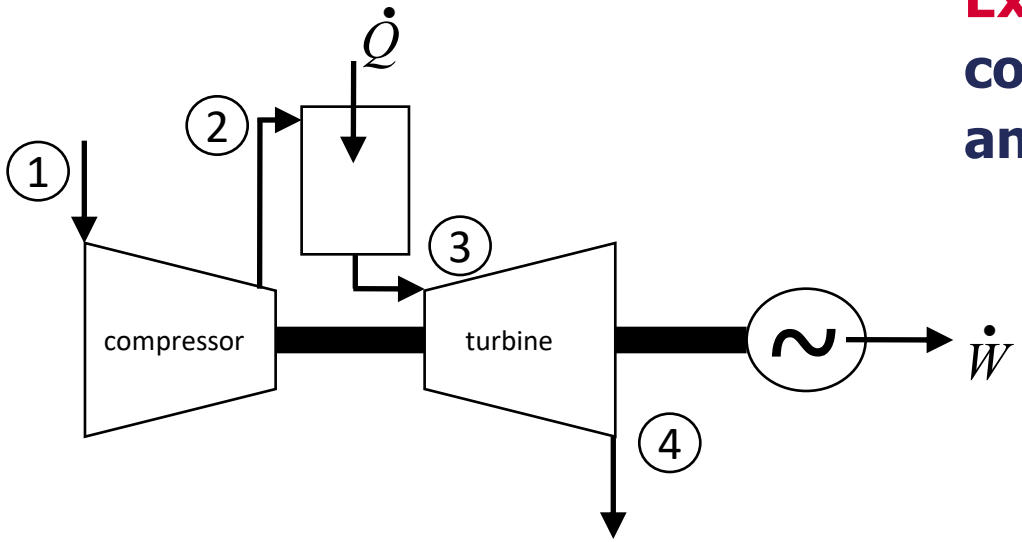


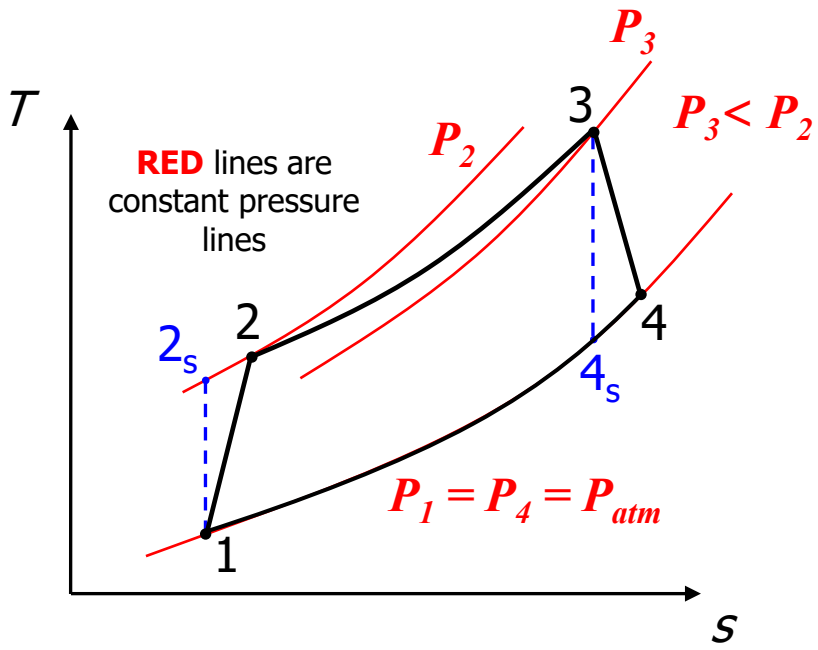


Combustor - pressure drop		Actual	Ideal
Compressor inlet pressure (P_1)	bar	1.013	1.013
Compressor pressure ratio		11.7	11.7
Compressor exit pressure (P_2)	bar	11.85	11.85
Pressure drop in combustor		2%	0%
Pressure after combustor (P_3)	bar	11.61	11.85
Turbine pressure ratio		11.46	11.70

Note:
Given quantities or **intermediate calculations**
To be determined

Example 3.2: Analysis of gas turbine using compressor and turbine isentropic efficiencies and combustor pressure drop





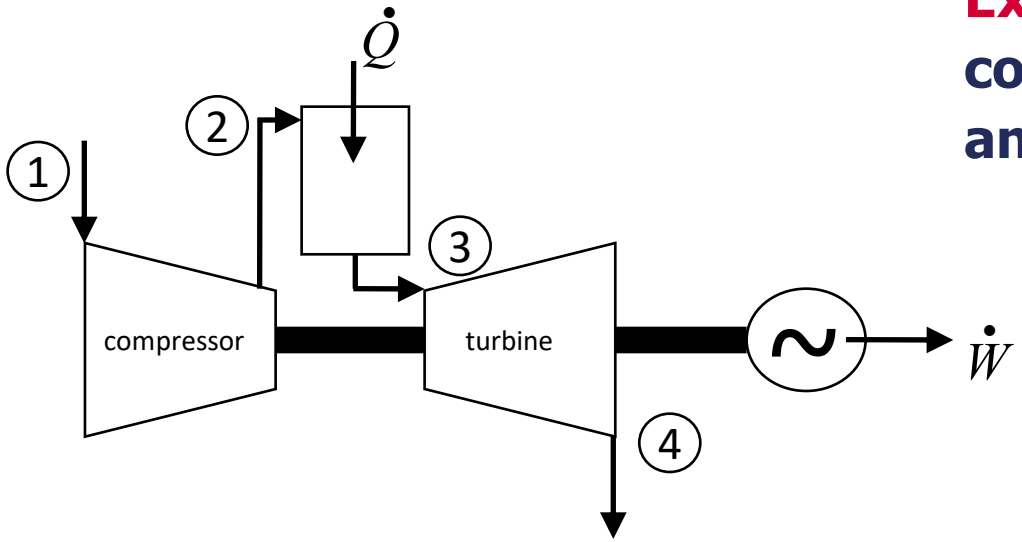
Turbine

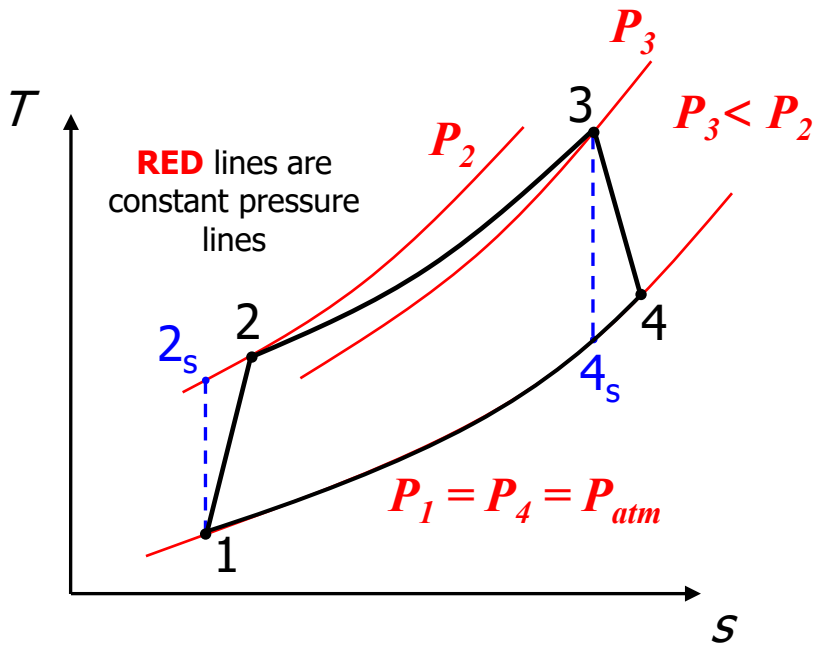
Turbine pressure ratio		**11.46**	**11.70**
Isentropic efficiency		**90%**	**100%**
Estimated turbine inlet temperature (T_3)	**K**	**1372.4**	**1372.4***
Isentropic exit temperature (T_{4s})	**K**	**746.6**	**742.82**
Isentropic temperature drop	**K**	**625.8**	**629.53**
Turbine exit temperature (T_4)	**K**	**809.15**	**742.82**

* This is the value calculated to give the correct turbine exit temperature, given all the other assumptions

Note:
Given quantities or **intermediate calculations**
To be determined

Example 3.2: Analysis of gas turbine using compressor and turbine isentropic efficiencies and combustor pressure drop



