Fluid Mechanics 4

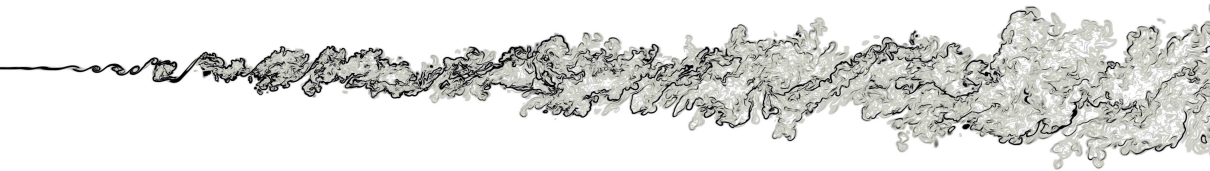
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Lecture 1

Definition of fluid and continuum hypothesis

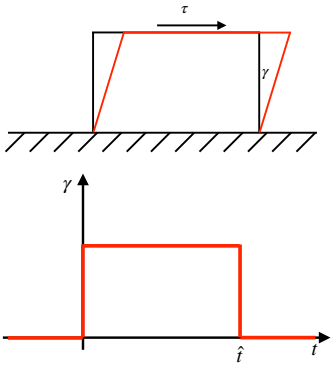


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Definition of fluid

Fluid Mechanics deals with "fluids". What is a fluid?

- Easy to define intuitively what a fluid is:
  - Water and air are fluids
  - A block of steel is a solid
- A formal definition is more difficult, since it is based on the way the material reacts to external actions.
- The difference between solids, liquids and gases is related to the properties of the material.
- **Particle Arrangement**
  - **Solids**: Particles are closely packed together in a fixed, orderly arrangement.
    The particles have very little freedom to move, so solids maintain a definite shape and volume.
  - **Liquids**: Particles are also close together but not in a fixed position.
    They can move past each other, so liquids can flow and take the shape of their container while maintaining a definite volume.
  - **Gases**: Particles in a gas are far apart and move freely in all directions.
    Gases do not have a definite shape or volume and will expand to fill the entire space available to them.
- **Intermolecular Forces**
  - **Solids**: Strong intermolecular forces hold the particles together.
  - **Liquids**: The intermolecular forces in liquids are weaker than in solids but still significant enough to keep the particles close.
    This allows liquids to flow while still being relatively incompressible.
  - **Gases**: The intermolecular forces in gases are very weak or negligible, allowing the particles to move independently. This results in the ability of gases to expand and compress more easily than liquids.



⟵ Linearly Elastic Solid

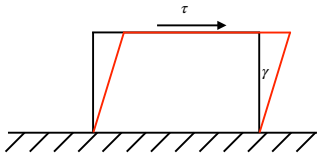
- If we apply a tangential force

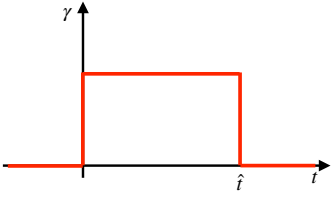
$$\tau = F/A$$

  we get an angular deformation γ.
- The deformation γ is proportional to the sheer stress

$$\tau = G\gamma$$

  where G depends on the material (steel, concrete, wood...) and the state of the material (temperature, pressure).
- If we apply the tangential force at $t = 0$ and maintain the sheer stress τ the deformation γ remains constant until we remove it at $t = \hat{t}$





Newtonian Fluid $\longrightarrow$

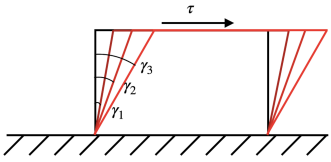
- If we a apply a tangential force

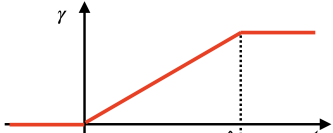
$$\tau = F/A$$

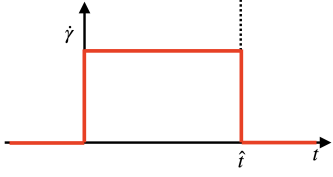
  we get an angular deformation γ which keeps increasing with time.
- When we remove the force τ the deformation γ remains at it latest angular deformation.
- What remains constant in this case during the application of the force is the angular deformation rate $\dot{\gamma}$:

$$\tau = \mu\dot{\gamma}$$

  where μ is the dynamic viscosity of a fluid and depends on the material and the state of the fluid.
- You don't recover the original configuration.

