Fluid Mechanics 4 (Mechanical)
Tutorial week 6

Question 1

Viscous oil flows steadily between two horizontal, stationary, parallel plates, as shown in Fig. 1. The flow between the plates is fully developed and steady. The gap width between the plates is $h = 3$ mm. The oil viscosity is 0.5 Ns/m^2 and the pressure gradient is -1200 Pa/m.

- Determine the magnitude of shear stress acting on the top and bottom plates.
- Determine the volume flow rate per unit width between the plates. (The volume flow rate is the volume of fluid which passes per unit time.)

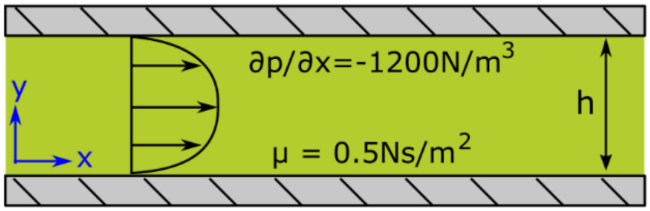


Figure 1: Oil flow in a channel

Question 2

An incompressible, viscous fluid is placed between two horizontal, infinitely long flat plates (see Fig. 2). The two plates move in opposite directions with constant velocities U_1 and U_2. The fluid flow between the plates is considered to be steady. The pressure gradient in the x-direction is zero, and the only body force is due to the fluid weight. Derive an expression for the velocity distribution $u(y)$ between the two plates.

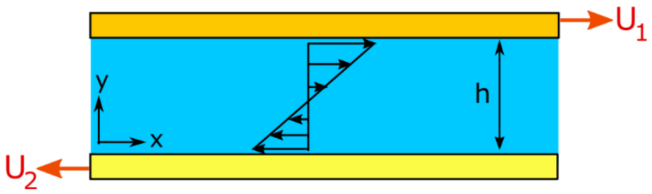


Figure 2: Flow between two plates.

Question 3

The stream function for an incompressible, two-dimensional flow field is given as:

$$\psi = ay^2 - bx$$

Where a and b are constants. Is this flow irrotational? Can you determine the velocity potential ϕ?

Question 4

In planar polar coordinates the velocity can be expressed in the following way:

$$v_r = \frac{\partial \phi}{\partial r} = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \quad \text{and} \quad v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -\frac{\partial \psi}{\partial r},$$

The velocity components in polar coordinates for an incompressible, planar flow are:

$$v_r = Ar^{-1} + Br^{-2}\cos\theta$$

$$v_\theta = Br^{-2}\sin\theta$$

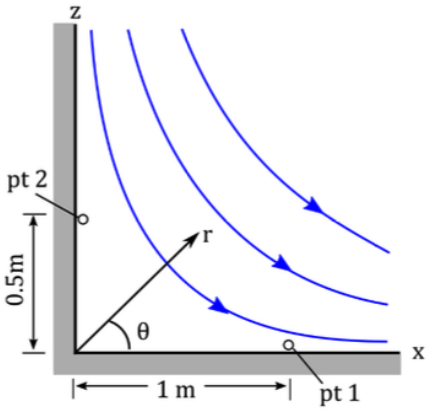
Where A and B are constants. Determine the corresponding stream function.

Question 5

A two-dimensional inviscid, incompressible fluid that flows in the vicinity of a 90° corner is described by the stream function:

$$\psi = 2r^2 \sin 2\theta$$

Where ψ has units of m^2/s and r has units of m. The flow in the x-z plane is horizontal (this is not a vertical flow).



(a) Determine the corresponding velocity potential function ϕ.

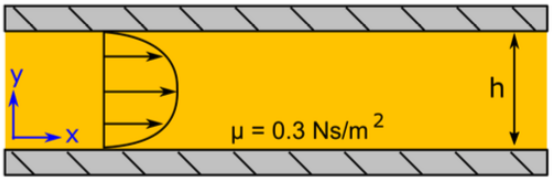
(b) Determine the pressure at point 2 near the wall if the pressure at point 1 is 30 kPa. Assume that the fluid density is $1000\,\mathrm{kg/m}^3$ and that there is no elevation difference between points (1) and (2).