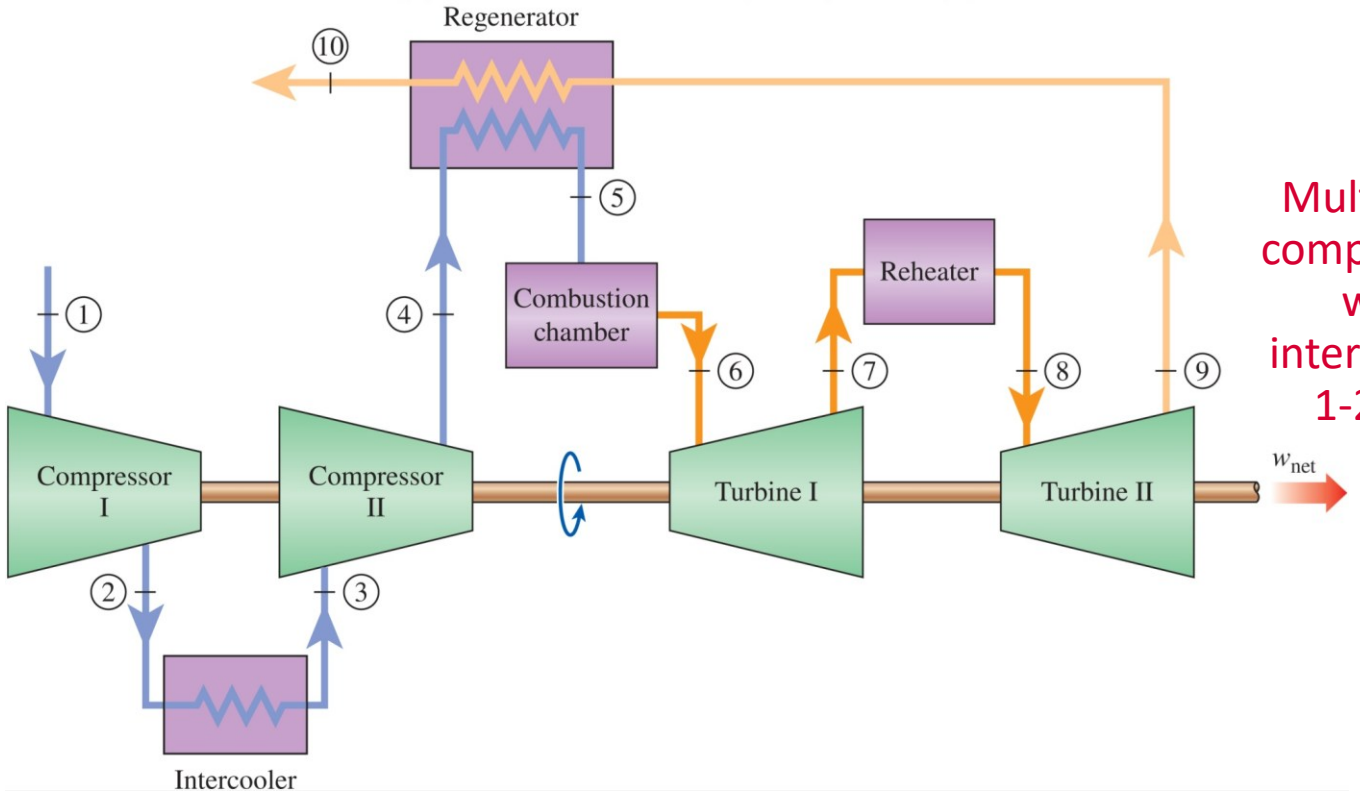


Combustor heat input and overall performance		Actual	Ideal
Combustor heat input	MW	486.69	512.94
Turbine work output	MW	343.95	384.42
Overall work output	MW	168.55	235.3
Efficiency	%	34.6	45.9

Note:
Given quantities or **intermediate calculations**
To be determined

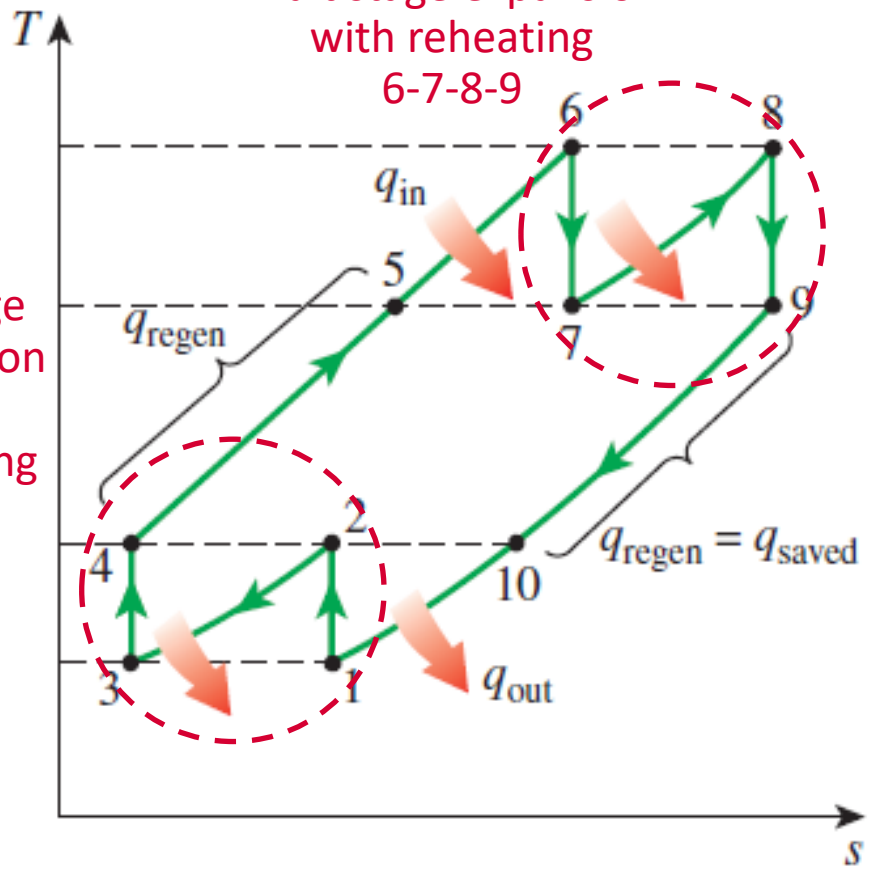
Ways to Improve Brayton Cycle Performance

- Intercooling, reheating, regeneration...



Multistage compression with intercooling 1-2-3-4

Multistage expansion with reheating 6-7-8-9

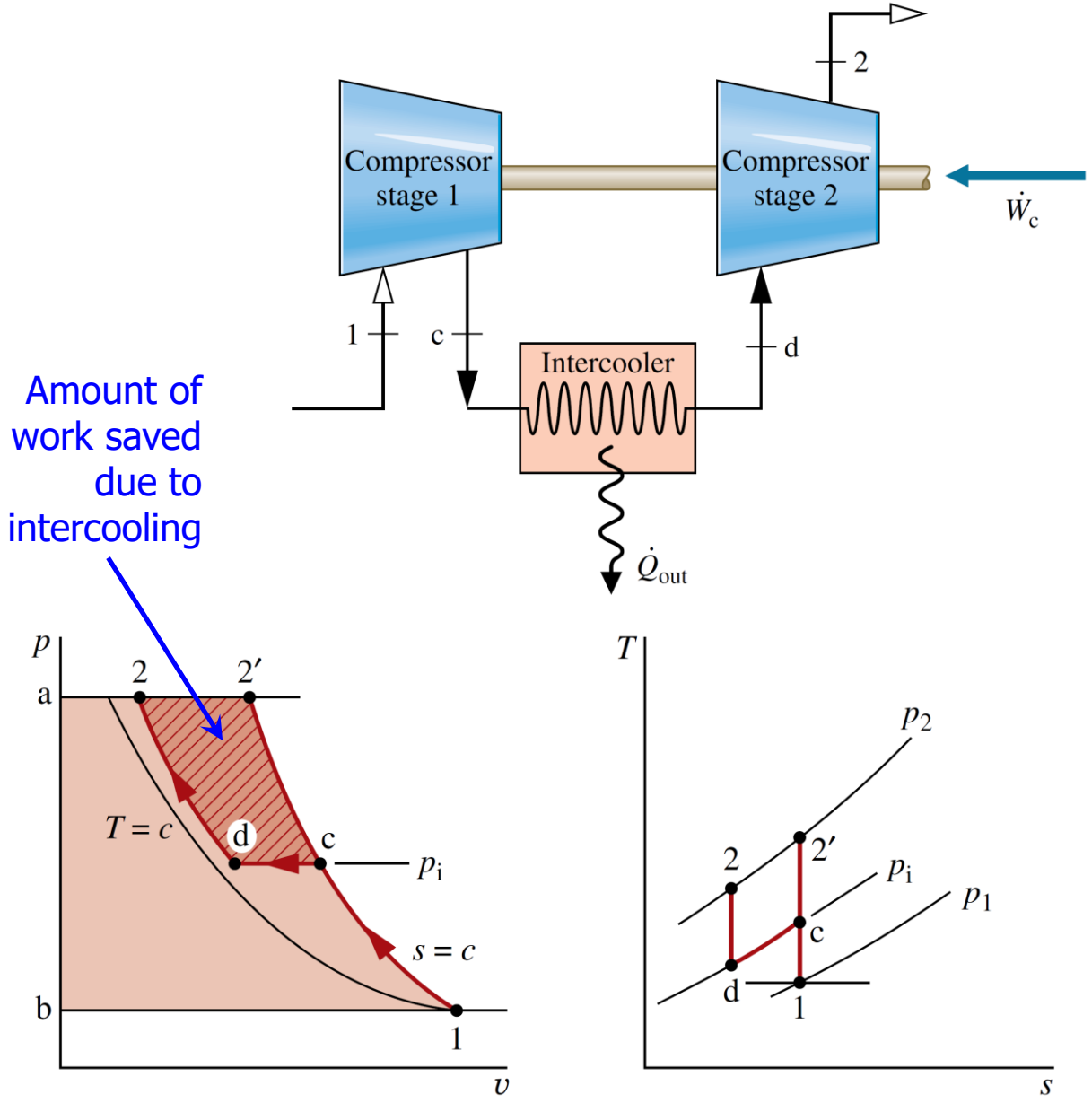


$$\eta = \frac{\dot{W}_t \uparrow - \dot{W}_c \downarrow}{\dot{Q}_{in} \downarrow}$$

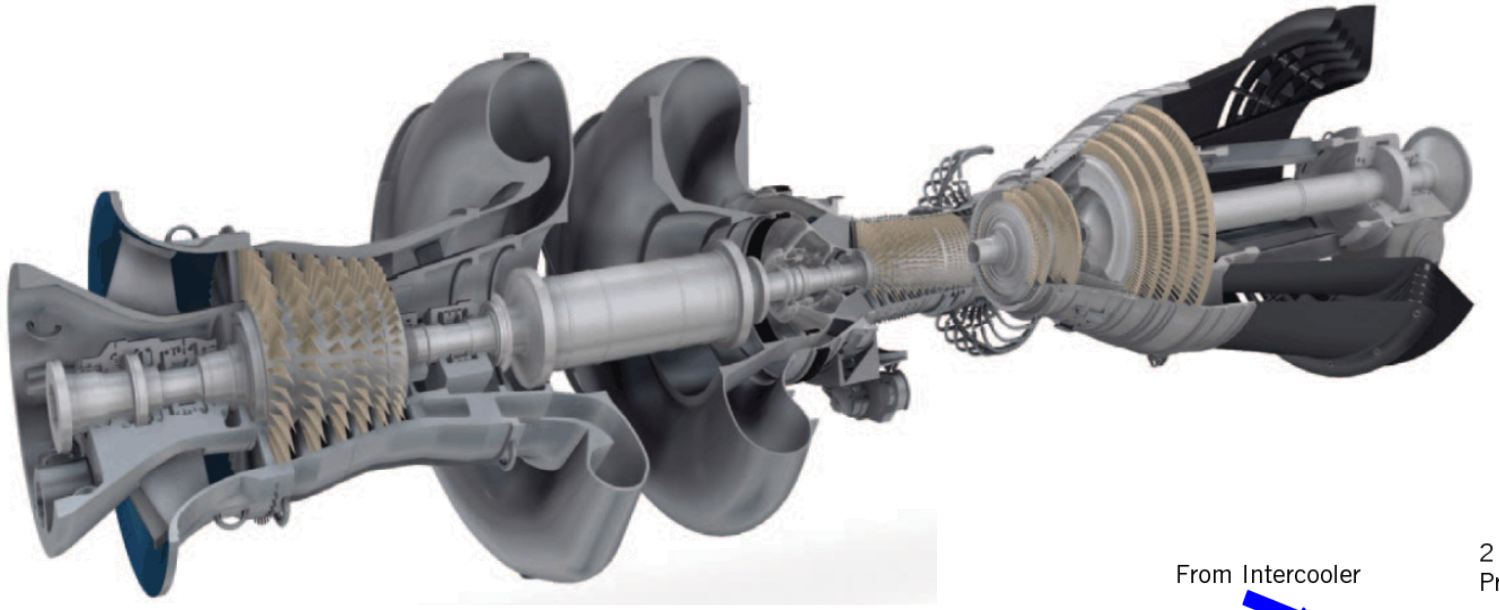
Intercooling and reheating always decreases the thermal efficiency unless they are accompanied by regeneration.