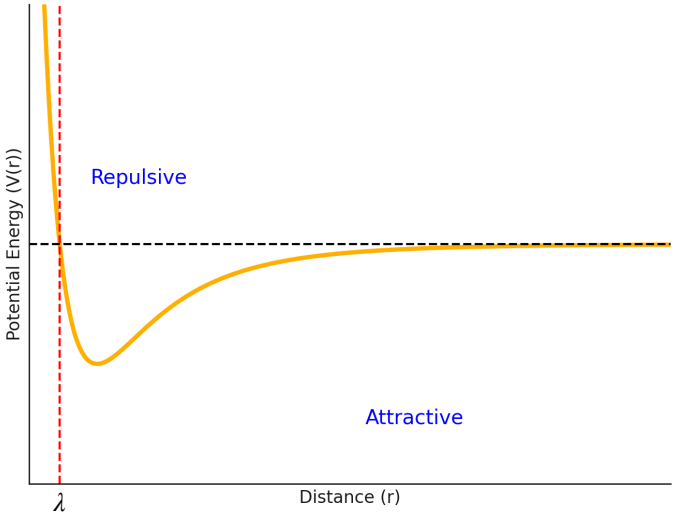






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Lennard-Jones potential

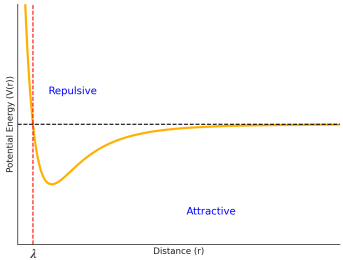


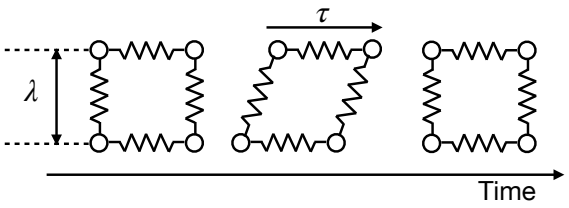
- The Lennard-Jones potential describes the interaction between a pair of neutral atoms or molecules.
- It has a repulsive component at short ranges and an attractive component at longer ranges:

$$V(r) = 4\epsilon \left[\left(\frac{\lambda}{r} \right)^{12} - \left(\frac{\lambda}{r} \right)^{6} \right]$$

where r is the distance between particles, ϵ is the depth of the potential well, and λ is the distance at which the potential is zero.

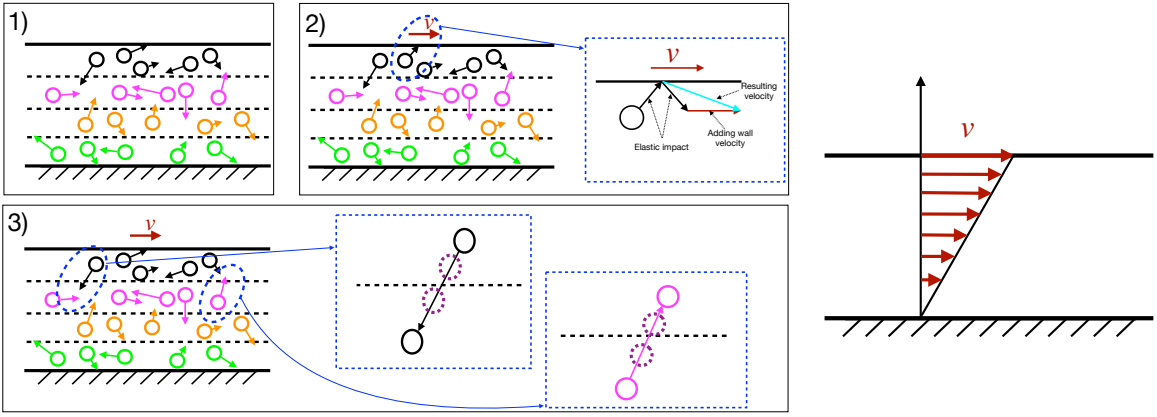
A simplified microscopic view of solids





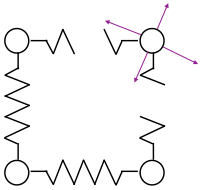
- Solids can be described as a collection of particles connected by strings
- If a force is applied, they are deformed and then go back to original shape

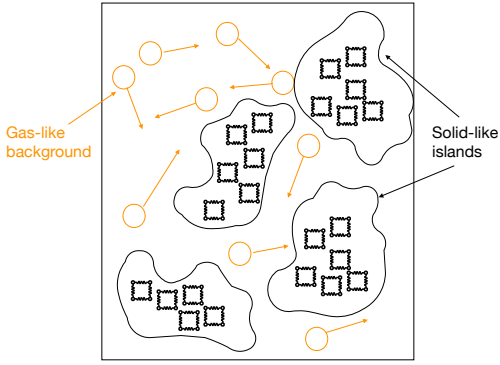
A simplified microscopic view of gases



- Consider a gas between two walls. The top wall moves with velocity v in the horizontal direction
- The velocity of particles in the top layer of the gas is affected by the moving wall when impact the wall during their chaotic motion
- Since particles move from one layer to the other, the information of the moving wall "propagates" down

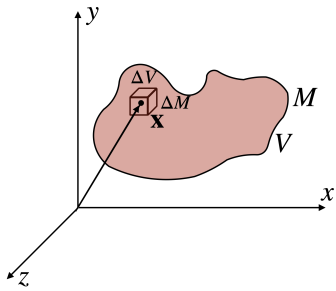
A (very) simplified microscopic view of liquids





