Formula Sheet

Dynamics 4 (MECE10002)
December 2025 (Main)

Analytical dynamics: Discrete systems

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i$$

$$T = \frac{1}{2}m\dot{x}^2, \quad T = \frac{1}{2}I\dot{\varphi}^2$$

$$V_\mathrm{g} = mgh, \quad V_\mathrm{e} = \frac{1}{2}k\Delta x^2$$

$$V_\mathrm{e} = \frac{1}{2}k_\mathrm{t}\Delta\alpha^2, \quad k_\mathrm{t} = \frac{G\pi d^4}{32L}$$

$$\delta W_i = Q_i \delta q_i$$

Analytical dynamics: Continuous systems

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i$$

$$T = \frac{\rho A}{2}\int_0^L \left(\frac{\partial w}{\partial t}\right)^2 \mathrm{d}x$$

$$V_\mathrm{e} = \frac{EI}{2}\int_0^L \left(\frac{\partial^2 w}{\partial x^2}\right)^2 \mathrm{d}x, \; V_\mathrm{g} = \rho A g \int_0^L w(x,t)\mathrm{d}x$$

$$w(x,t) = \sum_{k=1}^{n} \varphi_k(x) q_k(t)$$

$$Q_i(t) = \varphi_i(b)F(t), \quad Q_i(t) = \int_0^L \varphi_i(x) f(x,t)\mathrm{d}x$$

Longitudinal waves

$$\frac{\partial^2 \Phi}{\partial t^2} = c^2\left(\frac{\partial^2 \Phi}{\partial x^2}\right), \quad \frac{\partial^2 u}{\partial t^2} = \frac{E}{\rho}\left(\frac{\partial^2 u}{\partial x^2}\right)$$

$$\frac{\partial^2 w}{\partial t^2} = \frac{T}{\mu}\left(\frac{\partial^2 w}{\partial x^2}\right), \quad \frac{\partial^2 \theta}{\partial t^2} = \frac{G}{\rho}\left(\frac{\partial^2 \theta}{\partial x^2}\right)$$

$$u(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - ct)\right]$$

$$u(x,t) = f(x)g(t)$$

$$f(x) = A\cos\left(\frac{2\pi}{\lambda}x\right) + B\sin\left(\frac{2\pi}{\lambda}x\right)$$

$$\frac{\partial^2 g}{\partial t^2} + \omega^2 g = 0, \quad \frac{\partial^2 f}{\partial x^2} + \frac{\omega^2}{c^2}f = 0$$

$$\omega = \frac{2\pi c}{\lambda}, \quad f = \frac{c}{\lambda}, \quad c = \sqrt{\frac{E}{\rho}}$$

Transverse waves

$$\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(\frac{EI}{\rho A}\frac{\partial^2 w}{\partial x^2}\right) = 0$$

$$\rho A\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} = 0$$

$$w(x,t) = B\sin\left[\frac{2\pi}{\lambda}(x - ct)\right]$$

$$w(x,t) = \phi(x)\cos(\omega t)$$

$$\phi(x) = C_1\cos(kx) + C_2\sin(kx) + C_3\cosh(kx) + C_4\sinh(kx)$$

$$\frac{\partial^4 \phi(x)}{\partial x^4} - k^4\phi(x) = 0$$

$$\omega = \frac{2\pi c}{\lambda}, \quad f = \frac{c}{\lambda}, \quad c = \left(\frac{2\pi}{\lambda}\right)\sqrt{\frac{EI}{\rho A}}$$

$$k^4 = \frac{\rho A\omega^2}{EI}, \quad k = \frac{\omega}{c}$$

Sound and pressure waves

$$\left\langle (p - p_0)^2 \right\rangle = \frac{1}{T}\int_t^{t+T} \left[p(t) - p_0\right]^2 \mathrm{d}t$$

$$L_p = 10\log_{10}\left[\frac{\left\langle (p - p_0)^2\right\rangle}{p_\mathrm{ref}^2}\right]$$

Impulsive dynamics

$$\frac{A}{I}\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(\frac{E}{\rho}\frac{\partial^2 w}{\partial x^2}\right) = 0$$

$$\rho A\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} = 0$$

$$w(x,t) = X(x)(A_1\cos\omega_0 t + A_2\sin\omega_0 t)$$

$$w(x,t) = \sum_{j=0}^{\mathrm{inf}} A_j X_j(x)\sin(\omega_j t - \phi_j)$$

$$X(x) = C_1(\cos kx + \cosh kx) + C_2(\cos kx - \cosh kx) + C_3(\sin kx + \sinh kx) + C_4(\sin kx - \sinh kx)$$

$$\omega_i^2 = \int_0^L EI\left(\frac{\mathrm{d}^2 X_i}{\mathrm{d}x^2}\right)^2 \mathrm{d}x \Big/ \int_0^L \rho A X_i^2 \mathrm{d}x$$

$$\bar{\omega}_i^2 = \omega_i^2 \Big/ \left(1 + \frac{m|X_i(x_0)|}{\rho A\int_0^L X_i^2 \mathrm{d}x}\right)$$

$$\bar{A}_i = \frac{mv_0|X_i(x_0)|}{\bar{\omega}_i \rho A\int_0^L X_i^2 \mathrm{d}x}$$

$$T = \frac{1}{2}\int_0^L \rho A\dot{w}_j^2(x,t)\mathrm{d}x, \; U = \frac{1}{2}\int_0^L EI\left[\frac{\partial^2 w_j(x,t)}{\partial x^2}\right]^2 \mathrm{d}x$$