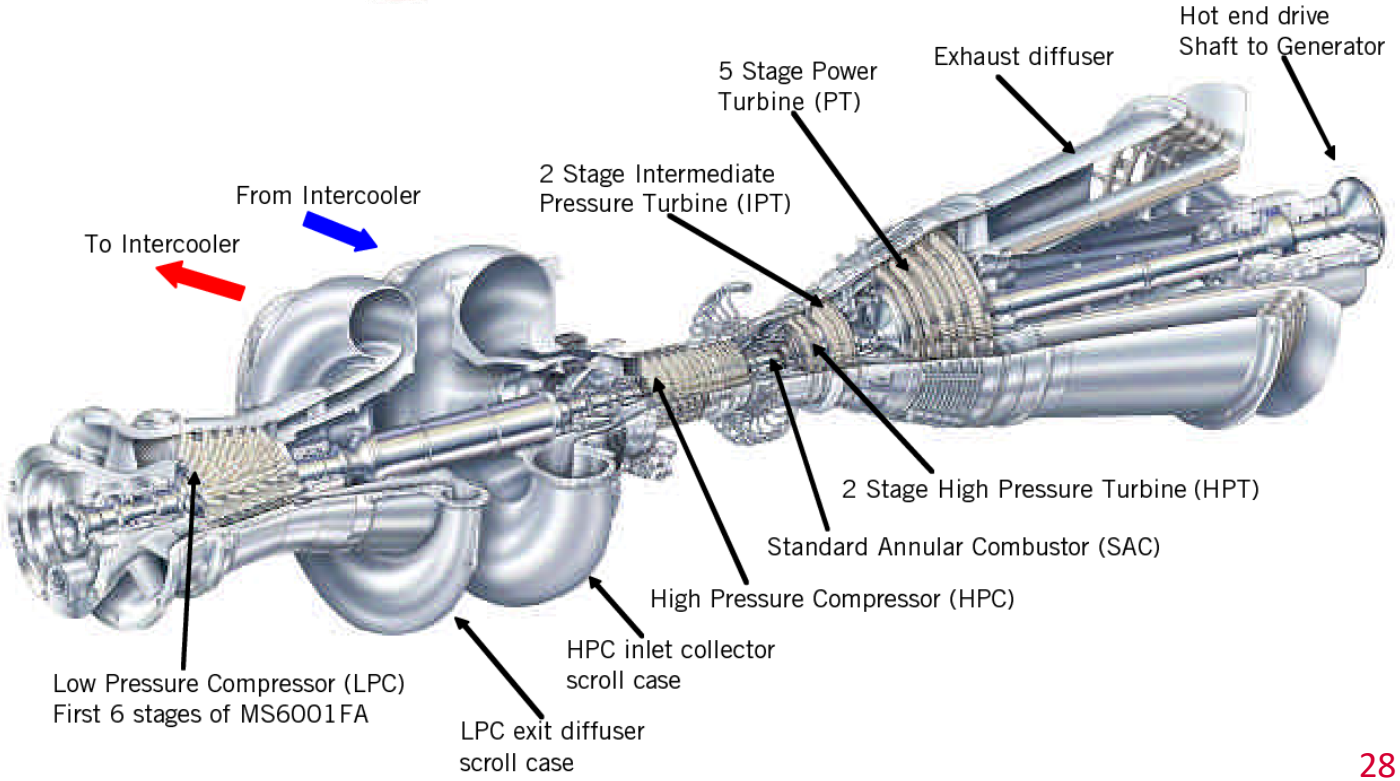
Intercooling

- Intercooling provides significant benefits to the Brayton cycle by reducing the work of compression for the high pressure compressor, which allows for higher pressure ratios, thus increasing overall efficiency.
  - The intercooler reduces specific volume and thus reduces work in the following compressor stage. It also reduces temperature into the combustor and thus reduces the peak temperature.



Intercooling



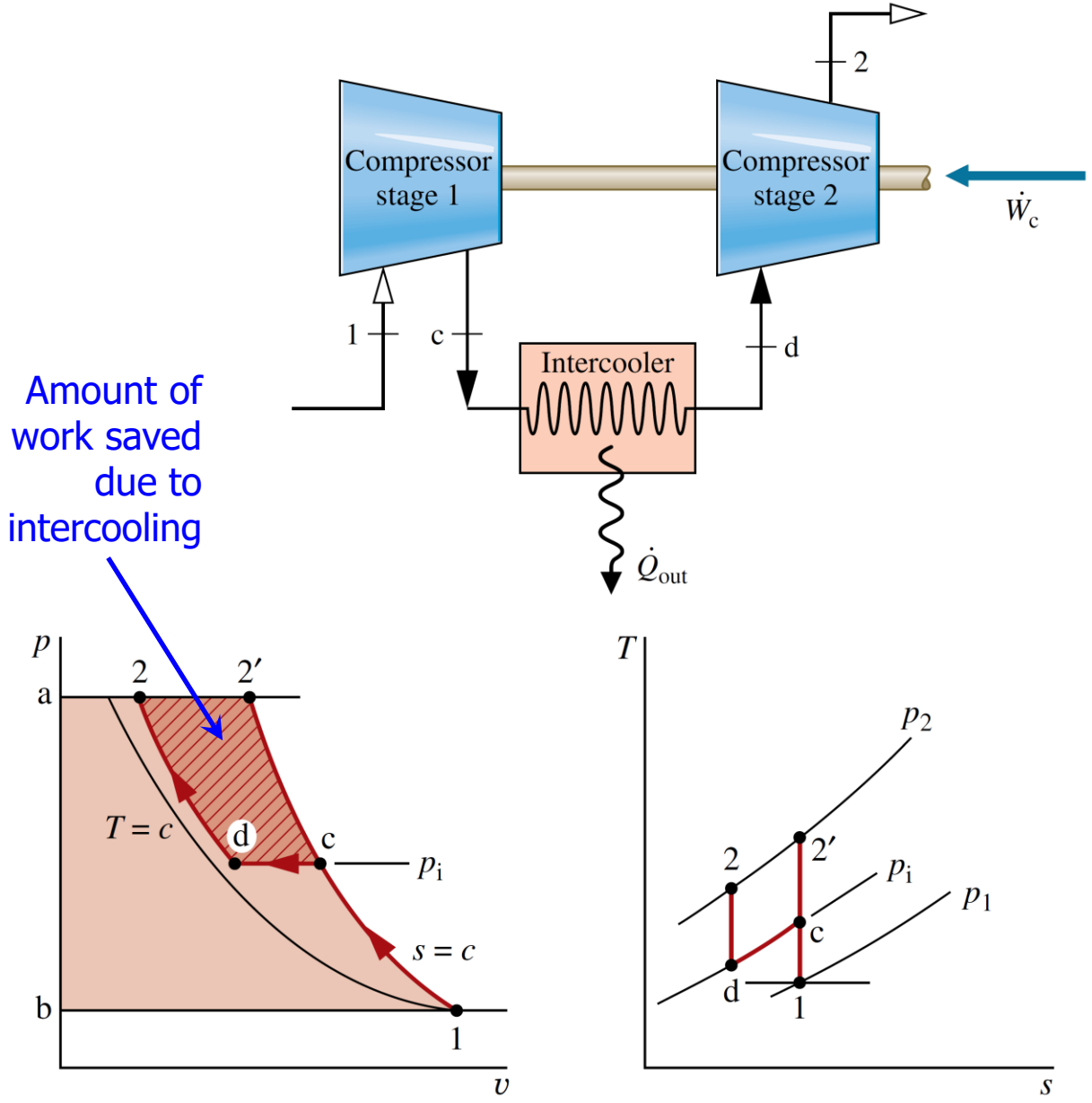


GE LMS100™ Gas Turbine

https://www.ge.com/content/dam/gepower-new/global/en_US/downloads/gas-new-site/resources/reference/ger-4222a-new-high-efficiency-simple-cycle-gas-turbine-lms100.pdf

Intercooling

- The net work output of a gas turbine can be increased by reducing the compressor work input through multistage compression with *intercooling*.
  - How to determine the intercooler pressure (p_i) for minimum compressor work?



