Fluid Mechanics 4

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Lecture 5

Streamlines, Pathlines, Streaklines



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Introduction

- Streamlines, Pathlines, and Streaklines are fundamental tools to visualize and interpret fluid motion.
- Appear in experiments (smoke, dye injection, particle tracking) and simulations (CFD).
- Crucial to distinguish between **steady** and **unsteady** flows.

- A flow is described by its velocity field:

$$\mathbf{u}(\mathbf{x}, t) = (u(x, y, z, t), v(x, y, z, t), w(x, y, z, t))$$

- **Steady flow:** $\mathbf{u}(\mathbf{x}, t) = \mathbf{u}(\mathbf{x})$
- **Unsteady flow:** $\mathbf{u}$ depends explicitly on t

Streamlines

- Streamlines are curves that represent the direction of flow in a fluid at every point. They are tangent to the velocity vector at all points and give a snapshot of the flow pattern at a particular instant in time.
- Mathematically, the velocity vector $\boldsymbol{u}$ is tangent to the streamline at every point, and their governing equation in 3D is:

$$\frac{d\boldsymbol{x}_S}{ds} \times \boldsymbol{u}(\boldsymbol{x}_S) = \boldsymbol{0},$$

  where "$\times$" denotes the vector product and $\boldsymbol{x}_S(s)$ is the parametric representation of just one streamline at one moment in time.
- If the components of the velocity are $\boldsymbol{u} = (u, v, w)$, and those of the streamline are $\boldsymbol{x}_S = (x_S, y_S, z_S)$:

$$\frac{dx_s}{u} = \frac{dy_s}{v} = \frac{dz_s}{w}$$

  or

$$\frac{dy_s}{dx_s} = \frac{v}{u} \quad \text{and} \quad \frac{dz_s}{dx_s} = \frac{w}{u}$$

Pathlines (trajectories)

- Pathlines represent the trajectories of individual fluid particles as they move through the flow. If you were to track a specific particle over time, the curve traced out by its movement would be a pathline.
- The position of a fluid particle $\mathbf{x}_p(t)$ at time t, which started at position $\boldsymbol{x}_0$ at time t_0, is given by:

$$\frac{d\boldsymbol{x}_p(t)}{dt} = \boldsymbol{u}_p(t), \quad \boldsymbol{x}_p(t_0) = \boldsymbol{x}_0$$

  with $\boldsymbol{u}_p(t) = \boldsymbol{u}(\boldsymbol{x}_p(t), t)$
- Lagrangian description: follow one marked particle over time.
- Example: tracking one dye spot in water.

Streaklines

- A streakline is the curve formed by all fluid particles that have passed through a particular point in the flow field over time.
- Experimental visualization: continuous dye injection or smoke lines in wind tunnels.
- Hybrid description: Eulerian (fixed location) + Lagrangian (all particles passing through).
- In unsteady flow, streaklines differ from streamlines because streaklines are influenced by the history of the flow. Streaklines give a **history** of the particles that passed through a specific point, while streamlines are instantaneous snapshots.

- **Equation for Streaklines**: If fluid particles are released at position $\boldsymbol{x}_0$ at time t_0, the position of these particles at a later time t is governed by:

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + \int_{t_0}^{t} \boldsymbol{u}(t')\, dt'$$

- The streakline is constructed by tracking the positions of all particles passing through $\boldsymbol{x}_0$ at different times.

Examples for steady and unsteady velocities

Steady flow (uniform):

$$u(x, y) = 1, \quad v(x, y) = 0$$

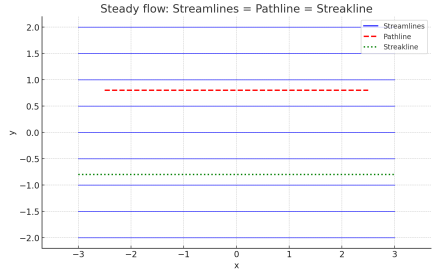
- Streamlines are horizontal straight lines.
- Pathlines and streaklines coincide with streamlines.

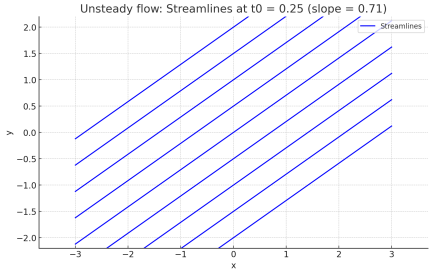
Unsteady flow (time-dependent vertical component):

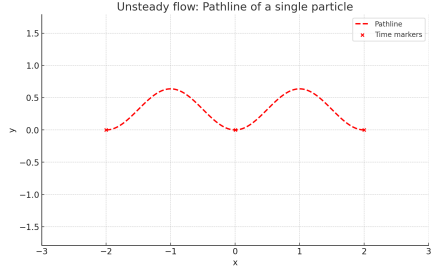
$$u(x, y, t) = 1, \quad v(x, y, t) = \sin(\pi t)$$

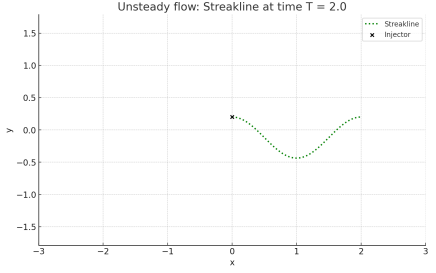
- Streamlines (at a certain time): instantaneous straight lines with slope $\sin(\pi t)$.
- Pathlines: obtained by integrating particle motion in time.
- Streaklines (at a certain time): locus of all particles released from a fixed injector.

Comparison of Flow Visualizations









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Key Takeaways

- **Streamlines:** Instantaneous flow structure.
- **Pathlines:** Actual trajectory of a particle.
- **Streaklines:** Experimental dye/smoke visualization.
- In steady flows, all three coincide. In unsteady flows, they differ.