Fluid Mechanics 4 (Mechanical)
Tutorial week 1

Question 1

Compute the derivative dy/dx of the function $y = x \sin x$

Question 2

Consider the 2-dimensional velocity field that is described by the equation:

$$\boldsymbol{V} = 4xy\boldsymbol{e}_1 + 2(x^2 - y^2)\boldsymbol{e}_2$$

Where $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ represent the unit vectors in the x- and y-directions. Is this flow field irrotational (zero vorticity)?

Question 3

Demonstrate that the cross product is not commutative:

$$\boldsymbol{u} \times \boldsymbol{v} \neq \boldsymbol{v} \times \boldsymbol{u}$$

where $\boldsymbol{u} = (u_1, u_2, u_3)$ and $\boldsymbol{v} = (v_1, v_2, v_3)$.

Question 4

Consider a Cartesian coordinate system with velocity components:

$$u = x^2 + y^2 + z^2$$

$$v = xy + yz + z$$

$$w = ?$$

Determine the form of the w velocity component if the flow has zero divergence.

Question 5

Consider the scalar field

$$\phi(x, y, z) = x^2 y + yz,$$

and the vector field

$$\mathbf{F}(x, y, z) = (xy,\ y^2 z,\ xz).$$

1. Compute the **gradient** of $\phi(x, y, z)$.
2. Compute the **divergence** of $\mathbf{F}(x, y, z)$.
3. Compute the **curl** of $\mathbf{F}(x, y, z)$.

Question 6

Let the velocity field be

$$\mathbf{u} = (u, v, w).$$

1. Write the divergence $\nabla \cdot \mathbf{u}$ in index notation.
2. Write the convective term $(\mathbf{u} \cdot \nabla)\mathbf{u}$ in index notation.

Question 7

The transport of a passive scalar, often denoted by ϕ, can be described by the advection-diffusion equation. The general form of this equation is given by:

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = \nabla \cdot (D \nabla \phi) + S$$

Where:

- ϕ is the passive scalar (e.g., concentration of a pollutant).
- t is time.
- $\mathbf{u}$ is the velocity field (advection).
- D is the diffusion coefficient (representing molecular diffusion).
- S is a source term (representing any additional sources or sinks of the scalar).

Write this equation in index form.

Question 8

Use the **divergence theorem** to compute the outward flux of the vector field

$$\mathbf{F}(x, y, z) = (x^2,\, y^2,\, z^2)$$

through the surface of the cube bounded by

$$x = 0,\ x = 1, \qquad y = 0,\ y = 1, \qquad z = 0,\ z = 1.$$

Exam-style questions

Question 9

Sketch the Lennard–Jones potential as a function of intermolecular distance, and indicate the region corresponding to gas-like behaviour.

Question 10

What is the fundamental difference between a solid and a fluid? Consider how each material responds when subjected to a constant shear stress.