Determining Intercooler Pressure for Minimum Total Compressor Work

- Consider a 'balloon' of gas flowing through an adiabatic, reversible expansion or compression process. The balloon itself expands or contracts adiabatically; the only work transfer is displacement work at the boundary (which in turn exerts a force on the turbine or compressor blades).
- By definition for the 'balloon' – a closed system:

$$dq + dw = du + (dKE \text{ and } dPE) \qquad \text{(conservation of energy)}$$

and $dw = -Pdv$ (reversible displacement work)

therefore: $dq = du + Pdv$

But for the special case of adiabatic processes $dq = 0 = du + Pdv$

$du = -Pdv$ (interpretation is that all the work comes from the internal energy)

By definition: $h = u + Pv$

Differentiating: $dh = du + d(Pv) = du + Pdv + vdP$

$= vdP$ for adiabatic, reversible and isentropic processes

Determining Intercooler Pressure for Minimum Total Compressor Work

- Now consider the control volume around an adiabatic, reversible compressor.
- The shaft work crossing the control surface is given by:

$$\cancel{dq}^{0} + dw = dh \;\; (+ dPE \;\; + dKE)$$

But from the previous result: $dw = vdP$

Also, for an adiabatic reversible process $Pv^k = constant$

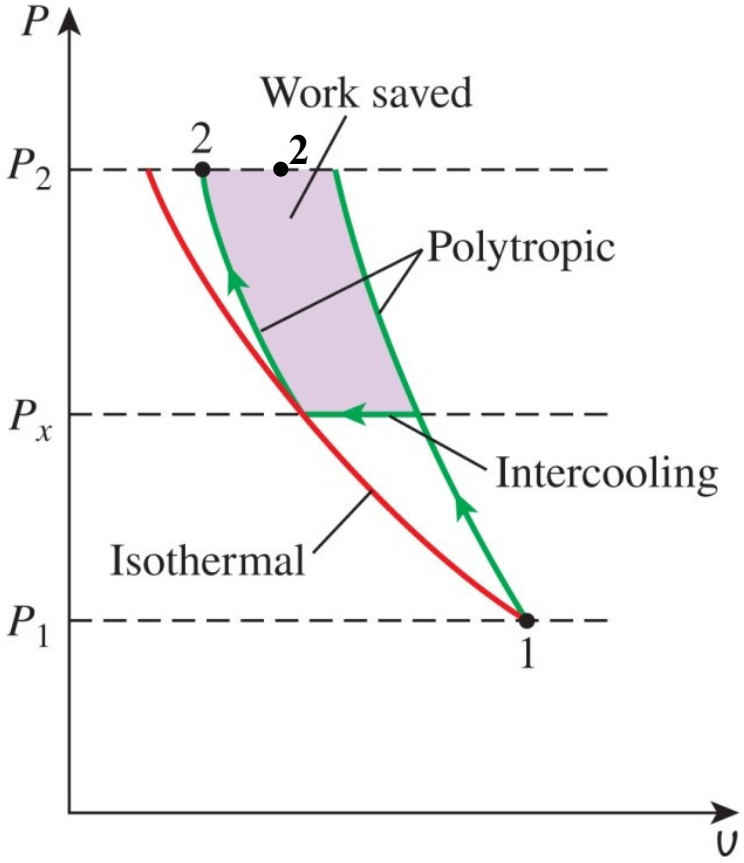
$$\Rightarrow v = (constant/P)^{1/k} = (Pv^k/P)^{1/k}$$

constant

For one-stage: $w_{12} = \int vdP = (Pv^k)^{1/k} \int P^{-1/k} dP$

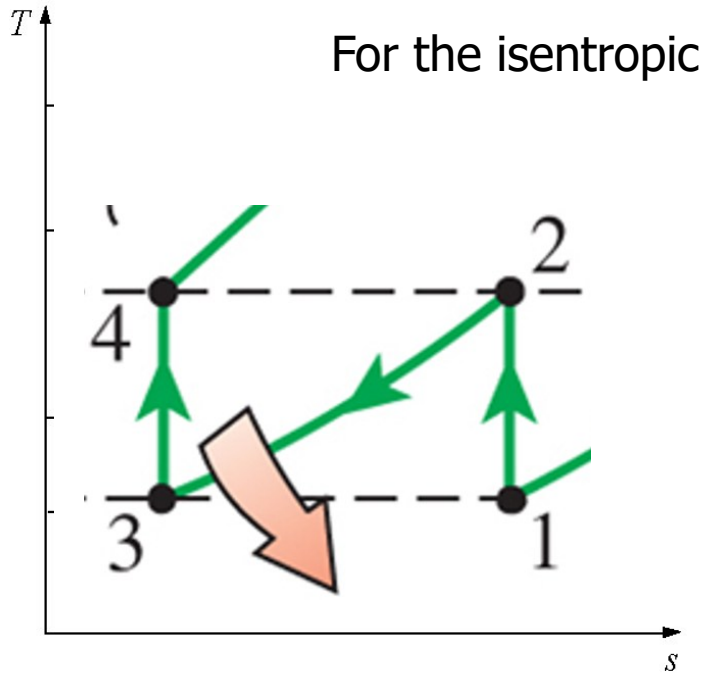
$$= \frac{[(Pv^k)^{1/k} P^{(k-1)/k}]_2}{(k-1)/k} - \frac{[(Pv^k)^{1/k} P^{(k-1)/k}]_1}{(k-1)/k}$$

$$= \frac{k}{(k-1)} (P_2 v_2 - P_1 v_1)$$



$$w_{comp} = \int_1^2 vdP + \int_3^4 vdP$$

Note: cannot use $dh = vdP$ for 2-3 since not adiabatic process, but no work transfer.



For the isentropic compression processes:

$$w_{comp} = \frac{k}{k\text{-}1}(P_2v_2 - P_1v_1) + \frac{k}{k\text{-}1}(P_4v_4 - P_3v_3)$$

$$= \frac{k}{k\text{-}1}R(T_2 - T_1) + \frac{kR}{k\text{-}1}(T_4 - T_3)$$

$$= \frac{k}{k\text{-}1}R\left[T_1(T_2/T_1 - 1) + T_3(T_4/T_3 - 1)\right]$$

$$= \frac{k}{k\text{-}1}R\left[T_1\left(\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right) + T_3\left(\left(\frac{P_4}{P_3}\right)^{(k-1)/k} - 1\right)\right]$$

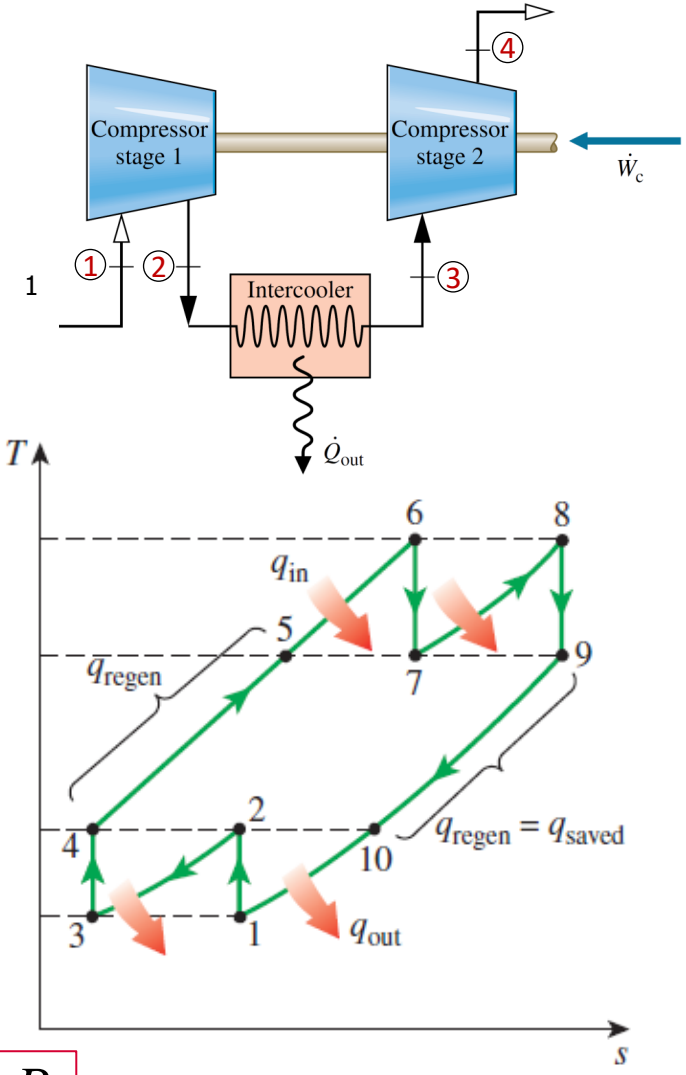
Note that the fraction $kR/(k\text{-}1) = C_p$; therefore, $w_{comp} = C_p\left[T_1\left(\left(\frac{P_2}{P_1}\right)^{(k-1)/k} - 1\right) + T_3\left(\left(\frac{P_4}{P_3}\right)^{(k-1)/k} - 1\right)\right]$

Determining Intercooler Pressure for Minimum Total Compressor Work

- For two-stage compression, assuming that intercooling takes place at constant pressure and the gases can be cooled to the inlet temperature for the compressor, such that $P_3 = P_2$ and $T_3 = T_1$.
- The total work supplied to the compressor becomes

$$w_{comp} = C_p T_1 \left[\left(\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right) + \left(\left(\frac{P_4}{P_2} \right)^{(k-1)/k} - 1 \right) \right]$$

$$= C_p T_1 \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} + \left(\frac{P_4}{P_2} \right)^{(k-1)/k} - 2 \right]$$



- To find the unknown pressure P_2 that gives the minimum work input for fixed compressor inlet conditions T_1, P_1, and exit pressure P_4, we set

$$\frac{dw_{comp}(P_2)}{dP_2} = 0 \qquad \text{This yields} \qquad P_2 = \sqrt{P_1 P_4}$$

or, the pressure ratios across the two compressors are equal.

$$\frac{P_2}{P_1} = \frac{P_4}{P_2} = \frac{P_4}{P_3}$$

or, $W_{comp\ I,in} = W_{comp\ II,in}$

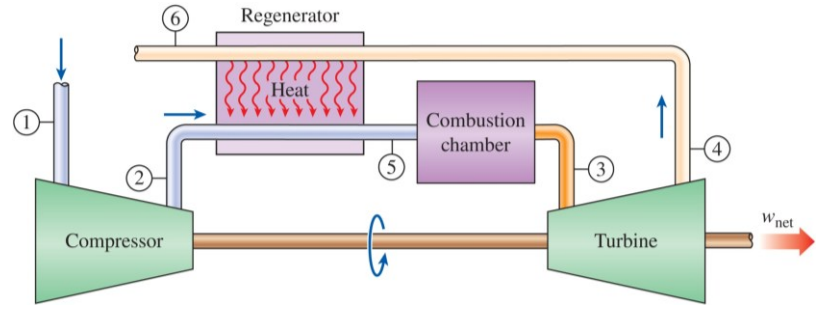
Regeneration

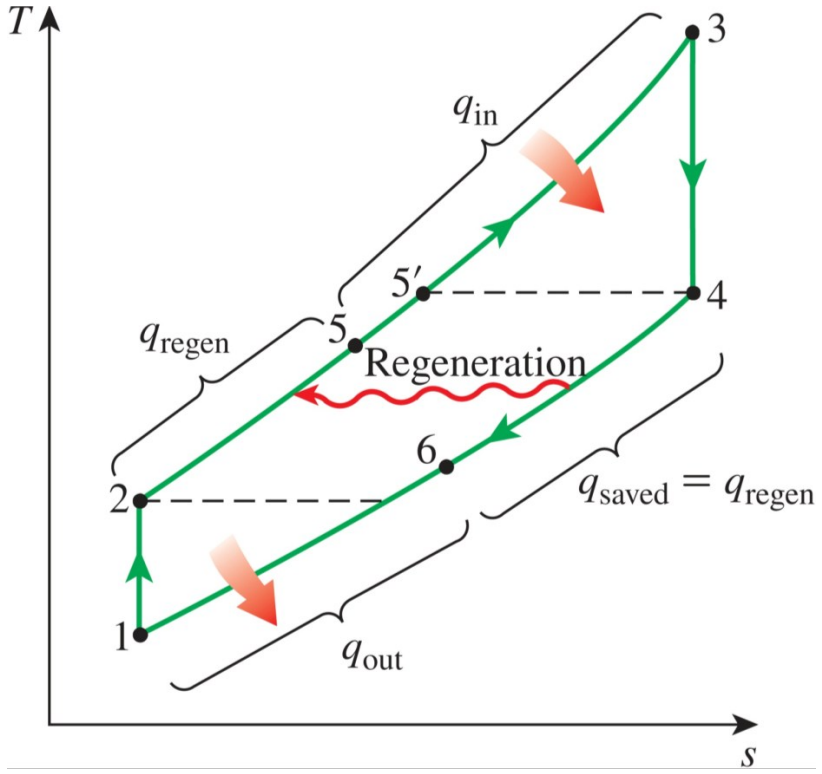
- The temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor.
  - Therefore, the high-pressure air leaving the compressor can be heated by transferring heat from the hot exhaust gases in a counterflow heat exchanger, also known as a **regenerator**.
  - Preheating reduces the heat added per unit of mass flowing (and thus the amount of fuel that must be burned).