Question 6

Two fixed, horizontal, parallel plates are spaced 4 mm apart. A viscous liquid with $\mu = 0.3\,\mathrm{Ns/m}^2$ flows between the plates with a mean velocity of $u = 0.2\,\mathrm{m/s}$.

(a) If the mean velocity between the plates is expressed as $\overline{u} = \frac{q}{h}$, where $q = \int_0^h u(y)\,dy$ is the volume flow rate per unit width, determine the pressure drop per unit length (in kPa/m) in the direction of the flow.

(b) Determine the maximum velocity (in m/s) within the channel.



Exam-style questions

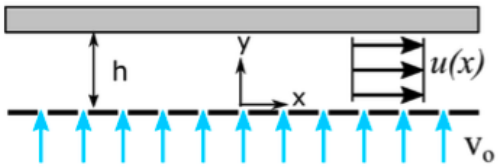
Question 7

Air flows steadily between two closely spaced parallel plates separated by height h by entering through a porous surface on the bottom plate. Assume that the viscous effects are negligible such that the horizontal velocity is uniform in the y direction, but varies as a function of x. Assume that the air is incompressible.

(a) Using a control volume, determine an expression for $u(x)$.

(b) Determine an expression for the y-velocity component $v(y)$ in the narrow passage.

(c) Determine an expression of the acceleration components of the fluid in between the narrow passage.



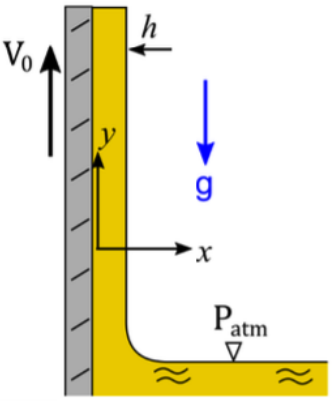
Question 8

A wide moving belt passes through a container of viscous fluid. The belt moves vertically upward with a constant velocity, V_0. Due to the viscous forces, the belt picks up a liquid film of thickness h. Gravity tends to make the fluid flow down the belt. Use the governing equations of fluid mechanics to determine:

(a) An expression of the velocity $v(x)$.

(b) The shear stress acting at the interface between the belt and fluid if the viscosity and density of the fluid is $\mu = 0.095\,\mathrm{Ns/m^2}$ and $\rho = 860\,\mathrm{kg/m^3}$ and the thickness of the fluid is $h = 3\,\mathrm{mm}$.

(c) The volume flow rate per unit width $q = \int v(x)\,dx$ if the belt velocity is $V_0 = 2\,\mathrm{m/s}$.



Question 9

A thin film of water with constant height h flows steadily down an inclined roof after a rainfall (see Fig. 3). In this thin layer of water, viscous effects are important to describe the fluid motion. The top of the film is exposed to the air (i.e., open atmosphere), such that the interface between the film and the air is considered to be a free surface. The film height relative to the roof dimensions is such that the roof can be assumed to be an infinitely long plate.

1. Using logical reasoning, determine the pressure gradient along the direction of the flow (i.e., $\partial p/\partial x$) for this thin film with free surface and shallow angle α. (Tip: don't use equations here, just make a reasonable assumption.)

2. If the primary flow direction is in the x-direction, determine an expression for $u(y)$.

3. Determine the volume flow rate per unit width (q, defined below) if the roof is inclined with $\alpha = 15°$ and $h = 1\,\mathrm{mm}$. The properties of water are: $\rho = 1000\,\mathrm{kg/m^3}$ and $\mu = 0.00152\,\mathrm{Ns/m^2}$.

$$q = \int u(y)\,dy$$

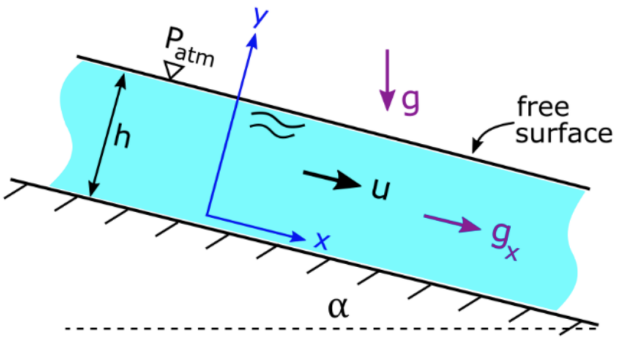


Figure 3: Film of water on a roof.