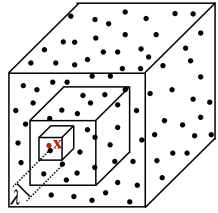
- Background of gas-like particles with islands of solid-like material
- Liquids exist only in a certain range of temperature and pressure
- Oscillations of molecules are strong enough to break bonds
- Or, in other words, bonds are not strong enough to resist to oscillations everywhere
- Solid-like islands are formed and destroyed continuously
- Thanks to the presence of the gas-like particles, the solid-like islands can slide on each other and the material behaves as a fluid

Continuum hypothesis



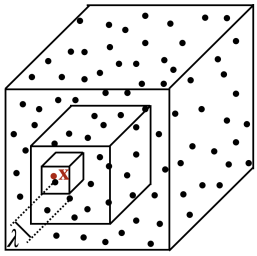
- (Average) density $\bar{\rho}$ of an object with volume V and mass M: $\bar{\rho} = M/V$
- How can we define the local density in the point $\mathbf{x}$? It is the limit:

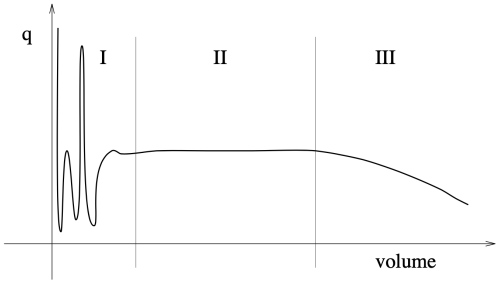
$$\lim_{\Delta V \to 0} \frac{\Delta M}{\Delta V}$$



- However, in the microscopic point of view we used before, there is an issue when $\Delta V \to 0$, because ΔV will not contain any particle when it is very small.
- Molecules are at a finite distance. This characteristic distance is usually defined by the mean free path.
- The mean free path is the average distance a particle travels between collisions with other particles in a given medium.
- For air at ambient conditions, $\lambda \approx 70\ nm$.

Continuum hypothesis and definition of quantities





- For intermediate ΔV, the ratio $\Delta M/\Delta V$ can be properly defined.
- If ΔV is too small, there are issues because there are not enough particles in the volume to properly define an average
- For very large ΔV, the definition might become inaccurate when there are spatial variations of the property we are defining.
- With the continuum hypothesis, we assume that all the lengthscales of our system are much larger then λ, so we can define the quantity everywhere.

Equivalence between the discrete and continuum systems

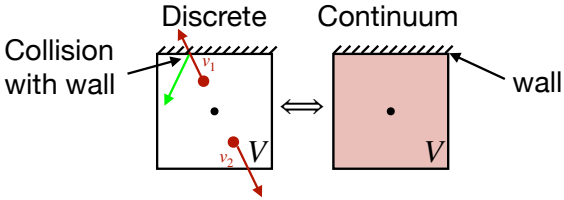


- We want the continuum representation to be equivalent to the discrete system.
  - The density in the continuum system is defined as $\rho = M/V$
  - For the velocity, we can use the average $\langle v \rangle = \sum_i v_i$
  - In the simplified example in the figure, we have only two particles with opposite velocities, so the velocity in the continuum system is zero.
  - However, we are missing something. The kinetic energy of the discrete system in the figure is

$$k = \frac{1}{2}\left(\frac{M}{2}v_1^2 + \frac{M}{2}v_2^2\right)$$

but in the continuum system we defined $v = 0$, so $k = 0$.
To preserve the equivalence of the two system, we need to introduce an additional quantity to represent the **kinetic energy of the particles**. We introduce the **temperature** $T = m\langle v^2 \rangle / (3k_B)$ (m is the mass of each particle and k_B the Boltzmann constant).

Equivalence between the discrete and continuum systems



- If we introduce a wall, the particles impact the wall and bounce back, applying a force on the wall.
- Nothing happens in the continuum system since the velocity is zero.
- We are again missing something in the continuum system.
- We need to introduce the pressure p.
- Most of the thermodynamics variables (entropy, surface tension, ...) need to be introduced to preserve the equivalence between the discrete and continuum systems.

When is the continuum hypothesis appropriate? The Knudsen Number

The **Knudsen number** (Kn) is a dimensionless number defined as the ratio of the mean free path (λ) to a characteristic length scale (L) of the system:

$$\mathrm{Kn} = \frac{\lambda}{L}$$

The Knudsen number indicates the relative importance of molecular motion compared to the system's size.

- Low Knudsen numbers ($\mathrm{Kn} \ll 1$) imply continuum behaviour so a continuum fluid dynamics model is appropriate.
- High Knudsen numbers ($\mathrm{Kn} \gg 1$) indicate free molecular flow, where particle collisions are less frequent, so a discrete model should be used.

Typical Situations for High Knudsen Numbers

- Rarefied gas dynamics (e.g., in high-altitude atmospheric conditions)
- Micro- and nano-scale fluid flows (e.g., in microfluidic devices)
- Vacuum systems (e.g., inside vacuum chambers or outer space)
- Gas flows in small pores or through membranes