$$q_{in,\ with\ regen} = h_3 - h_5$$

  - The net work per unit of mass flowing is not altered with the inclusion of a regenerator. Accordingly, since the heat added is reduced, thermal efficiency increases.

Gas-turbine engine with regenerator





Regeneration

We define the regenerator effectiveness (ε_{regen}) as the ratio of the heat transferred to the compressor gases in the regenerator to the maximum possible heat transfer to the compressor gases.

$$q_{regen,\,act} = h_5 - h_2$$

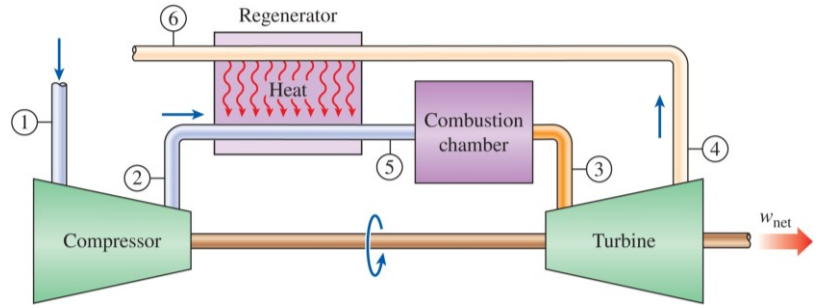
$$q_{regen,\,\max} = h_{5'} - h_2 = h_4 - h_2$$

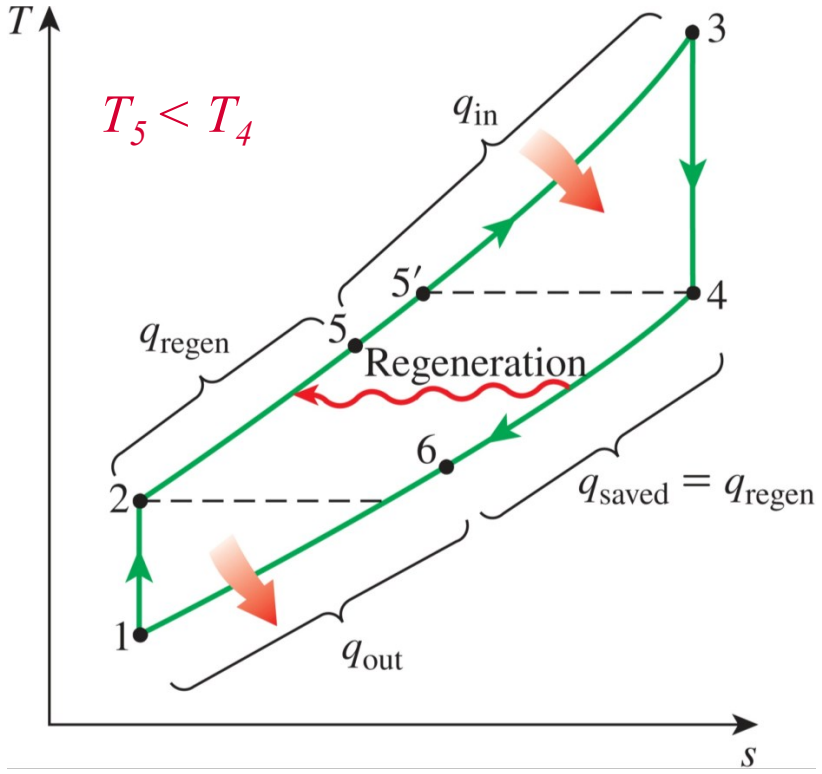
$$\varepsilon_{regen} = \frac{q_{regen,\,act}}{q_{regen,\,\max}} = \frac{h_5 - h_2}{h_4 - h_2}$$

For ideal gases using the cold-air-standard assumption with constant specific heats, the regenerator effectiveness becomes:

$$\varepsilon_{regen} \cong \frac{T_5 - T_2}{T_4 - T_2}$$

Gas-turbine engine with regenerator





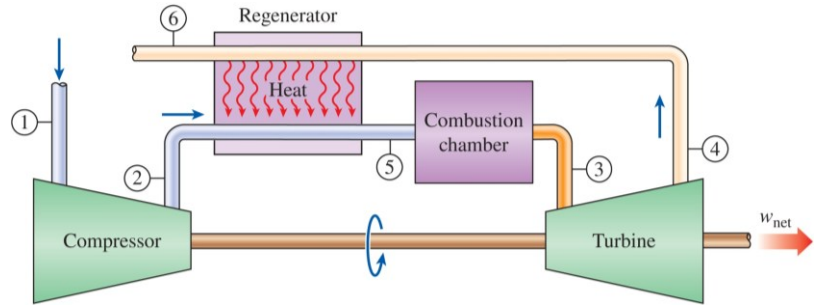
Thermal Efficiency with Regeneration

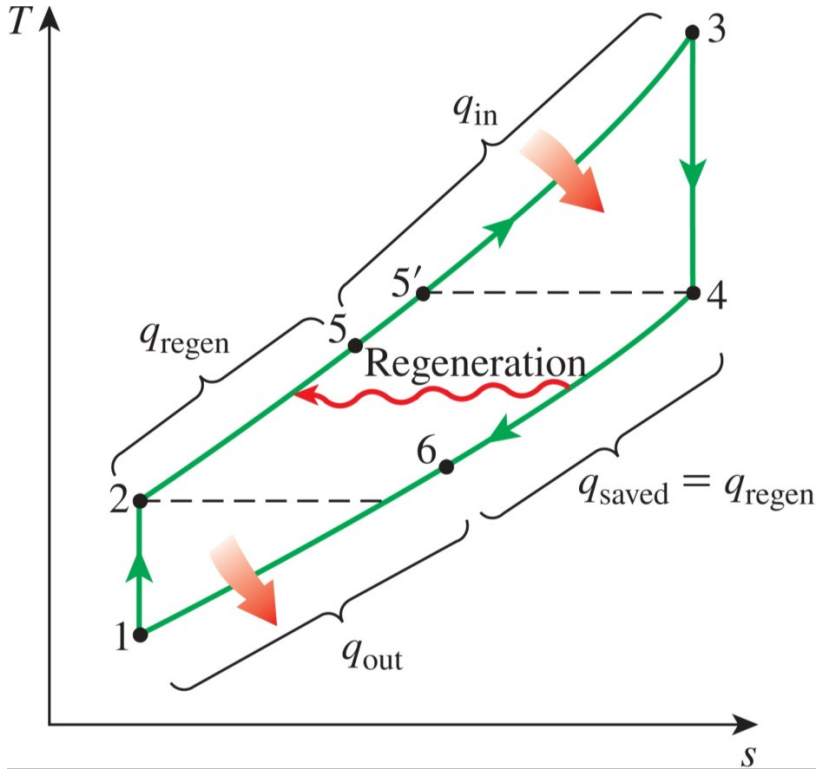
Using the closed cycle analysis and treating the heat addition and heat rejection as steady-flow processes, the regenerative cycle thermal efficiency is

$$\eta_{th,\,regen} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{h_6 - h_1}{h_3 - h_5}$$

Notice that the heat transfer occurring within the regenerator is **not** included in the efficiency calculation because this energy is not heat transferred across the cycle boundary.

Gas-turbine engine with regenerator





Thermal Efficiency with Regeneration

Assuming an ideal regenerator $\varepsilon_{regen} = 1$ and constant specific heats, the thermal efficiency ($\eta_{th,\,regen}$) becomes:

$$\eta_{th,\,regen} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

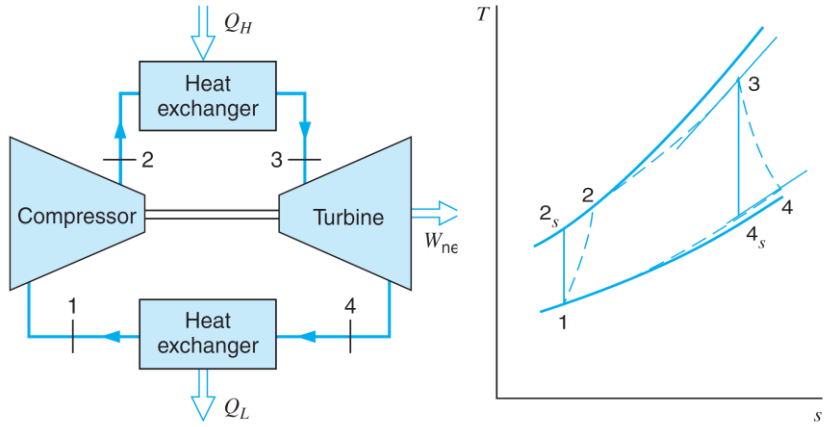
$$= 1 - \frac{T_1}{T_3}(r_p)^{(k-1)/k}$$

Thermal efficiency of an ideal Brayton cycle with regeneration depends on the ratio of the minimum to maximum temperatures as well as the pressure ratio.

[Section 10.3 of Borgnakke and Sonntag (8th Ed.)]

