Fluid Mechanics 4 (Mechanical)
Tutorial week 7

Question 1

The velocity distribution for a two-dimensional flow of a viscous fluid between two wide parallel plates is parabolic and is expressed as:

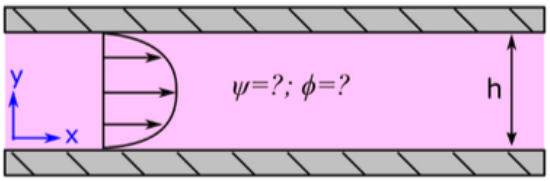
$$u(y) = U_c \left[1 - \left(\frac{y}{h}\right)^2\right]$$

Where U_c is the centerline velocity and the vertical velocity component $v = 0$.

(a) Determine, if possible, an expression for the stream function for the fluid flow field between the plates.

(b) Determine, if possible, an expression for the potential function for the fluid flow field between the plates.

(c) Is the flow rotational or irrotational?



Question 2

Consider the incompressible, two-dimensional flow of an inviscid fluid between the boundaries shown in the figure below. The velocity potential for this flow is:

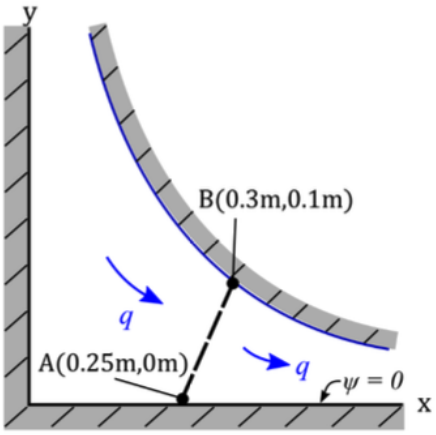
$$\phi = x^2 - y^2$$

(a) Determine the corresponding stream function for this flow.

(b) The volume flow rate per unit width (q) can be calculated by the following:

$$q = \int_{\psi_1}^{\psi_2} d\psi = \psi_2 - \psi_1.$$

Determine the volume flow rate passing through the line AB if the streamline at the bottom wall is considered to be $\psi = 0$.

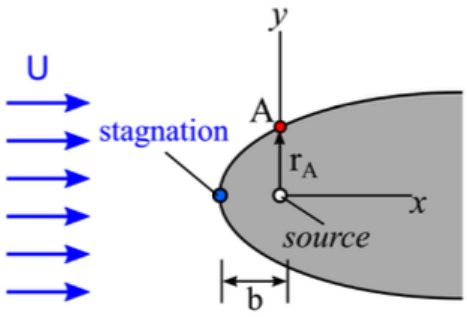


Question 3

A body having a shape of a half-body is placed in an inviscid fluid stream. Far upstream the half-body, the velocity is a horizontal uniform velocity with magnitude U. Consider point A, which is directly above the location where the source would be placed to create a half-body.

(a) Use the Bernoulli equation to derive an expression for the stagnation pressure (P_{stag}) as a function of the pressure at point A (P_A) and the velocity at point A (V_A). Ignore the elevation difference between the stagnation point and anywhere along the half-body.

(b) Determine an expression for pressure differential $P_{\text{stag}} - P_A$ as a function of the free stream velocity U.



Question 4

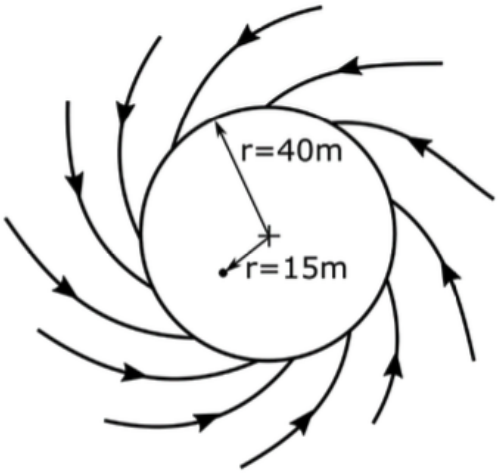
A tornado can be modelled as a sink + free vortex. The stream function is expressed as:

$$\psi = \psi_{\text{sink}} + \psi_{\text{free vortex}} = -\frac{q}{2\pi}\theta - \frac{\Gamma}{2\pi}\ln(r)$$

The circulation around the tornado is $\Gamma = 8500\,\text{m}^2/\text{s}$. The pressure at 40 m radius from the center is measured to be 2200 Pa less than the pressure in the far-field (i.e., a point far from the origin, where the velocity is negligible). Assume that the flow is inviscid with sea level air with $\rho = 1.225\,\text{kg/m}^3$ and $P_{\text{atm}} = 101\,\text{kPa}$.

