Fluid Mechanics 4 (Mechanical)
Tutorial week 3

Question 1

Consider the steady two-dimensional velocity field

$$\mathbf{u}(x, y) = (u, v) = (2x,\ -2y).$$

1. Derive the equation of the streamlines.
2. Sketch the family of streamlines in the xy-plane.

Question 2

Consider the two-dimensional velocity field

$$\mathbf{u}(x, y) = (u, v) = \big(1.5,\ -0.6\, y\big),$$

with SI units (m, s). A fluid particle starts at $(x(0), y(0)) = (x_0, y_0)$.

1. Write the ODEs for the particle trajectory $(x(t), y(t))$.
2. Solve for $x(t)$ and $y(t)$ for arbitrary x_0, y_0.
3. Eliminate t to obtain the particle path in the (x, y)–plane.

Question 3

Consider a horizontal pipe of cross-sectional area $A = 0.05$ m^2 carrying a **gas** with density $\rho = 1$ kg/m^3 at the time considered here (ρ is not necessarily constant in time). The velocity field is uniform across the section.
At the pipe inlet, the velocity is $u_{\text{in}} = 2$ m/s. At the pipe outlet, the velocity is $u_{\text{out}} = 3$ m/s.

1. Using the **Reynolds Transport Theorem** with $b = 1$ (so B is the total mass), write the mass conservation equation for a fixed control volume enclosing the pipe segment.
2. Compute the net rate of mass flow out of the control volume (in kg/s).
3. State whether the mass inside the control volume increases, decreases, or remains constant.

Exam-style questions

Question 4

For a fixed control volume, the Reynolds Transport Theorem can be written as

$$\frac{dB}{dt} = \int_{V_0} \frac{\partial(\rho b)}{\partial t}\, dV + \int_{S_0} \rho b\, \mathbf{u} \cdot \mathbf{n}\, dS.$$

Briefly explain the physical meaning of the two terms on the right-hand side.