When does the efficiency of the air-standard Brayton cycle equal the efficiency of the air-standard regenerative Brayton cycle?

Therefore, $\eta_{th} = \eta_{th,\,regen}$



$$\eta_{th} = 1 - \frac{1}{\left(r_p\right)^{\frac{k-1}{k}}}$$

Thermal Efficiency with Regeneration

Assuming an ideal regenerator $\varepsilon_{regen} = 1$ and constant specific heats, the thermal efficiency ($\eta_{th,\,regen}$) becomes:

$$\eta_{th,\,regen} = 1 - \frac{T_1}{T_3}\left(\frac{P_2}{P_1}\right)^{(k-1)/k}$$

$$= 1 - \frac{T_1}{T_3}(r_p)^{(k-1)/k}$$

Thermal efficiency of an ideal Brayton cycle with regeneration depends on the ratio of the minimum to maximum temperatures as well as the pressure ratio.

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When does the efficiency of the air-standard Brayton cycle equal the efficiency of the air-standard regenerative Brayton cycle?

Therefore, $\eta_{th} = \eta_{th,\,regen}$

$$1 - \frac{1}{(r_p)^{(k-1)/k}} = 1 - \frac{T_1}{T_3}(r_p)^{(k-1)/k}$$

$$r_p = \left(\frac{T_3}{T_1}\right)^{k/[2(k-1)]}$$

What happens if the regenerative Brayton cycle operates at a pressure ratio larger than this value?

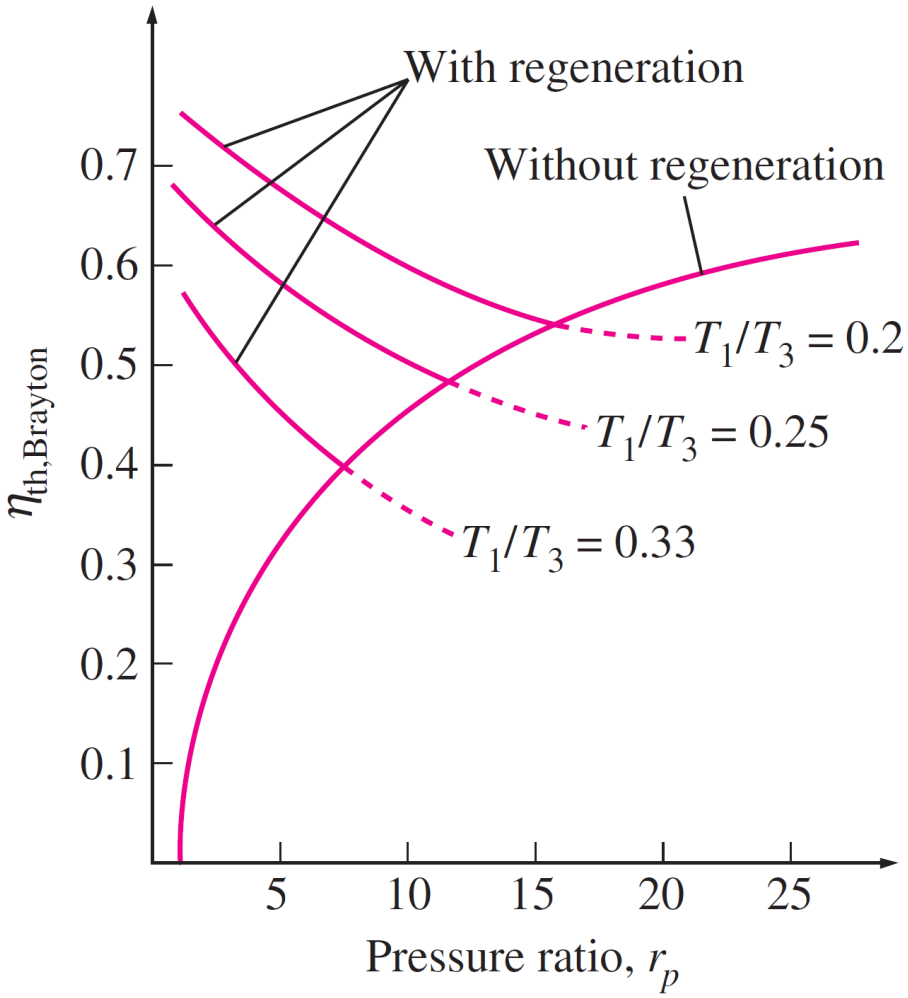
Thermal Efficiency with Regeneration

$$r_p = \left(\frac{T_3}{T_1} \right)^{k/[2(k-1)}$$

For fixed T_3 and T_1, pressure ratios greater than the above value cause T_4 to be less than T_2, and the regenerator is not effective.

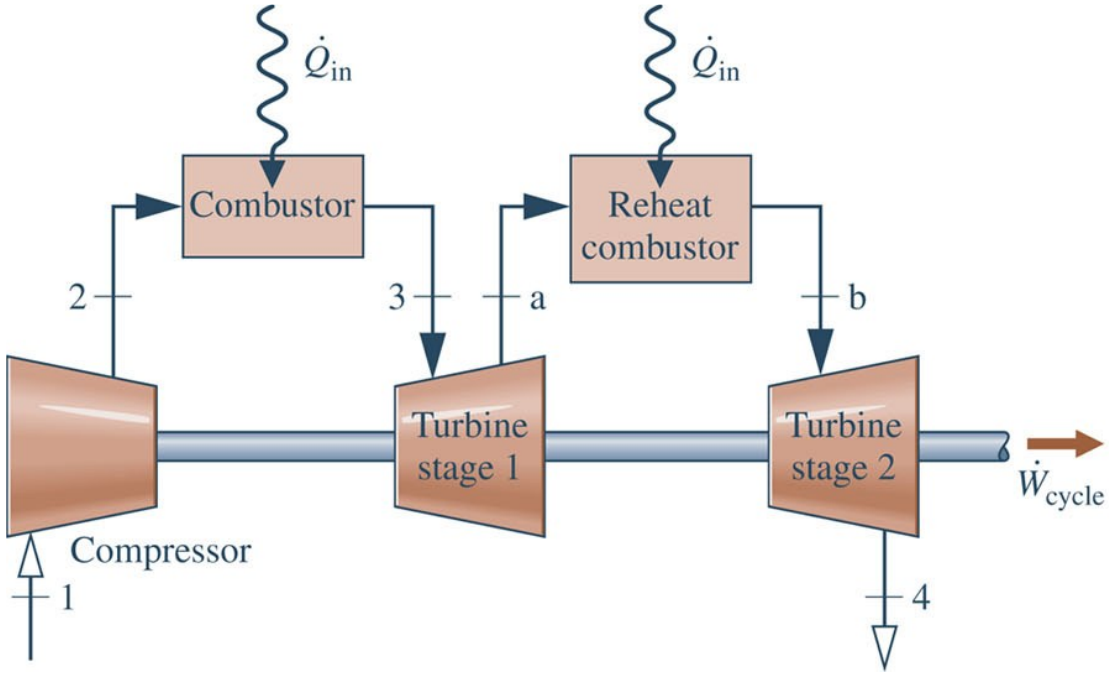
The figure shows a plot of the regenerative Brayton cycle efficiency as a function of the pressure ratio and minimum to maximum temperature ratio, T_1/T_3.

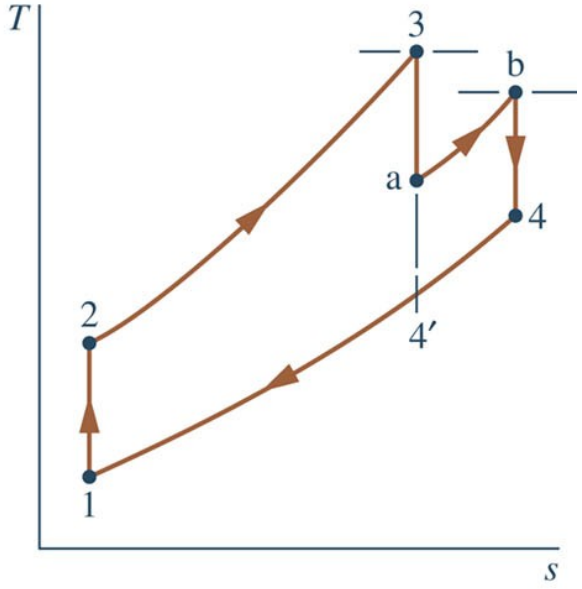
This figure also shows that regeneration is most effective at lower pressure ratios and low minimum-to-maximum temperature (T_1/T_3) ratios.



Reheating

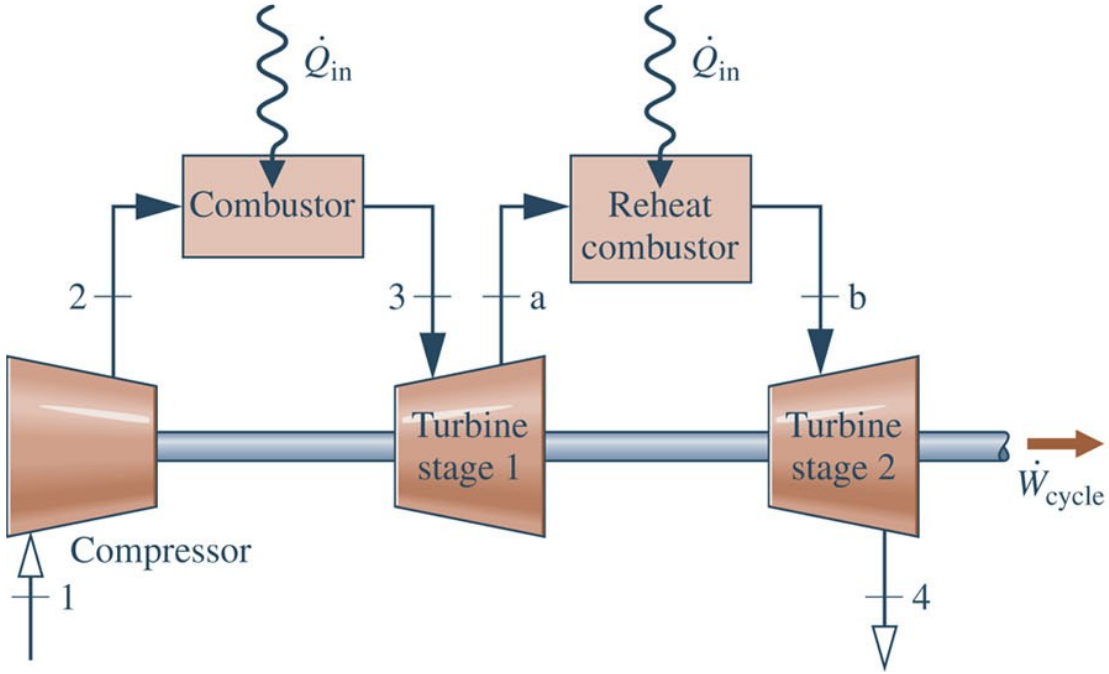
- The work output of a turbine operating between two pressure levels can be increased by expanding the gas in stages and reheating it in between—that is, utilising multistage expansion with reheating. This is accomplished without raising the maximum temperature in the cycle.