(a) Determine the value of the sink strength (q) in m^2/s.

(b) Determine the pressure at $r = 15\,\text{m}$ from the vortex center.



Exam-style questions

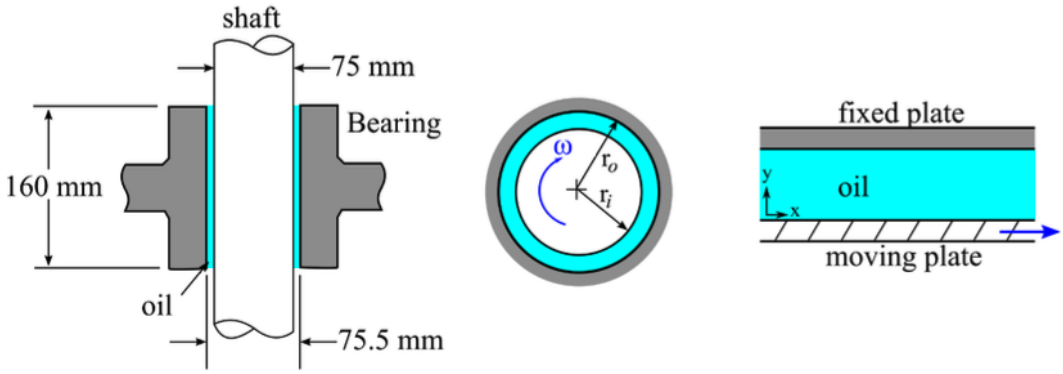
Question 5

Consider an unloaded journal bearing, where a shaft passing through the bearing rotates at a speed of $\omega = 60$ revolutions per minute. A lubricating oil with viscosity of $\mu = 0.2\,\mathrm{Ns/m^2}$ is placed between the rotating shaft and the bearing housing to reduce wear of the journal bearing. The rotating shaft has a diameter of 75 mm and the bearing has a diameter of 75.5 mm and length of 160 mm as shown in the figure. Model the journal bearing as a Couette flow with the bottom wall moving (i.e., "unravel" the cylinders and model them as infinitely long plates). Assume that there is no pressure gradient in the direction of the flow.

(a) Determine an expression of the oil velocity distribution between the shaft and bearing housing.

(b) Determine the shear stress in $\mathrm{N/m^2}$ acting on the rotating shaft.

(c) Determine the torque required to overcome the viscous resistance when the shaft is rotating at $\omega = 60$ revolutions per minute.



Question 6

An ideal fluid flows past an infinitely long semicircular cylinder as shown in the figure. The horizontal, uniform free stream velocity far from the cylinder is U and the pressure is P_o. Assume that this flow can be modeled as an ideal flow around a cylinder.

(a) Determine the maximum and minimum pressure along the cylinder surface if the fluid is air with $\rho = 1.2\,\mathrm{kg/m^3}$, $P_o = 1 \times 10^5\,\mathrm{Pa}$ and uniform velocity $U = 30\,\mathrm{m/s}$. Ignore the elevation difference along the surface of the cylinder.

By choosing the solid surface as the $\psi = 0$ streamline, the stream function can be expressed as: $\psi = Ur\left(1 - \frac{R^2}{r^2}\right)\sin\theta$. The radius of the semicircular cylinder is $R = 0.5\,\text{m}$.

(b) Determine the maximum velocity along the cylinder.

(c) Determine the velocity at $\theta = \frac{\pi}{2}$, $r = 2R$.

(d) Determine the velocity along the cylinder at $\theta = \frac{\pi}{4}$ and $\theta = \frac{3\pi}{4}$.

