# Fluid Mechanics 4 (Mechanical)
# Tutorial week 2

## Question 1

An anemometer is located at a point:

$$\vec{P}_1 = (0, 0) \text{ m},$$

near an airplane wing and measured a constant velocity of:

$$\vec{u} = (10, 10) \text{ m/s} \quad \text{(i.e., 2D flow)}$$

When the anemometer was moved 0.1 m in the x-direction to point:

$$\vec{P}_2 = (0.1, 0) \text{ m},$$

it measured a velocity of:

$$\vec{u} = (10.1, 10.1) \text{ m/s}.$$

When the anemometer was moved 0.1 m in the y-direction to point:

$$\vec{P}_3 = (0.0, 0.1) \text{ m},$$

it measured a velocity of:

$$\vec{u} = (10.2, 10.2) \text{ m/s}.$$

Estimate:
(a) the local acceleration,
(b) the convective acceleration, and
(c) the total acceleration.

## Question 2

Consider the two-dimensional velocity field

$$\mathbf{u}(x, y) = (\gamma y,\ 0),$$

where $\gamma$ is a constant.

1. Compute the velocity gradient tensor $\nabla \mathbf{u}$.
2. Decompose $\nabla \mathbf{u}$ into:
   - the volumetric dilation (compression/expansion),
   - the symmetric strain rate tensor $E$,
   - the antisymmetric rotation tensor $\Omega$.
3. Determine the vorticity vector $\boldsymbol{\omega} = \nabla \times \mathbf{u}$.
4. Briefly describe the type of fluid deformation taking place.

## Question 3

Consider the scalar field $\phi(x, y, z) = x^2 + y^2 + z^2$ and the vector field $\mathbf{F}(\mathbf{x}) = (2xy, x^2 - z, 3z^2)$, where $\mathbf{x} = (x, y, z)$.

1. Calculate the gradient of the scalar field $\phi$.
2. Calculate the divergence of the vector field $\mathbf{F}$.
3. Calculate the curl of the vector field $\mathbf{F}$.
4. Calculate the gradient of the vector field $\mathbf{F}$.
5. For all the quantities computed above, say if they are scalars, vectors, or second order tensors.

## Question 4

Consider a fluid with the following velocity field in three-dimensional space:

$$\mathbf{u}(x, y, z, t) = (yz, x^2 - z, 2xy)$$

where $\mathbf{u} = (u_1, u_2, u_3)$ represents the velocity components in the $x$, $y$, and $z$ directions, respectively.
Calculate the components of the rate of strain tensor $\mathbf{S}$ for the given velocity field.

## Question 5

A fluid particle in a steady flow moves along the x-axis. Its distance from the origin is $r_0$ at time $t_0$, and its trajectory is $r(t) = \left[K(t - t_0) + r_0^3\right]^{1/3}$, where $K$ is a positive constant with units of volume/time. Determine the flow Eulerian velocity and acceleration, $u(x)$ and $a(x)$, and show that $a(x)$ may also be obtained from $\frac{Du(x)}{Dt}$.

## Question 6

Consider the two-dimensional velocity field

$$\mathbf{u}(x, y) = (ax,\ -ay),$$

where $a$ is a constant.

1. Compute the velocity gradient tensor $\nabla\mathbf{u}$.
2. Decompose $\nabla\mathbf{u}$ into:
   - the volumetric dilation (compression/expansion),
   - the symmetric strain rate tensor $E$,
   - the antisymmetric rotation tensor $\Omega$.
3. Determine the vorticity vector $\boldsymbol{\omega} = \nabla \times \mathbf{u}$.
4. Briefly describe the type of fluid deformation taking place.

## Question 7

Consider the (steady) 3D velocity field (units: $x, y, z$ in m, $t$ in s)

$$\mathbf{u}(x, y, z) = \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} 1.20\,x + 0.40\,y - 0.10\,z \\ -0.30\,x + 0.90\,y + 0.20\,z \\ 0.00\,x - 0.50\,y + 0.60\,z \end{pmatrix}.$$

1. Compute the velocity gradient tensor $\nabla\mathbf{u}$ and the divergence $\nabla\cdot\mathbf{u}$ (give numerical matrices/scalars).

2. Form the symmetric strain–rate tensor $E$ and the antisymmetric rotation tensor $\Omega$.

3. Compute the vorticity vector $\boldsymbol{\omega}$ (numerical values).

4. A small cubic fluid element of edge length $L_0 = 1\,\text{mm}$, centered at $(x_0, y_0, z_0) = (0, 0, 0)$, has initial edge vectors aligned with the axes:

$$\mathbf{e}_x = (L_0, 0, 0), \quad \mathbf{e}_y = (0, L_0, 0), \quad \mathbf{e}_z = (0, 0, L_0).$$

   Using the linearized kinematics $\Delta\mathbf{e}_i \approx (\nabla\mathbf{u})\,\mathbf{e}_i\,\Delta t$ with $\Delta t = 0.02\,\text{s}$,

   (a) compute the updated edge vectors $\mathbf{e}_i^{\text{new}} = \mathbf{e}_i + \Delta\mathbf{e}_i$,

   (b) compute their new lengths,

   (c) compute the angle (degrees) between $\mathbf{e}_x^{\text{new}}$ and $\mathbf{e}_y^{\text{new}}$.

5. Using $\dot{V}/V = \nabla\cdot\mathbf{u}$ for small times, estimate the element's volume after $\Delta t$ and compare it to the volume obtained from the triple product $V^{\text{new}} \approx \mathbf{e}_x^{\text{new}} \cdot (\mathbf{e}_y^{\text{new}} \times \mathbf{e}_z^{\text{new}})$ (percent difference).

6. Classify the motion (dominantly extensional, shear, rotational?) and justify your answer using your $E$, $\Omega$, and $\boldsymbol{\omega}$.

# Exam-style questions

# Question 8

Explain the difference between **vorticity** and **angular deformation (shear strain)** of a fluid element.
Can a fluid element:

1. have non-zero vorticity without undergoing angular deformation?

2. undergo angular deformation while having zero vorticity?

Briefly justify your answers.

# Question 9

Consider the two-dimensional velocity field

$$\mathbf{u}(x, y) = (ay, ax),$$

where $a$ is a positive constant.

1. Calculate the divergence $\nabla \cdot \mathbf{u}$.

2. Calculate the vorticity $\boldsymbol{\omega} = \nabla \times \mathbf{u}$.

3. Calculate the strain-rate tensor $\mathbf{S}$.

4. Describe how an initially square fluid element deforms.