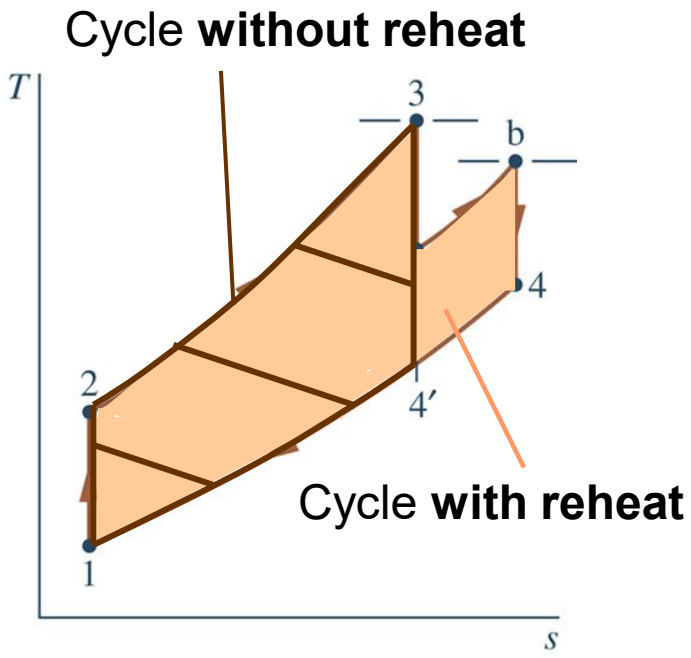




Reheating

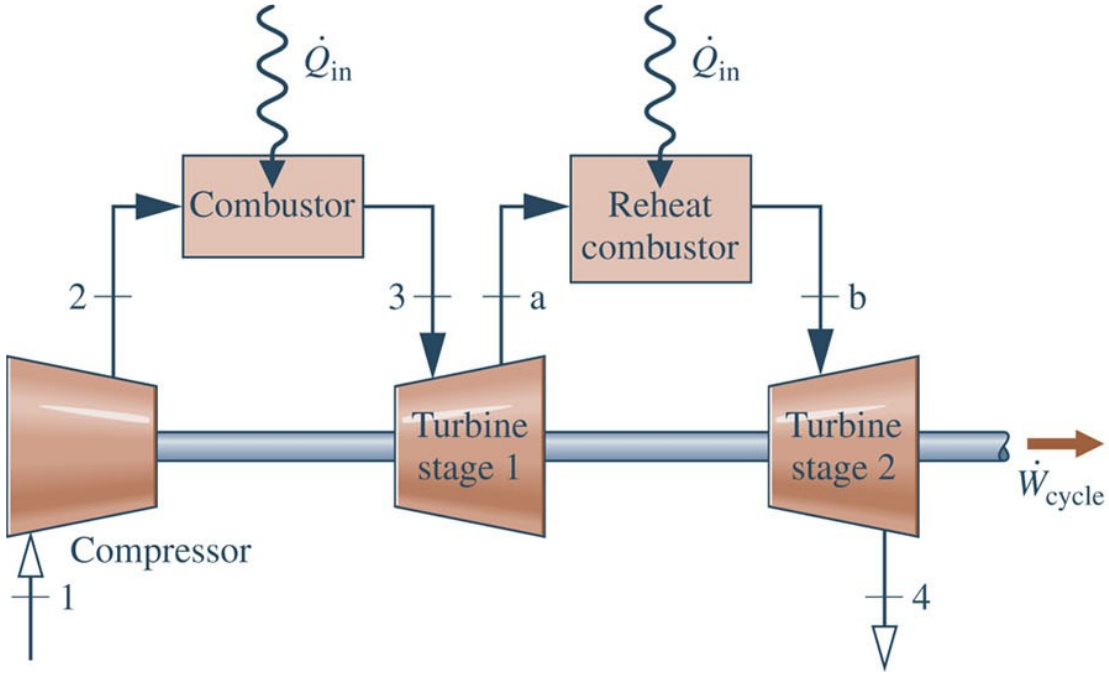
- The ideal Brayton cycle with reheat is 1-2-3-a-b-4-1. The ideal Brayton cycle without reheat is 1-2-3-4′-1.
- The reheat cycle has a larger enclosed area than the cycle without reheat and thus a greater net work developed per unit of mass flowing, which is the aim.

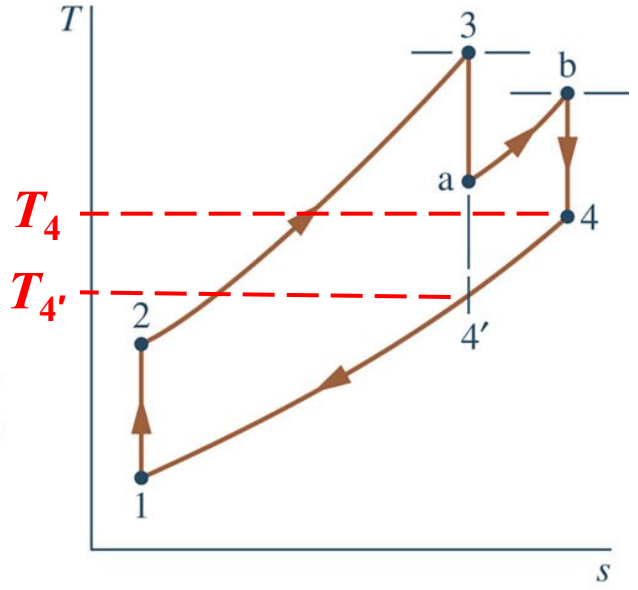




Reheating

- The T-s diagram also shows that the temperature at the exit of the second-stage turbine (state 4) is greater than at the exit of the single turbine of the cycle without reheat (state 4′). Accordingly, with reheat the potential for regeneration is also enhanced.
- When reheat and regeneration are used together, the thermal efficiency can increase significantly over that for the cycle without reheat.





Reheating

When using multistage expansion through two or more turbines, reheating between stages will increase the net work done (it also increases the required heat input unless it is accompanied by regeneration).

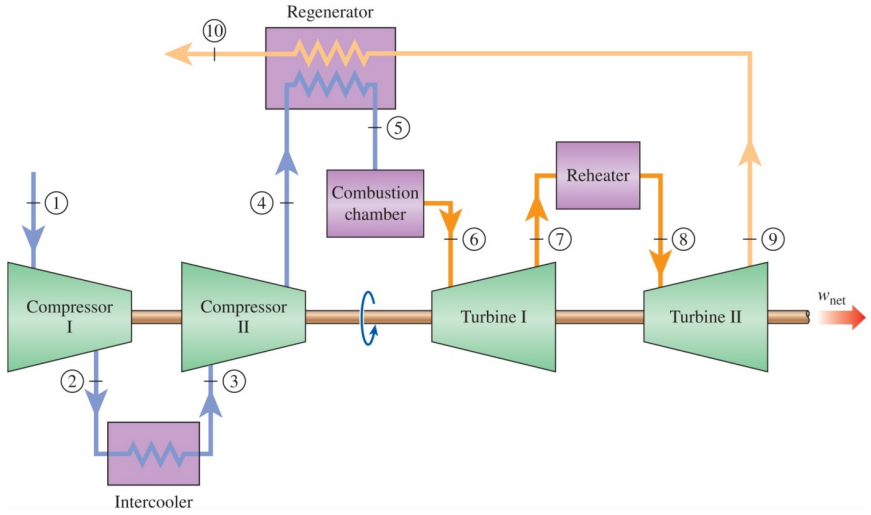
The optimum intermediate pressure for reheating is the one that maximizes the turbine work. Following the development given above for intercooling and assuming reheating to the high-pressure turbine inlet temperature in a constant pressure steady-flow process, we can show the optimum reheat pressure to be

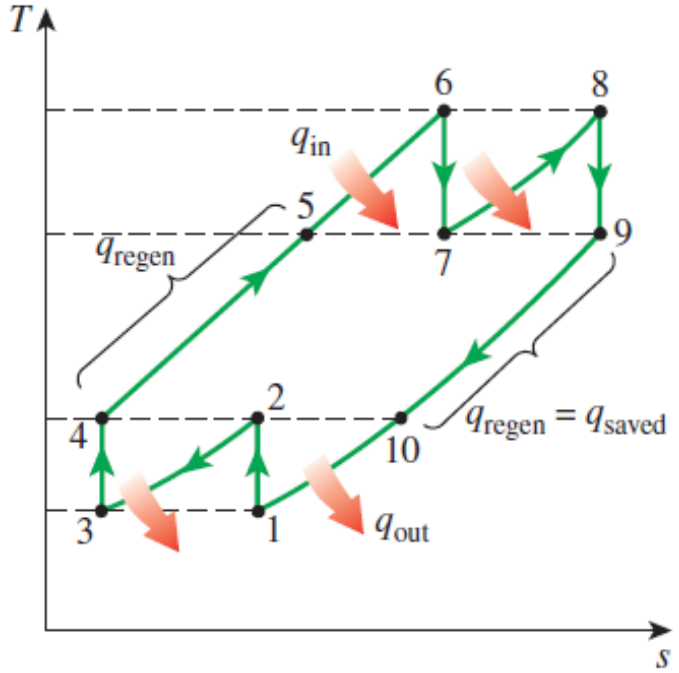
$$P_7 = \sqrt{P_6 P_9}$$

or the pressure ratios across the two turbines are equal.

