Fluid Mechanics 4 (Mechanical)
Tutorial week 8

Question 1

In addition to the velocity field, we might be interested in modelling the transport of a conserved passive scalar $\phi(\boldsymbol{x}, t)$. The equation for a scalar can be written in the following way:

$$\frac{\partial \phi}{\partial t} + \boldsymbol{u} \cdot \nabla \phi = D \nabla^2 \phi$$

where D is the diffusivity (constant and uniform). The scalar is:

- **Conserved**: if there is no source or sink of ϕ in the equation.
- **Passive**: if it has no effect on the fluid properties (ρ, ν, D) and hence no effect on momentum.

If ϕ is the temperature ($\phi = T$) then D is the thermal diffusivity. If the range of temperature is very narrow, there are no effects on the fluid properties (ρ, ν, D), and the temperature does not have effects on the flow. If ϕ is the concentration of a trace species (again, negligible influence on ρ, ν and D) then D is the molecular diffusivity.

1. Derive the equation for the mean scalar $\langle \phi \rangle$.
2. Identify unclosed terms in the mean scalar equation.

Question 2

The measurements in Table 1 have been performed in a fluid flow laboratory for a turbulent flow downstream of a grid. These are measurement of one component of the velocity in one point of the flow $\boldsymbol{x}_0$ for different realisation of the same experiment.

1. Sketch an histogram of the velocity measurements. Tip: use a relatively small number of bins (intervals) to evaluate the histogram. A good value for the number of bins could be 6, distributed in the range $-3 : 3$.
2. Normalise the histogram to obtain a probability density function.
3. Compute the mean in the point $\boldsymbol{x}_0$.

   [Ans: $-0.066\ m/s$]

4. Compute the variance in the point $\boldsymbol{x}_0$.

   [Ans: $1.13\ m^2/s^2$]

Exam-style questions

Question 3

Derive the following expression for the variance $\langle u'^2 \rangle$ of one component of the velocity:

$$\langle u'^2 \rangle = \langle u^2 \rangle - \langle u \rangle^2 .$$

Realisation	Velocity measurement (m/s)
1	-1.23
2	-2.07
3	0.74
4	0.39
5	-0.31
6	-1.31
7	0.27
8	0.79
9	0.64
10	-0.56
11	0.88
12	1.02
13	-1.25
14	1.84
15	1.31
16	-0.74
17	1.12
18	-1.01
19	-0.96
20	-0.88

Table 1: Velocity measurements in a turbulent flow experiment.