$$\frac{P_6}{P_7} = \frac{P_7}{P_9} = \frac{P_8}{P_9}$$

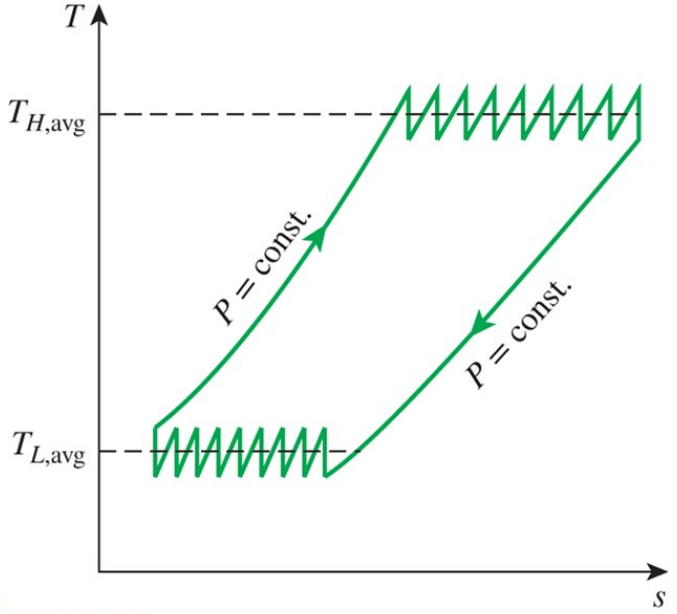
Gas-turbine engine with two-stage compression with intercooling, two-stage expansion with reheating, and regeneration



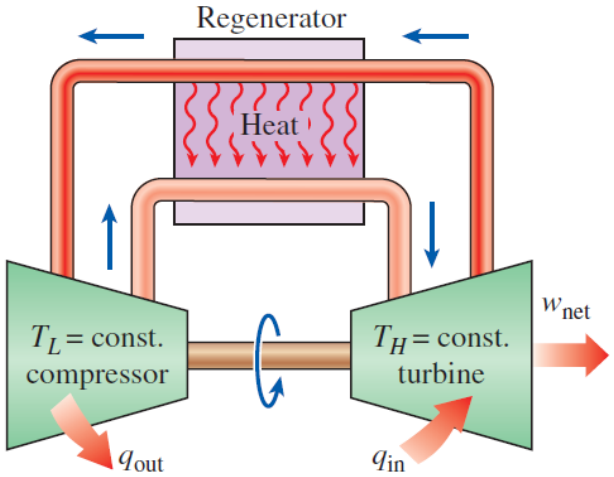


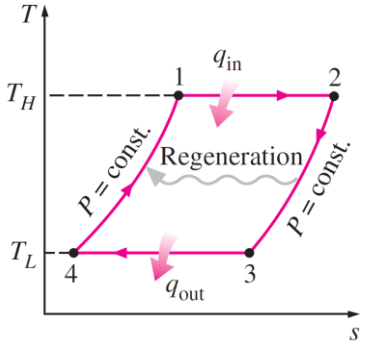
Intercooling, Reheating, and Regeneration Stages

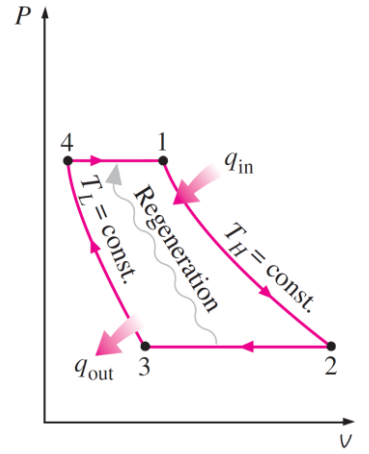
- As the number of compression and expansion stages increases, the gas-turbine cycle with intercooling, reheating, and regeneration approaches the Ericsson cycle and the thermal efficiency approaches the theoretical limit (the Carnot efficiency). However, the contribution of each additional stage to the thermal efficiency is less and less, and the use of more than two or three stages cannot be justified economically.



Steady-flow Ericsson engine







The Stirling and Ericsson cycles are totally reversible, as is the Carnot cycle, and thus according to the Carnot principle, all three cycles must have the same thermal efficiency when operating between the same temperature limits

$$\eta_{th,Stirling} = \eta_{th,Ericsson} = \eta_{th,Carnot} = 1 - \frac{T_L}{T_H}$$

Intercooling, Reheating, and Regeneration

Key Points:

Multistage compression with intercooling: The work required to compress a gas between two specified pressures can be decreased by carrying out the compression process in stages and cooling the gas in between. This keeps the specific volume as low as possible.

Multistage expansion with reheating: It keeps the specific volume of the working fluid as high as possible during expansion, maximizing work output.

Intercooling and reheating always decrease the thermal efficiency unless they are accompanied by regeneration, as temperature at the exit of the compressor reduces and reheating needs additional heat.

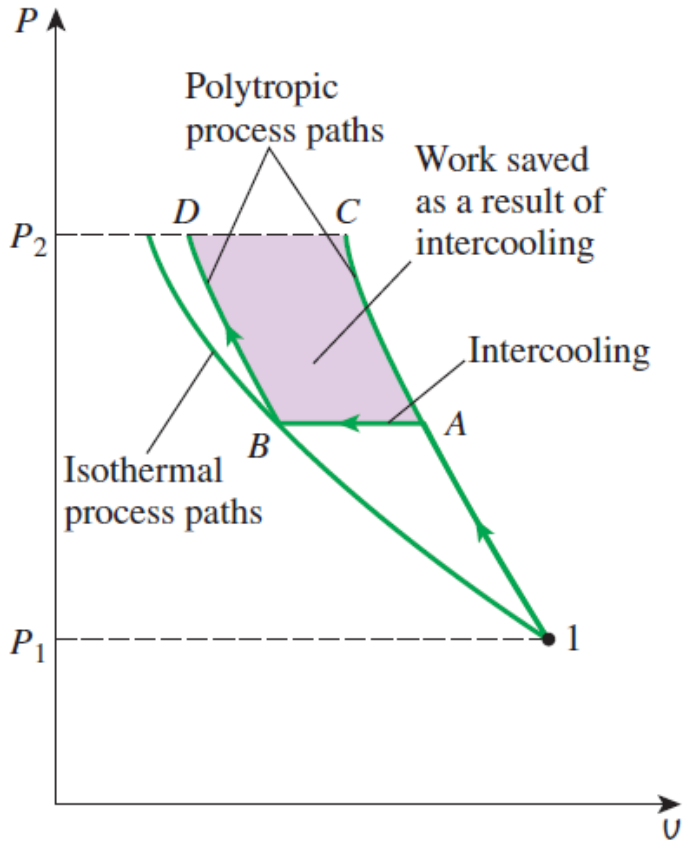
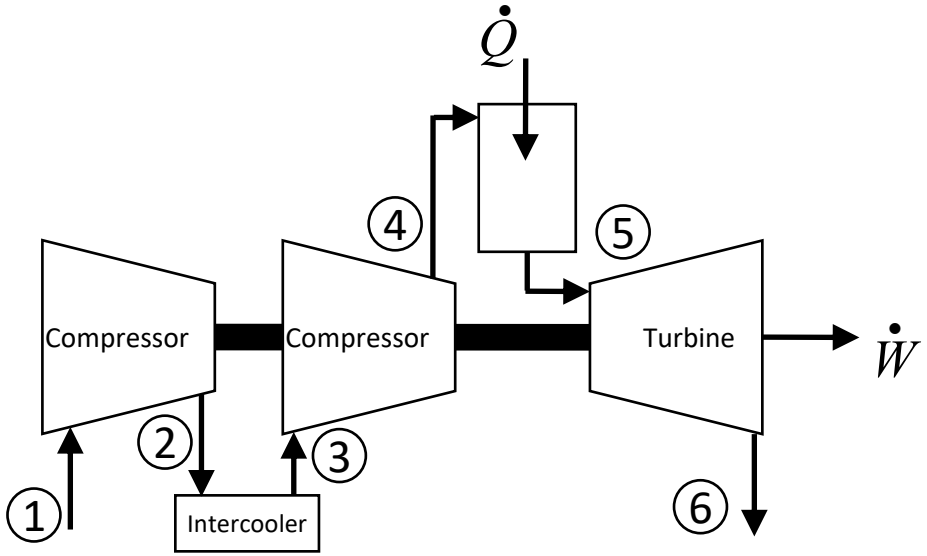


FIGURE 9–43
Comparison of work inputs to a single-stage compressor (1*AC*) and a two-stage compressor with intercooling (1*ABD*).

Example 3.3

An intercooled gas turbine (GT) has the following characteristics

Ambient conditions	15°C, 1.013 bar
Compressor isentropic efficiency	87%
Turbine isentropic efficiency	90%
Compressor stage pressure ratio	6.6
Air flow (assumed constant)	200 kg/s
Firing temperature	1400°C
Temperature after intercooler	15 K above inlet temperature
Pressure drop in intercooler	0.15 bar
Pressure drop in combustor	1%
Effective air properties:	C_p = 1.15 kJ/kg·K and k = 1.333



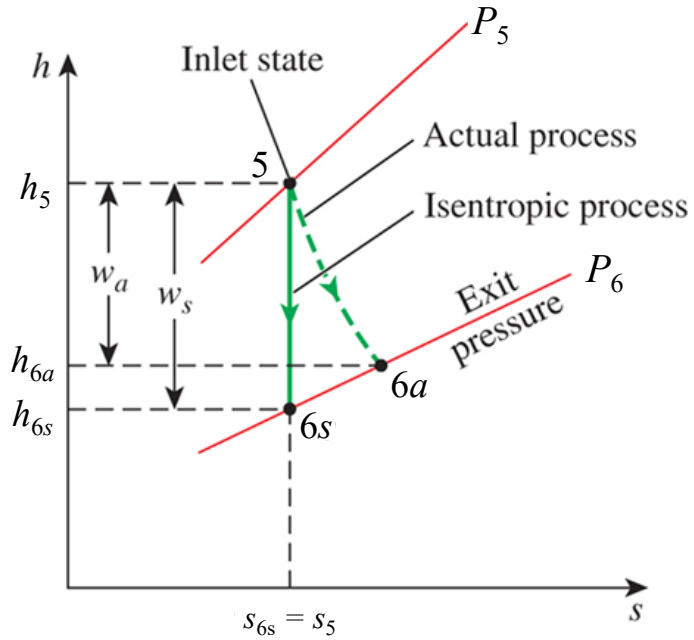
What are the compressor work, turbine work, combustor heat input and efficiency for:

a) The above configuration?

b) When a regenerator is added that heats the air to within 150 K of the maximum possible value?

c) With a single stage compressor with the same exit pressure?

d) If the firing temperature of the GT in (c) is reduced to 1150°C?



Summary

- Simple gas turbine energy balance
- Turbine and compressor efficiencies
- Options for improving Brayton cycle performance

Next: **Rankine and Combined Cycles**